NANOSYSTEMS: PHYSICS, CHEMISTRY, MATHEMATICS, 2013, 4 (4), P. 570–576

MODEL OF THE INTERACTION OF POINT SOURCE ELECTROMAGNETIC FIELDS WITH METAMATERIALS

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PACS 81.05.Xj, 78.67.Pt

We consider Green's function for layered system. We express it in terms of the well-known scalar s and p ones. For a single NIM layer in vacuum and with a single dispersive Lorentz form for equal electric and magnetic permeabilities $\varepsilon(\omega)$ and $\mu(\omega)$, we obtain an explicit form for Green's function. Also we find Green's function for multilayered system and obtain recurrence relations for its coefficients.

Keywords: metamaterial, point perturbation, refraction, NIM, Maxwell's equations.

1. Introduction

Metamaterials are artificial materials engineered to have properties that may not be found in nature. In particular, they may have negative refractive index. Such materials are called negative index materials (NIMs). In general, a NIM system is defined by the property that for certain frequencies ω the electric permeability $\varepsilon(\omega)$ or the magnetic permeability $\mu(\omega)$ becomes negative. The NIM situation is the case where both at the same frequency $\hat{\omega}$ become negative and are equal –1. Recently, NIMs have come under increased scrutiny (see [1], [2]).

The existence of NIMs has been debated in previous theoretical literature (see [3–8]). In particular, the sign of the index of refraction, which involves taking a square root $n = \pm\sqrt{\varepsilon\mu}$, has been the subject of discussion. Naively it equals +1, in both vacuum and a NIM system but this result is challenged for the NIM situation. The use of the phenomenological Maxwell's equations should solve possible ambiguities.

2. Model

As in [9], where the following model is fully described, the starting point is the set of phenomenological Maxwell's equations for the case where the permanent polarization and magnetization are absent ($\varepsilon_0 = \mu_0 = 1$),

$$\begin{gathered} \partial_t D(x,t) = \partial_x \times H(x,t), \quad \partial_t B(x,t) = -\partial_x \times E(x,t), \\ \partial_x \cdot D(x,t) = 0, \quad \partial_x \cdot B(x,t) = 0, \end{gathered} \tag{1}$$

with the constitutive equations

$$D(x,t) = E(x,t) + P(x,t), P(x,t) = \int_{t_0}^{t} ds\chi_e(x,t-s)\cdot E(x,t),$$

$$H(x,t) = B(x,t) - M(x,t), \quad M(x,t) = \int_{t_0}^{t} ds\chi_m(x,t-s)\cdot H(x,t),$$

where $\chi_e(x,t)$ and $\chi_m(x,t)$ are the electric and magnetic susceptibility tensors. We assume that the system is dispersive, nonabsorptive and use causality and passivity conditions. Causality requires that the susceptibilities $\chi_e(x,t) = \chi_m(x,t) = 0$ for $t \leq t_0$ ($t_0 = 0$). Passivity means that the electromagnetic energy $\varepsilon_{em}(t) = \dfrac{1}{2}\displaystyle\int dx \left\{E(x,t)^2 + H(x,t)^2\right\}$ cannot increase as a function of time. We use the Fourier transform,

$$\hat{f}(z) = \int_0^\infty dt \exp[izt]\, f(t), \quad f(t) = \frac{1}{2\pi}\int_\Gamma dz \exp[-izt]\, \hat{f}(t),$$

where Γ is a path running from $-\infty$ to $+\infty$ at some distance $\delta > 0$ parallel to the real axis, $z = \omega + i\delta$, $\delta \to 0$ ($\Im z > 0$). We consider the isotropic system, $\hat{\chi}(x,z) = \hat{\chi}(x,z)U$ where U is the unit matrix 3×3, and we are dealing with a single dispersive Lorentz contribution $\varepsilon(\omega) = \mu(\omega) = 1 - \dfrac{\Omega^2}{\omega^2 - \omega_0^2}$. Then $\hat{\omega} = \sqrt{\omega_0^2 + \dfrac{\Omega^2}{2}}$ is the NIM frequency as $\varepsilon(\pm\hat{\omega}) = \mu(\pm\hat{\omega}) = -1$.

