# ABSORPTION OF ELECTROMAGNETIC RADIATION IN THE QUANTUM WELL PLACED IN A MAGNETIC FIELD

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The absorption of electromagnetic radiation by electrons of the quantum well is investigated. These calculations take into account the spin-orbit interaction (SOI) in the Rashba model. Analytical expression of the absorption coefficient is obtained. Resonant frequencies and positions of the resonance peaks were found.

Keywords: quantum well, spin-orbit interaction, the absorption coefficient.

### 1. Introduction

Optical properties of low-dimensional systems is a major branch of nanostructurebased physics. The interest in the study of the absorption coefficient of electromagnetic radiation in nanostructures has increased greatly in recent years. The emerging field of spintronics has generated interest in the effects of the spin-orbit interaction (SOI) in lowdimensional semiconductor heterostructures.

The absorption of electromagnetic radiation in semiconductors while taking into account the SOI have been investigated previously [1]. The spin-orbit interaction of 2D electrons were investigated previously [2,3], where the value of the coupling constant  $\alpha$ , and the expression for the magnetic susceptibility were found.

Splitting of the cyclotron resonance for two-dimensional electrons in InAs quantum wells was observed [4]. This effect can be explained by the use of the SOI in the Rashba model.

The electron transport and the cyclotron resonance in a one-sided, selectively doped HgTe/CdHgTe (013) heterostructure with a 15-nm quantum well with an inverted band structure have been investigated [5]. Modulation of the Shubnikov-de Haas oscillations was observed, and the spin splitting in a zero magnetic field was found to be about 30 meV. A large  $\Delta m_c/m_c \simeq 0.12$  splitting of the cyclotron resonance line was discovered and shown to be due to both the spin splitting and the strong nonparabolicity of the dispersion relation in the conduction band.

The Rashba effect in the ring was previously considered [6]. The behavior of the energy spectrum was analyzed in detail. The energy spectrum depended on the width of the ring and the Rashba parameter of the spin-orbit interaction.

Inelastic light scattering by a two-dimensional system of electrons in a conduction band with Rashba spin-orbit coupling was studied theoretically for the resonance case where the frequencies of the incident and scattered light were close to the effective distance between the conduction band and spin-split band in a III-V semiconductor [7].

### 2. Absorption coefficient

In this paper we studied the absorption coefficient of electromagnetic radiation in the quantum well with (SOI). The Hamiltonian has the form [3]

$$H = \frac{1}{2m^*} (p_x + \frac{e}{c} By)^2 + \frac{p_y^2}{2m^*} + \frac{p_z^2}{2m^*} + \alpha \mathbf{v} \left[ \boldsymbol{\sigma} \times \frac{\mathbf{p} - \frac{e}{c} \mathbf{A}}{\hbar} \right],$$

the vector potential of magnetic field  $\mathbf{A}$  is  $\mathbf{A} = (-By, 0, 0)$ .

The electron energy spectrum of the problem has the form [3]

$$E_s^{\pm} = \hbar\omega_c [s \pm \sqrt{(1/2 - g\beta/2)^2 + \gamma^2 s}] + \varepsilon_0, s \ge 1$$
$$E_0 = \hbar\omega_c (1/2 - g\beta/2) + \varepsilon_0, s = 0,$$

where  $\beta = m^*/2m_0, \gamma^2 = \alpha^2/A\hbar\omega_c, A = \hbar^2/2m^*, \varepsilon_0$  — is the lower level of the electron energy in the well,  $\alpha$  — coupling constant,  $m^*$  — effective electron mass,  $m_0$  — free electron mass,  $\omega_c = eB/m^*c$  — cyclotron frequency, g — g factor of electrons.

The electronic transitions with the change s, but without changing the sign (plusplus, or minus-minus transitions) correspond to the cyclotron resonance [3]. Also introduced was the concept of the combined resonance responsible for transitions with a change of sign and the change s [3].

