NANOSYSTEMS: PHYSICS, CHEMISTRY, MATHEMATICS, 2013, 4 (3), P. 313–319

HEAT TRANSPORT IN MARANGONI LAYERS WITH NANOPARTICLES

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PACS 47.55.Nb, 47.55.Pf

This study investigates the influence of nanoparticle concentration on the Marangoni effect in the boundary layer near the free boundary of an incompressible fluid with small kinematic viscosity and thermal conductivity. The study was conducted on the basis of a single-phase model derived from the Navier-Stokes equations by replacing thermal parameters for their effective values. Two cases of stationary axisymmetric fluid flow are considered. In the first case, the fluid is cooled on the free surface near the symmetry axis, and in the second case, the fluid is heated. In the first case, a rotation of the fluid in a thin boundary layer appears near the free boundary, while there is no rotation outside the layer. In both cases, as the concentration of nanoparticles increases, the heat flux and the fluid velocity at the free boundary decrease.

Keywords: nanoparticles, a free boundary, the Marangoni effect, the boundary layer, thermal conductivity.

1. Introduction

The idea of the heat transport by fluid with micron-sized particles was proposed by Maxwell in the late nineteenth century [1]. However, at that time, the method failed to develop. The model of heat transport by a fluid containing metal nanoparticles was offered by Choi S. and Estman J. in 1995 [2]. It was thought that the thermal conductivities of some metals are hundreds of times greater than the thermal conductivities of some liquids. In an initial paper, several calculations were made for heat transport in the convective motion of a fluid with nanoparticles in areas with solid boundaries [2]. It was shown that the heat flux can be changed by tens of percent, depending on the concentration and type of nanoparticles. The calculations were based on single-phase and two-phase models. These studies indicated that the quantitative differences in these models were small, so, in this paper, a simple single-phase model was used. Recently, a number of reviews and articles on the convection of fluid with nanoparticles have been published [3–5]. The experimental results on the problem [5] confirmed the theoretical calculations. The heat transport in Marangoni layers in the plane case was previously studied [6]. In this article, the axisymmetric case was studied using constant thermal parameters. The flow of fluid at a given longitudinal temperature gradient on the free surface was calculated. Two cases were considered; when the temperature of the free boundary either increased or decreased with distance from the axis of symmetry. In the first case, a rotation of the fluid in the boundary layer appears near the free boundary with an absence of rotation outside this layer. In both cases, the heat flux on the free boundary decreased with increasing nanoparticle concentration and may be reduced by tens of percent, depending on the concentration of the nanoparticles and their composition.

2. The Equations of the Model

This study investigates the stationary axisymmetric flow of a viscous heat-conducting incompressible fluid in a layer of infinite thickness, limited to the top by the free boundary Γ. Along Γ, a temperature gradient is given which is positive in the first case and negative in the second. For small values of the kinematic viscosity and thermal diffusivity near the free boundary, there occurs a thin boundary Marangoni layer, outside of which, the fluid flow is slow and in the first approximation is described by the equations of an inviscid fluid. The study explores cases when metallic nanoparticles such as copper, silver, alumina and titanium oxide are placed in the base fluid, water. In these calculations, the single-phase model of nanofluids is used. It is assumed that the base fluid and the nanoparticles are in thermodynamic equilibrium, with no slide occurring between the fluid and the nanoparticles. The nanoparticles are spherical in shape and of uniform size. The equations of motion in the case of single-phase fluid issue from the Navier-Stokes equations by replacing the physical parameters on their effective values:

$$(\mathbf{v},\,\nabla)\,\mathbf{v} = -\rho_{nf}^{-1}\nabla p + \mu_{nf}\rho_{nf}^{-1}\nabla^2\mathbf{v},$$
$$(\mathbf{v},\,\nabla)\,T = \chi_{nf}\nabla^2 T, \quad \operatorname{div}\mathbf{v} = 0,$$