Maxwell's equations (1) can be expressed in terms of Fourier transforms,

$$L^e(z) \cdot \hat{E}(x,z) = g^e(x,z), \quad L^m(z) \cdot \hat{H}(x,z) = g^m(x,z),$$

where

$$L^e(z) = z^2\varepsilon(x,z) + (\in \cdot p) \cdot \mu(x,z)^{-1} \cdot (\in \cdot p),$$
$$L^m(z) = z^2\mu(x,z) + (\in \cdot p) \cdot \varepsilon(x,z)^{-1} \cdot (\in \cdot p),$$
$$g^e(x,z) = izE(x,0) + i(\in \cdot p) \cdot \left\{\mu(x,z)^{-1} \cdot H(x,0)\right\},$$
$$g^m(x,z) = izH(x,0) - i(\in \cdot p) \cdot \left\{\varepsilon(x,z)^{-1} \cdot E(x,0)\right\}.$$

Here $L^e(z)$ and $L^m(z)$ are the electric and magnetic Helmholtz operators, $\in$ is the Levi-Civita symbol, and $p = -i\partial_x$ so $(\in \cdot p) \cdot f = i\partial_x \times f$. Let now

$$R^e(z) = L^e(z)^{-1}, \quad R^m(z) = L^m(z)^{-1}.$$

Then

$$\hat{E}(x,z) = R^e(z) \cdot g^e(x,z), \quad \hat{H}(x,z) = R^m(z) \cdot g^m(x,z).$$

Next we introduce Green's functions

$$G^{e,m}(x,y,z) = \langle x|R^{e,m}|y\rangle,$$
$$L^{e,m}(z) \cdot G^{e,m}(x,y,z) = \delta(x-y)U.$$

Then $E(x,t)$ is given by the inverse Fourier transform of

$$\hat{E}(x,z) = \int dy G(x,y,z) \cdot g(y,z),$$

where $g(y,z)$ is some integrable initial field configuration or an external current density.

We only consider the electric Green's function and drop the superscript e. We also assume that the system is layered, and layers are parallel to the X_1X_2-plane and there is the translation invariance in the X_1 and X_2 directions (the three Cartesian axes are denoted by X_1, X_2 and X_3 with corresponding unit vectors e_1, e_2 and e_3). Then the permeabilities only depend on x_3,

$$\varepsilon(x,z) = \varepsilon(x_3,z) = \varepsilon_j(z), \quad \mu(x,z) = \mu(x_3,z) = \mu_j(z).$$

We denote $x = x_3$, $y = y_3$ and let $k = (k_1,k_2,k_3)$, $\kappa = (k_1,k_2,0) = \kappa e_\kappa = k^\perp \perp e_3$,

$$\zeta^2(x,\kappa,z) = z^2\varepsilon(x,z)\mu(x,z) - \kappa^2.$$

We obtain

$$G(x,y,z) = \frac{1}{2\pi}\int d\kappa \exp[-i\kappa \cdot (x^\perp - y^\perp)]\, G_\kappa(x,y,z),$$
$$G_\kappa(x,y,z) = G_s(x,y,z,\kappa) + G_p(x,y,z,\kappa),$$

where

$$G_s(x,y,z,\kappa) = G_s(x,y,z,\kappa)\, e_3 \times e_\kappa e_3 \times e_\kappa,$$
$$G_p(x,y,z,\kappa) = \left(e_\kappa + \frac{i\kappa}{\zeta(x)^2}\partial_x e_3\right)\left(e_\kappa - \frac{i\kappa}{\zeta(y)^2}\partial_x e_3\right) G_p(x,y,z,\kappa),$$

s-polarization part G_s and p-polarization part G_p of Green's function are scalar and satisfy

$$\left\{z^2\varepsilon(x,z) - p\frac{z^2\varepsilon(x,z)}{\zeta(x,\kappa,z)^2}p\right\} G_p(x,y,z,\kappa) = \delta(x-y),$$
$$\left\{\frac{\zeta(x,\kappa,z)^2}{\mu(x,z)} - p\frac{1}{\mu(x,z)}p\right\} G_s(x,y,z,\kappa) = \delta(x-y).$$