The wave functions has the form: [1]

 $\times$ 

$$\left(\begin{array}{c}f_1\\f_2\end{array}\right) = \left(\begin{array}{c}\sum\limits_{s=0}^{\infty}a_s\psi_s\\\sum\limits_{s=0}^{\infty}b_s\psi_s\end{array}\right),$$

where  $\psi_s$  — oscillator functions,  $a_s$ ,  $b_s$  — coefficient of expansion  $f_1$  and  $f_2$  respectively.

$$a_{s-1}^{(s)} = \pm i \sqrt{\frac{\sqrt{(\frac{1}{2} - \beta)^2 + \gamma^2 s} \mp (\frac{1}{2} - \beta)}{2\sqrt{(\frac{1}{2} - \beta)^2 + \gamma^2 s}}},$$
$$b_s^{(s)} = \sqrt{\frac{\sqrt{(\frac{1}{2} - \beta)^2 + \gamma^2 s} \pm (\frac{1}{2} - \beta)}{2\sqrt{(\frac{1}{2} - \beta)^2 + \gamma^2 s}}}.$$

The remaining coefficients  $a_t^{(s)}$  and  $b_t^{(s)}$  equal to zero [1]. Then the transition matrix elements due to electromagnetic radiation has the form

$$\langle (f_1 f_2) | H_R | \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \rangle = \langle (f_1 f_2) \begin{pmatrix} \sum_{s'=1}^{\infty} a_{s'} H_R \psi_{s'} \\ \sum_{s'=1}^{\infty} b_{s'} H_R \psi_{s'} \end{pmatrix} \rangle = \frac{ie\hbar}{m^* l_0} \sqrt{\frac{2\pi\hbar N_f}{\epsilon\omega}}$$
$$\left( \sum_{s=0}^{\infty} \sum_{s'=1}^{\infty} a_s^* a_{s'} \left( \sqrt{\frac{s+1}{2}} \delta_{s,s'-1} - \sqrt{\frac{s}{2}} \delta_{s,s'+1} \right) + b_s^* b_{s'} \left( \sqrt{\frac{s+1}{2}} \delta_{s,s'-1} - \sqrt{\frac{s}{2}} \delta_{s,s'+1} \right) \right),$$
(1)

 $l_0$  — is a magnetic length,  $\delta_{s,s'}$  — Kronecker delta symbol,  $N_f$  — number of photons,  $H_R$  — operator of the electron-photon interaction.

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We consider the case of a non-degenerate electron gas. Expression for the absorption due to these transitions are given by

$$\Gamma(\omega) = \frac{2\pi\sqrt{\epsilon(\omega)}n}{c\hbar N_{\mathbf{f}}} \left[ 1 - \exp\left(-\frac{\hbar\omega}{T}\right) \right]$$
$$\times \sum_{\alpha} \sum_{\beta} f_0(E_{\alpha}) |\langle (f_1 f_2) | H_R | \begin{pmatrix} f_1 \\ f_2 \end{pmatrix} \rangle |^2 \delta\left(E_{\alpha} - E_{\beta} + \hbar\omega\right), \tag{2}$$

where  $\alpha, \beta$  — quantum numbers of the electron to initial and final states,  $f_0$  — is the electronic-Boltzmann distribution function,  $\epsilon(\omega)$  — real part of the dielectric constant,  $\omega$  — frequency of the photon,  $(1 - \exp(-\hbar\omega/T))$  — takes into account the stimulated emission of photons,  $\delta(x)$  — Dirac delta function, n — the concentration of electrons. Under the condition  $\hbar\omega \gg T$  this term will be neglected. This expression (2) is similar to that used in [8].

We obtain the following expression for the absorption coefficient using (1) and (2):

$$\Gamma(\omega) = \frac{2\pi\sqrt{\epsilon}n}{c\hbar N_f} \frac{e^2\hbar^2}{m^{*2}l_0^2} \frac{2\pi\hbar N_f}{\epsilon\omega} \sum_{s=0}^{\infty} \sum_{s'=1}^{\infty} \left(\frac{s+1}{2}\delta_{s,s'-1} - \frac{s}{2}\delta_{s,s'+1}\right) \times (a_s a_{s'} a_s^* a_{s'}^* + b_s a_{s'} a_s^* b_{s'}^* + a_s b_s^* a_{s'}^* b_{s'} + b_s b_{s'} b_s^* b_{s'}^*) f_0(E_s^{\pm}) \delta(E_s^{\pm} - E_{s'}^{\pm} + \hbar\omega).$$
(3)

It is clear that the least s' is equal to one. It is the lowest level on which an electron can be found after the absorption of a photon.