where $\mathbf{v} = (v_r,\, v_\theta,\, v_z)$ is the velocity vector, p is the pressure, T is the temperature. $(r,\,\theta,\,z)$ are cylindrical coordinates. Parameters $\rho_{nf},\ \mu_{nf},\ \chi_{nf}$ are the effective values of density, dynamic factor of viscosity, thermal diffusivity of fluid with nanoparticles. The fluid motion is axially symmetric, i.e. velocity vector, pressure and temperature are independent of the circumferential coordinate θ. It is assumed that the surface tension is linearly dependent on the temperature $\sigma = \sigma_0 - |\sigma_T|\,(T - T_*)$, where σ_0, $|\sigma_T|$, T_* are constant. Deformability of the free boundary is to be neglected. On the free surface Γ, there are satisfied dynamic conditions for shear stresses, the kinematic condition and the temperature is set [6]:

$$2\mu_{nf}\,(\Pi\mathbf{n} - (\mathbf{n}\Pi\mathbf{n})\mathbf{n}) = \nabla_\Gamma\sigma,$$
$$\mathbf{v}\mathbf{n} = 0, \quad T = T_\Gamma(r,\,z), \quad (r,\,z) \in \Gamma,$$

where Π is the strain rate tensor, $\mathbf{n}$ is the normal vector to the free boundary Γ. ∇_Γ is the gradient along Γ, T_Γ is the set temperature of the free boundary. Along Γ, a temperature gradient satisfies the condition $\nabla_\Gamma T_\Gamma \neq 0$ at $r \leq L$ and $T_\Gamma = const$ at $r > L$. We turn to the dimensionless variables in equations of motion and boundary conditions, having selected as a scale of length, velocity, pressure and temperature the following parameters $L,\ U,\ \sigma_0/L,$ $A_T L$. Where $U = \left(|\sigma_T|^2 A_T^2 L \rho_f^{-2}\nu_f^{-1}\right)^{1/3}$ and A_T are the scale of the temperature gradient along the surface Γ. Parameters $\rho_f,\ \nu_f$ indicate the density and viscosity kinematic factor of the base fluid. With large temperature gradients along the free surface and small diffusion coefficients, the boundary layer that arises near Γ is characterized by a large value for the velocity gradient across the boundary layer. We introduce a small parameter of the formula $\varepsilon = \left(\rho_f\nu_f^2\,|\sigma_T|^{-1} L^{-2} A_T^{-1}\right)^{1/3}$ and note that the order of the thickness of the boundary layer is εL.

The effective values of the physical parameters for a fluid with nanoparticles $\rho_{nf},\ \mu_{nf},$ $\chi_{nf},\ k_{nf}$ are expressed by the parameters of the base fluid $\rho_f,\ \mu_f,\ \chi_f,\ k_f$ and the parameters of metal particles – density ρ_S and coefficient of thermal conductivity k_S of the known formulas [7, 8]:

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_S, \quad \mu_{nf} = \mu_f(1-\varphi)^{-5/2},$$
$$\chi_{nf} = k_{nf}/(\rho c_p)_{nf}, \quad (\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \rho(\rho c_p)_s,$$

where φ is the volume concentration of nanoparticles in the mixture. The coefficient k_{nf} is determined by the formula [9]:

$$k_{nf} = k_f \frac{k_S + 2k_f - 2\varphi(k_f - k_S)}{k_S + 2k_f + \varphi(k_f - k_S)},$$

where k_{nf}, k_f denote the thermal conductivity coefficients of nanofluids and base fluid. The thermal parameters are considered constant. The dynamic viscosity of nanofluids is shown in [7], published by Brinkman H.C. in 1952.

3. Asymptotic Method

The solution to the problem is built though the boundary layer method. The top of the cylindrical coordinate system is put on a free surface whose equation is in the form $z = 0$. Furthermore, in the boundary layer D_Γ we introduce a stretching transformation, $z = \varepsilon s$. Note that $s \leq 0$ is in the field D_Γ. Asymptotic expansion of the velocity and temperature components are built in the form of series in powers of a small parameter ε with $\varepsilon \to 0$ [10]:

$$v_r = h_{ro} + \varepsilon(h_{r1} + v_{r1}) + \ldots, \quad v_z = \varepsilon(h_{z1} + v_{z1}) + \ldots.$$
$$v_\theta = h_{\theta 0} + \varepsilon(h_{\theta 1} + v_{\theta 1}) + \ldots, \quad T = \theta_0 + \varepsilon\theta_1 + \ldots.$$