3. Results

3.1. Single NIM layer

In [9] the simplest layered system, i.e., two half spaces filled with NIM and vacuum was considered and the expressions for Green's function were found. In our investigation we considered the single NIM layer in a vacuum,

$$\varepsilon(x,z) = \begin{cases} \varepsilon(z), & x \in (a,b) \\ 1, & x \notin (a,b) \end{cases}, \quad \mu(x,z) = \begin{cases} \mu(z), & x \in (a,b) \\ 1, & x \notin (a,b) \end{cases}$$

with point perturbation located in a vacuum ($y < a$). For G_s, G_p with frequencies $z = \pm\hat{\omega}$ we find explicit expressions for case $\hat{\omega} > \kappa$ (radiative regime) and expressions in asymptotic form for case $\hat{\omega} < \kappa$ (evanescent regime) the same way [9]. We denote

$$\rho(\hat{\omega}) = \sqrt{\left|\omega^2\left(1 - \Omega^2/\left(\omega^2 - \omega_0^2\right)\right)^2 - \kappa^2\right|}.$$

In the reflection case ($x, y < a < b$), the receiver located in x and the point perturbation located in y are on the one side of the layer (see Fig. 1), and for $\hat{\omega} > \kappa$,

$$G_p(x,y,\pm\hat{\omega}) = \pm\frac{\rho(\hat{\omega})}{2i\hat{\omega}^2}\exp[\pm i\rho(\hat{\omega})|x-y|],$$
$$G_s(x,y,\pm\hat{\omega}) = \pm\frac{1}{2i\rho(\hat{\omega})}\exp[\pm i\rho(\hat{\omega})|x-y|],$$

where the term responsible for reflection is absent, i.e., there is no reflection at the frequencies $\pm\hat{\omega}$ for which $\varepsilon(\pm\hat{\omega}) = \mu(\pm\hat{\omega}) = -1$.

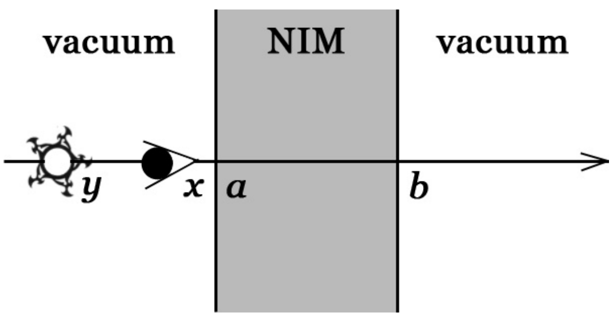

FIG. 1. Reflection case ($x > y$)

For $\hat{\omega} < \kappa$,

$$G_p(x, y, z) \overset{z \to \pm\hat{\omega}}{\sim} \frac{\rho(\hat{\omega})}{2\hat{\omega}^2} \exp\left[-\rho(\hat{\omega})|x-y|\right] + \frac{\Omega^2}{4\hat{\omega}^2\kappa^2} \frac{\rho(\hat{\omega})^3 \left(1 - 4\rho(\hat{\omega})^2\right)}{(z-\hat{\omega})(z+\hat{\omega})} \exp\left[-\rho(\hat{\omega})(a-x+a-y)\right],$$

$$G_s(x, y, z) \overset{z \to \pm\hat{\omega}}{\sim} -\frac{1}{2\rho(\hat{\omega})} \exp\left[-\rho(\hat{\omega})|x-y|\right] + \frac{\Omega^2}{4\kappa^2} \frac{\rho(\hat{\omega}) \left(1 - 4\rho(\hat{\omega})^2\right)}{(z-\hat{\omega})(z+\hat{\omega})} \exp\left[-\rho(\hat{\omega})(a-x+a-y)\right],$$

where the reflection term is still present, but we encounter dampening behavior, typical for the evanescent situation.

In the refraction case ($y < a < x < b$), the receiver is in the NIM layer (see Fig. 2), and for $\hat{\omega} > \kappa$,

$$G_p(x, y, \pm\hat{\omega}) = \pm\frac{\rho(\hat{\omega})}{2i\hat{\omega}^2} \exp\left[\pm i\rho(\hat{\omega})(a-x+a-y)\right],$$
$$G_s(x, y, \pm\hat{\omega}) = \pm\frac{1}{2i\rho(\hat{\omega})} \exp\left[\pm i\rho(\hat{\omega})(a-x+a-y)\right],$$