After calculating the sum from s we obtain:

$$\Gamma(\omega) = \frac{2\pi\sqrt{\epsilon}n}{c\hbar N_f} \frac{e^2\hbar^2}{m^{*2}l_0^2} \frac{2\pi\hbar N_f}{\epsilon\omega} \sum_{s'=1}^{\infty} \frac{s'}{2}$$

$$(a_{s'-1}a_{s'}a_{s'-1}^*a_{s'}+b_{s'-1}a_{s'}a_{s'-1}^*b_{s'}+a_{s'-1}b_{s'-1}^*a_{s'}b_{s'}+b_{s'-1}b_{s'}b_{s'-1}b_{s'})f_0(E_{s'-1}^{\pm})\delta(E_{s'-1}^{\pm}-E_{s'}^{\pm}+\hbar\omega)$$

$$-\frac{2\pi\sqrt{\epsilon}n}{c\hbar N_f} \frac{e^2\hbar^2}{m^{*2}l_0^2} \frac{2\pi\hbar N_f}{\epsilon\omega} \sum_{s'=1}^{\infty} \frac{s'+1}{2}$$

$$(a_{s'+1}a_{s'}a_{s'+1}^*a_{s'}+b_{s'+1}a_{s'}a_{s'+1}^*b_{s'}+a_{s'+1}b_{s'+1}^*a_{s'}^*b_{s'}+b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'+1}b_{s'}b_{s'}b_{s'+1}b_{s'}b_{s'}b_{s'+1}b_{s'}b_{s'}b_{s'}b_{s'}b_{s'+1}b_{s'}b_{s'}b_{s'+1}b_{s'}$$

We take into account the broadening of delta peaks using the Lorentz formula [8]. The following expression of the absorption coefficient is given by:

$$\frac{\Gamma(\omega)}{\Gamma_{0}} = \sum_{s'=1}^{\infty} \frac{\omega_{c}}{\omega} \frac{s'}{2} \left( a_{s'-1} a_{s'} a_{s'-1}^{*} a_{s'}^{*} + b_{s'-1} a_{s'} a_{s'-1}^{*} b_{s'}^{*} + a_{s'-1} b_{s'-1}^{*} a_{s'}^{*} b_{s'} + b_{s'-1} b_{s'} b_{s'-1}^{*} b_{s'}^{*} \right) \\
\times f_{0}(E_{s'-1}^{\pm}) \frac{1}{1 + \tau^{2} ((E_{s'-1}^{\pm} - E_{s'}^{\pm})\hbar^{-1} + \omega)^{2}} - \\
- \sum_{s'=1}^{\infty} \frac{\omega_{c}}{\omega} \frac{s'+1}{2} \left( a_{s'+1} a_{s'} a_{s'+1}^{*} a_{s'}^{*} + b_{s'+1} a_{s'} a_{s'+1}^{*} b_{s'}^{*} + a_{s'+1} b_{s'+1}^{*} a_{s'}^{*} b_{s'} + b_{s'+1} b_{s'} b_{s'+1}^{*} b_{s'}^{*} \right) \\
\times f_{0}(E_{s'+1}^{\pm}) \frac{1}{1 + \tau^{2} ((E_{s'+1}^{\pm} - E_{s'}^{\pm})\hbar^{-1} + \omega)^{2}},$$
(5)

where we introduced

$$\Gamma_0 = \frac{4\pi}{c\sqrt{\epsilon}} \frac{e^2\tau}{m^*} n,$$

 $\tau$  — phenomenological relaxation time.

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FIG. 1. The dependence of the absorption of electromagnetic radiation on the radiation frequency.  $\gamma^2 = 11, \ \beta = 0.25, \ T = 100 \mathrm{K}$ 



FIG. 2. The dependence of the absorption of electromagnetic radiation on the radiation frequency.  $\gamma^2 = 7$ ,  $\beta = 0.25$ , T = 100K

The graph of the absorption coefficient as a function of electromagnetic radiation frequency is presented in fig. 1,2. The right wing of graph is almost smooth, however, the left wing contains a series of peaks due to the electronic transitions between discrete levels. The smooth form of the right wing at high-frequencies is stipulated by the distribution function  $f_0(E_{s'+1}^{\pm})$ .

The resonance curve in the absence of the SOI is shown in fig. 3.



FIG. 3. The dependence of the absorption of electromagnetic radiation on the radiation frequency.  $\gamma^2 = 0, \beta = 0.25, T = 100$ K

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