Similar series are built for the pressure as well. Note that the functions h_{r0}, h_{z1}, $h_{\theta 0}$ are defined in the field of the boundary layer D_Γ; as they depend on the coordinates s, r and disappear outside of D_Γ. Functions v_{r1}, v_{z1} are defined outside the boundary layer; depending on the cylindrical coordinates z, r and satisfying the Euler equations that describe the first approximation for an ideal fluid outside D_Γ. Asymptotic expansions are substituted in the Navier-Stokes equations, heat-conductivity equation, the boundary conditions, here we pass to the variables s, r in D_Γ, and the sum of the coefficients of the same powers of the parameter ε equate to zero. As a result, the leading asymptotic term satisfies the equations:

$$\begin{gathered} h_{r0}\frac{\partial h_{r0}}{\partial r} + H_{z1}\frac{\partial h_{r0}}{\partial s} - \frac{h_{\theta 0}^2}{r} = A\frac{\partial^2 h_{r0}}{\partial s^2}, \\ \frac{\partial h_{r0}}{\partial r} + \frac{h_{r0}}{r} + \frac{\partial H_{z1}}{\partial s} = 0, \\ h_{r0}\frac{\partial h_{\theta 0}}{\partial r} + H_{z1}\frac{\partial h_{\theta 0}}{\partial s} + \frac{h_{r0}h_{\vartheta 0}}{r} = A\frac{\partial^2 h_{\theta 0}}{\partial s^2}, \\ h_{r0}\frac{\partial \theta_0}{\partial r} + H_{z1}\frac{\partial \theta_0}{\partial s} = \frac{B}{\mathrm{Pr}}\frac{\partial^2 \theta_0}{\partial s^2}. \end{gathered} \tag{1}$$

Here, we introduce the designation: $H_{z1} = h_{z1} + v_{z1}|_\Gamma$.

For this system of equations, we give the boundary conditions:

$$\begin{gathered} \frac{1}{(1-\varphi)^{5/2}}\frac{\partial h_{r0}}{\partial s} = -\frac{\partial T_\Gamma}{\partial r}, \quad H_{z1} = 0, \quad \frac{\partial h_{\theta 0}}{\partial s} = 0, \quad \theta_0 = T_\Gamma \, (s = 0), \\ h_{r0} \to 0, \quad h_{\theta 0} \to 0, \quad \theta_0 \to T_\infty \quad (s \to -\infty). \end{gathered} \tag{2}$$

It should be noted that $T_\infty = const$ is the constant temperature at infinity. $\mathrm{Pr} = \nu_f/\chi_f$ is the Prandtl number for water. The coefficients A and B in the equations of the boundary layer are expressed via the parameters of nanoparticles and the base fluid by the following formulas:

$$A = D\,(1-\varphi)^{-5/2}, \quad D = (1 - \varphi + \varphi\rho_f/\rho_S)^{-1},$$

$$B = D\frac{k_S/k_f + 2 - 2\varphi(1 - k_S/k_f)}{k_S/k_f + 2 + \varphi(1 - k_S/k_f)}.$$

We consider the case when the temperature at the free surface is defined by the power law $T_\Gamma = T_\infty + \tau r^{n+1}/(n+1)$, $(n \neq -1)$. Here, the parameter τ takes only two values $\tau = \pm 1$. In the first case, for $\tau = 1$, the temperature is the lowest value on the free surface of Γ at $r = 0$ on the symmetry axis and the fluid temperature increases with distance from the axis. In the second case, at $\tau = -1$, the temperature reaches its highest value on the axis of symmetry on Γ and the fluid cools with distance from the axis. Here the self-similar solution of the problem (1), (2) is determined. We introduce the stream function $\psi(s, r)$ by the formulas: $h_{r0} = \partial\psi/\partial s$, $H_{z1} = -r^{-1}\partial(r\psi)/\partial r$. The solution of the problem is performed as:

$$\psi = -r^{(n+2)/3} f(\xi), \quad \theta_0 = T_\infty + \tau r^{n+1}\theta_c(\xi)/(n+1), \quad h_{\theta 0} = r^{(2n+1)/3} G(\xi), \tag{3}$$

where $\xi = -sr^{(n-1)/3}$ (the minus sign is chosen for the realization of the inequality $\xi \geq 0$, as here $s \leq 0$). The functions $f(\xi)$, $\theta_c(\xi)$, $G(\xi)$ are determined by the boundary value problem:

$$\begin{aligned}
&3Af''' = (2n+1)f'^2 - (n+5)ff'' - 3G^2, \\
&3AG'' = (2n+4)Gf' - (n+5)fG', \\
&B\,\mathrm{Pr}^{-1}\,\theta_c'' = (n+1)\theta_c f' - (n+5)f\theta_c'/3, \\
&f(0) = 0, \quad f''(0) = \tau(1-\varphi)^{5/2}, \quad \theta_c(0) = 1, \quad G'(0) = 0, \\
&f'(\infty) = 0, \quad G(\infty) = 0, \quad \theta_c(\infty) = 0.
\end{aligned} \tag{4}$$

The heat flux on the free boundary is determined after solving the problem (4)

$$q_\Gamma = -k_{nf}\partial T/\partial z = \varepsilon^{-1} k_{nf} r^{(4n+2)/3}\tau\theta_c'(0)/(n+1), \quad (z = 0)$$

and local Nussels number $Nu = -\varepsilon^{-1}k_{nf}k_f^{-1}r^{(n+2)/3}\theta_c'(0)$.

We note the particular case when $n = 4$. The system (4) admits an exact solution at $\tau = -1$, depending on the variable by exponential law $f(\xi) = a\,(1 - \exp(-\gamma\xi))$, $G \equiv 0$. Here we have $\gamma = \sqrt[3]{3/A}(1-\varphi)^{5/6}$, $a = A\gamma/3$. The temperature distribution is calculated numerically.

4. The Results of the Calculations

The boundary value problem (4) was solved numerically by the shooting method. We note that the problem (4) was divided into two boundary value problems: at first we count the function $f(\xi)$, $G(\xi)$ and then we determine the function $\theta_c(\xi)$. When nanoparticles were absent in liquid for $\varphi = 0$, $A = B = 1$ and at $\tau = -1$, the solution of the problem without rotation of a thin layer was found in paper [10] for different values of the parameter n. In this paper, the solution was constructed when $\varphi \neq 0$ for the four types of nanoparticles – copper (Cu) , silver (Ag), titanium oxide (TiO_2) and alumina ($Al_2\ O_3$). Concentration of the nanoparticles corresponding to the parameter φ was varied from zero to 0.2. The numerical values of thermodynamic parameters k, μ, ρ, c_p were given in paper [6]. The calculations were made at $n = 0$. It was shown that cooling at the point of $r = 0$ on the free boundary for $\tau = 1$, the rotation of the liquid appears in a thin boundary layer near the free surface. Additionally, there is no rotation outside this layer. The presence of nanoparticles in the liquid slows down the rotation of the layer, correspondingly, the higher the nanoparticle concentration, the larger the degree of inhibition. Liquid can rotate both clockwise and counterclockwise. The boundary value problem (4) for each fixed set of initial parameters admits two symmetric solutions: f,

$\pm G$, θ_c. At $n = 0$, $\tau = 1$, $\varphi = 0$ for system (4), we give numerical values $f'(0) = -0.5793$, $G(0) = \pm 0.9934$, $\theta_c'(0) = 1.1039$. In the case when $\tau = -1$ the rotational effect is absent. Here, $f'(0) = 1.0563$, $\theta_c'(0) = -3.3629$, $G = 0$ at $\varphi = 0$.