for $\hat{\omega} < \kappa$,

$$G_p(x, y, z) \overset{z \to \pm\hat{\omega}}{\sim} \frac{\Omega^2}{4\hat{\omega}^2\kappa^2} \frac{\rho(\hat{\omega})^3 \left(1 + 4\rho(\hat{\omega})^2\right)}{(z-\hat{\omega})(z+\hat{\omega})} \exp\left[-\rho(\hat{\omega})(x-y)\right] - \frac{2\rho(\hat{\omega})^3}{\hat{\omega}^2} \exp\left[-\rho(\hat{\omega})(a-x+a-y)\right],$$

$$G_s(x, y, z) \overset{z \to \pm\hat{\omega}}{\sim} -\frac{\Omega^2}{4\kappa^2} \frac{\rho(\hat{\omega}) \left(1 + 4\rho(\hat{\omega})^2\right)}{(z-\hat{\omega})(z+\hat{\omega})} \exp\left[-\rho(\hat{\omega})(y-x)\right] - 2\rho(\hat{\omega}) \exp\left[-\rho(\hat{\omega})(a-x+a-y)\right].$$

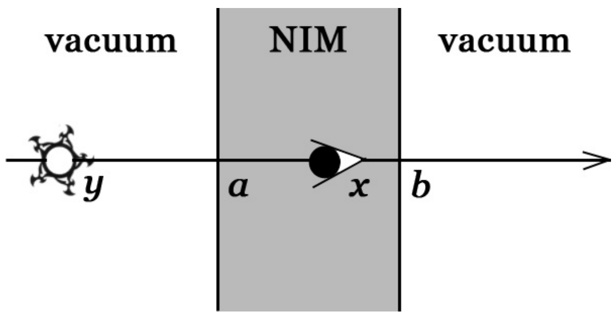

FIG. 2. Refraction case

In the transmission case ($y < a < b < x$), the receiver and point field source are located on different sides the NIM (see Fig. 3), for $\hat{\omega} > \kappa$,

$$G_p(x, y, \pm\hat{\omega}) = \pm\frac{\rho(\hat{\omega})}{2i\hat{\omega}^2} \exp\left[\pm i\rho(\hat{\omega})\,(x-y-2(b-a))\right],$$
$$G_s(x, y, \pm\hat{\omega}) = \pm\frac{1}{2i\rho(\hat{\omega})} \exp\left[\pm i\rho(\hat{\omega})\,(x-y-2(b-a))\right],$$

for $\hat{\omega} < \kappa$,

$$G_p(x,y,z) \overset{z\to\pm\hat{\omega}}{\sim} -\frac{2\rho(\hat{\omega})^3}{\hat{\omega}^2} \exp\left[-\rho(\hat{\omega})\left(x-y-2(b-a)\right)\right],$$
$$G_s(x,y,z) \overset{z\to\pm\hat{\omega}}{\sim} 2\rho(\hat{\omega}) \exp\left[-\rho(\hat{\omega})\left(x-y-2(b-a)\right)\right].$$

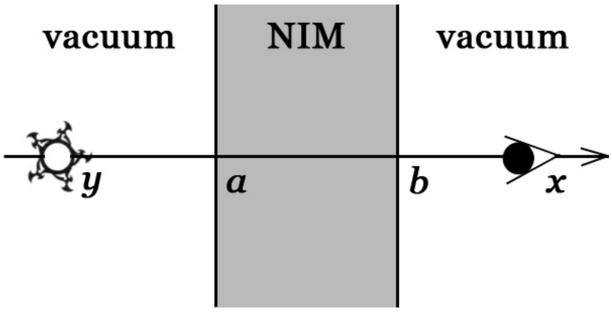

FIG. 3. Transmission case

In retrieving $E(x,t)$, the pole contributions in Green's function give rise to terms that oscillate in time according to $\exp[\pm i\hat{\omega}t]$, so no dampening occurs in a time dependent fashion, a property observed earlier in [2] for the single layer case.

3.2. Multilayered system

Also, we find Green's function for the multilayered system. The point perturbation is located in layer number 0. There are n layers in the positive direction of the x-axis and m layers in the negative, $m+n+1$ layers in total (see Fig. 4).

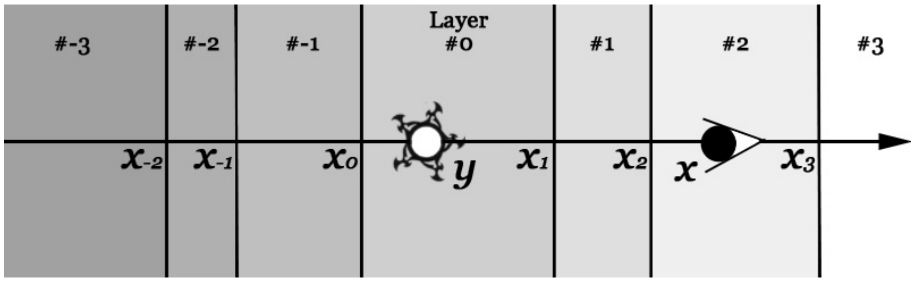

FIG. 4. Multilayered system

Let $\varepsilon(x,z) = \varepsilon_k(z)$ and $\mu(x,z) = \mu_k(z)$ if $x \in (x_k, x_{k+1})$, where $k = -m, \dots, n$, $x_{-m} = -\infty$, $x_{n+1} = +\infty$. We consider below only the p-polarized part Green's function, omitting subscript p,

$$G(x,y,z) =$$

$$= \begin{cases} D_{-m}e^{-i\zeta_{-m}x}, & x \in (-\infty,\ x_{-(m-1)}); \\ B_{-(m-1)}e^{-i\zeta_{-(m-1)}x} + C_{-(m-1)}e^{-i\zeta_{-(m-1)}x} + D_{-(m-1)}e^{-i\zeta_{-(m-1)}x}, & x \in (x_{-(m-1)},\ x_{-(m-2)}); \\ A_{-(m-2)}e^{-i\zeta_{-(m-2)}x} + B_{-(m-2)}e^{-i\zeta_{-(m-2)}x} + C_{-(m-2)}e^{-i\zeta_{-(m-2)}x} + & \\ +D_{-(m-2)}e^{-i\zeta_{-(m-2)}x}, & x \in (x_{-(m-2)},\ x_{-(m-3)}); \\ \dots & \dots \\ A_{-1}e^{-i\zeta_{-1}x} + B_{-1}e^{-i\zeta_{-1}x} + C_{-1}e^{-i\zeta_{-1}x} + D_{-1}e^{-i\zeta_{-1}x}, & x \in (x_{-1},\ x_0); \\ A_0e^{-i\zeta_0x} + B_0e^{-i\zeta_0x} + C_0e^{-i\zeta_0x} + D_0e^{-i\zeta_0x} + E_-e^{-i\zeta_0x}, & x \in (x_0,\ y); \\ A_0e^{-i\zeta_0x} + B_0e^{-i\zeta_0x} + C_0e^{-i\zeta_0x} + D_0e^{-i\zeta_0x} + E_+e^{i\zeta_0x}, & x \in (y,\ x_1); \\ A_1e^{-i\zeta_1x} + B_1e^{-i\zeta_1x} + C_1e^{-i\zeta_1x} + D_1e^{-i\zeta_1x}, & x \in (x_1,\ x_2); \\ \dots & \dots \\ A_{n-2}e^{-i\zeta_{n-2}x} + B_{n-2}e^{-i\zeta_{n-2}x} + C_{n-2}e^{-i\zeta_{n-2}x} + D_{n-2}e^{-i\zeta_{n-2}x}, & x \in (x_{n-2},\ x_{n-1}); \\ A_{n-1}e^{-i\zeta_{n-1}x} + B_{n-1}e^{-i\zeta_{n-1}x} + C_{n-1}e^{-i\zeta_{n-1}x}, & x \in (x_{n-1},\ x_n); \\ A_ne^{-i\zeta_nx}, & x \in (x_n,\ +\infty). \end{cases}$$

Here coefficients $A_\bullet$ are for waves that come from left outside of current layer, coefficients $B_\bullet$ are for waves reflected from the nearest left interface, coefficients $C_\bullet$ are for waves reflected from the nearest right interface and coefficients $D_\bullet$ are for waves that come from right outside of current layer. Coefficients $E_\pm$ are for waves that come direct from point perturbation located in y. We denote

$$\zeta_i^2(\kappa, z) = z^2\varepsilon_i(z)\mu_i(z) - \kappa^2,$$
$$K_0 = \frac{\zeta_0}{2iz^2\varepsilon_0}, \quad \lambda_{i,j}^{\pm} = \frac{\varepsilon_i\zeta_j \pm \varepsilon_j\zeta_i}{\varepsilon_i\zeta_j}$$

and introduce the Fresnel reflection coefficients

$$r_{i,j} = -\frac{\varepsilon_i\zeta_j - \varepsilon_j\zeta_i}{\varepsilon_i\zeta_j + \varepsilon_j\zeta_i} \quad \text{or} \quad r_{i,j} = -\frac{\lambda_{i,j}^-}{\lambda_{i,j}^+}.$$