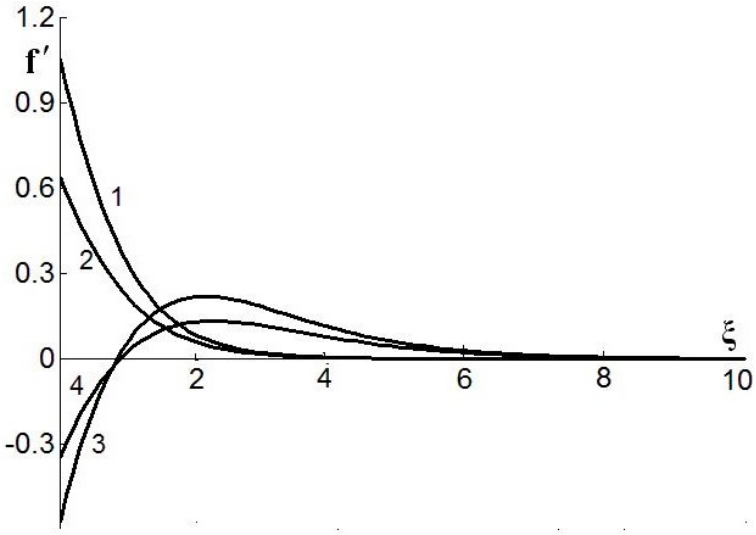

FIG. 1. The dependence of the amplitude of the radial velocity component on the transverse coordinate. Curves 1 and 2 correspond to the fluid flow without rotation, curves 3 and 4 – to the flow with rotation.

Figure 1 shows graphs of the radial velocity components' amplitude $f'(\xi)$ depending on a function of the transverse coordinate ξ in the boundary layer at various concentrations φ of copper nanoparticles. The flow without rotation corresponds to curves 1 and 2. The absence of nanoparticles in the layer ($\varphi = 0$) corresponds to curve 1, and value $\varphi = 0.2$ corresponds to curve 2. The radial component of the velocity decreased monotonically with distance from the free boundary. The presence of nanoparticles slowed down the flow of a liquid, with the braking effect appearing most prominently at the free surface. Curves 3 and 4 correspond to the values $\varphi = 0$ and $\varphi = 0.2$ for the flow of liquid with rotation. Without rotation, the velocity of the liquid decreased monotonically with the increased ξ and was positive, in this case, the rotational speed is not monotonic and a zone of countercurrent appears by the free surface (where $v_r < 0$).

Calculations showed that the circumferential velocity component $h_{\theta 0}$ decreased monotonically with distance from the free surface. When the concentration of nanoparticles increased, fluid flow inhibition occurred and the rotational effect weakened.

The graph of the function $\theta_c(\xi)$, which influences the temperature distribution in the boundary layer, in accordance with (3), is shown on Fig. 2. Curves 1 and 2 correspond to the flow of liquid with rotation, and curves 3 and 4 correspond to the flow of liquid without rotation. Curves 1 and 4 represent a nanoparticle concentration of 0 ($\varphi = 0$), while curves 2 and 3 correspond to a concentration of $\varphi = 0.2$. The temperature of the liquid with rotationless flow was defined by the function $\theta_0(r, \xi)$ in (3) and increased monotonically in the direction of the free surface inside the boundary layer with $r > 0$ and remained constant equal to T_∞ on the rotation axis $r = 0$. With higher nanoparticle concentrations, the temperature decreased monotonically in fixed section within the layer $\xi = const$. With a rotating boundary layer ($\tau = 1$), the temperature distribution inside the layer was not monotonous and differed from the one without rotation. A thin sublayer appeared inside the boundary layer D_Γ near the free boundary Γ, where the temperature increased with distance from Γ, and then fell outside this sublayer to a value of T_∞ on leaving D_Γ. Depending on the parameter φ, the temperature

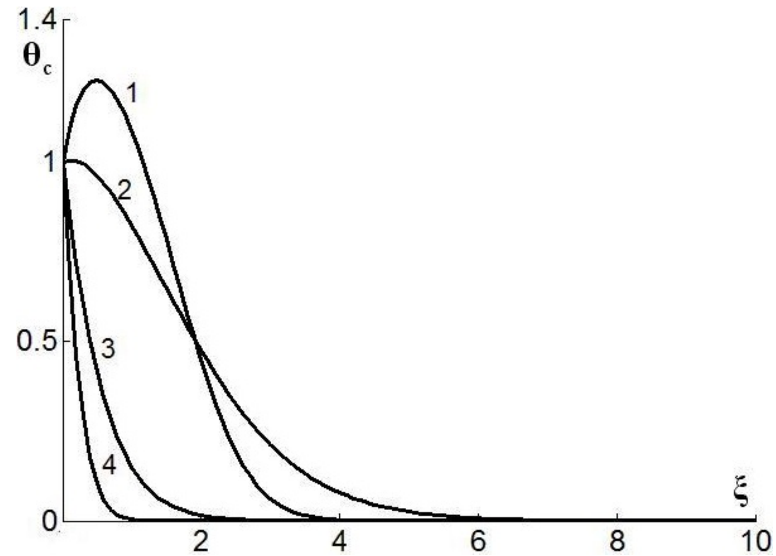

FIG. 2. The dependence of the function θ_c on the transverse coordinate in the boundary layer. Curves 1 and 2 correspond to the fluid flow with rotation, curves 3 and 4 – to the flow without rotation.

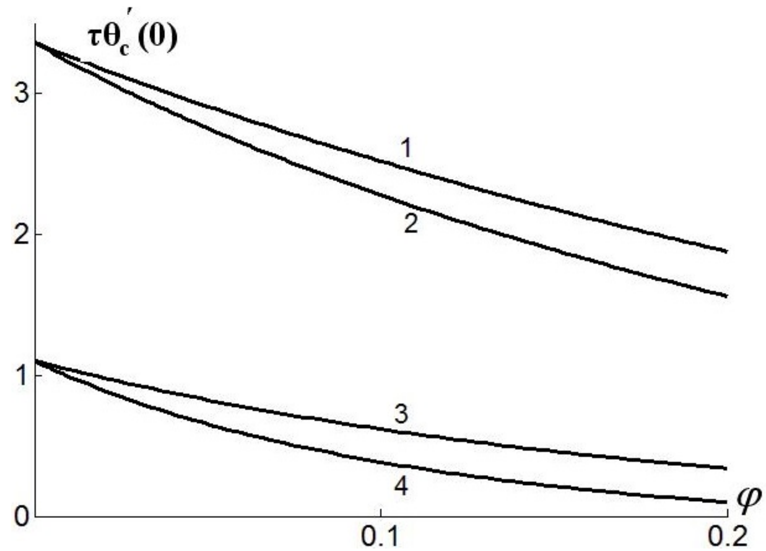

FIG. 3. The dependence of the amplitude of the heat flow on the volume concentration of nanoparticles on the free boundary. Curves 1 and 2 correspond to the fluid flow without rotation, curves 3 and 4 – to the flow with rotation.

distribution inside the layer was not monotonic at fixed transverse coordinate ξ. The value ξ_* exists for each value of the parameter φ. The temperature of the liquid decreased at $0 < \xi < \xi_*$ with an increase of the concentration parameter, φ. The temperature of the liquid increased at $\xi > \xi_*$.

Figure 3 shows graphs of the amplitude of the heat flux $\tau\theta_c'(0)$ on the free surface depending on the nanoparticle concentration φ of titanium oxide (curves 1 and 3) and copper (curves 2 and 4). The flow of liquid without rotation corresponds to curves 1 and 2, while curves 3 and 4 were calculated for a rotating layer. In all cases, increased nanoparticle concentrations reduced the amount of heat flux monotonically. Moreover, the heat flux reduction was dependent upon the nanoparticle composition.

5. Conclusion

The influence of nanoparticle concentration on heat transfer in a thin Marangoni boundary layer near a free, nondeformable boundary with given uneven temperature distribution along this boundary was researched in this paper. It was shown that depending on the temperature gradient's direction along the boundary, there might be a flow either with or without rotation. During the rotation of the liquid, a countercurrent zone appears near the free surface. It was shown that in both cases, with higher nanoparticle concentrations, the heat flux on the free surface decreased monotonically on the order of several tens of percent, depending on the concentration and composition of the nanoparticles. The liquid velocity inside the Marangoni layer decreased with higher nanoparticle concentrations. The rotational effect was weakened with the presence of nanoparticles in the liquid.

Acknowledgment

This work was supported by the RFBR grant No. 12-01-00582-a.

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