As is easy to see $r_{j,i} = -r_{i,j}$. After some calculations, we obtain $E_\pm = K_0e^{\mp i\zeta_0 y}$ that means $E_\pm e^{\pm i\zeta_0 x} = K_0e^{i\zeta_0|x-y|}$ and Green's function is the same for $x \in (x_0,\ x_1)$. Denoting

$$a_k = 2e^{-i\zeta_kx_k}, b_k = \lambda_{k,k-1}^+e^{-i\zeta_{k-1}x_k}, \quad c_k = 2e^{i\zeta_kx_k}, d_k = \lambda_{k,k-1}^-e^{-i\zeta_{k-1}x_k},$$
$$e_k = 2e^{i\zeta_kx_{k+1}}, f_k = \lambda_{k,k+1}^+e^{i\zeta_{k+1}x_{k+1}}, \quad g_k = 2e^{-i\zeta_kx_{k+1}}, h_k = \lambda_{k,k+1}^-e^{i\zeta_{k+1}x_{k+1}}$$

we obtain for $k = 1, \dots, (n-1)$,

$$A_k = \alpha_kA_n, \quad B_k = \frac{d_k}{c_k}\gamma_kA_n, \quad C_k = \frac{h_k}{g_k}\alpha_{k+1}A_n, \quad D_{k-1} = \alpha_kA_n,$$

for $k = -(m-1), \dots, 0$,

$$A_k = \alpha_kA_n + \beta_k, \quad B_k = \frac{d_k}{c_k}(\gamma_kA_n + \delta_k), \quad C_k = \frac{h_k}{g_k}(\alpha_{k+1}A_n + \delta_{k+1}), \quad D_{k-1} = \alpha_kA_n + \delta_k,$$

but $C_0 = \dfrac{h_0}{g_0}\alpha_1A_n$ and $A_{-(m-1)} = 0$, where α_k, β_k, γ_k, δ_k satisfy the following recurrence relations

$$\alpha_k = \tilde{\alpha}_k \alpha_{k+1} - \left(\frac{a\,d}{b\,c}\right)_k \gamma_{k+1} \quad \text{and} \quad \alpha_{n-1} = \tilde{\alpha}_{n-1},$$

where

$$\tilde{\alpha}_k = \frac{f_k}{e_k}\left(1 - \left(\frac{a\,d\,e\,h}{b\,c\,f\,g}\right)_k\right),$$

$$\gamma_k = \frac{a_k}{b_k}\left(\frac{h_k}{g_k}\alpha_{k+1} + \gamma_{k+1}\right) \quad \text{and} \quad \gamma_{n-1} = \left(\frac{a\,h}{b\,g}\right)_{n-1},$$

$$\beta_k = \tilde{\alpha}_k \beta_{k+1} - \left(\frac{a\,d}{b\,c}\right)_k \delta_{k+1} \quad \text{and} \quad \beta_0 = K_0\left(e^{-i\zeta_0 y} - \left(\frac{a\,d}{b\,c}\right)_0 e^{i\zeta_0 y}\right),$$

$$\delta_k = \frac{a_k}{b_k}\left(\frac{h_k}{g_k}\beta_{k+1} + \delta_{k+1}\right) \quad \text{and} \quad \delta_0 = K_0\frac{a_0}{b_0}e^{i\zeta_0 y}.$$

Here we use the notation $\left(\frac{a\,d}{b\,c}\right)_k = \frac{a_k d_k}{b_k c_k}$. Hence coefficients $A_\bullet$, $B_\bullet$, $C_\bullet$, and $D_\bullet$ depend on A_n. In our investigation we obtain

$$A_n = -\frac{\beta_{-(m-1)}}{\alpha_{-(m-1)}}.$$

Solving these recurrence relations and finding the explicit expressions for Green's function is the actual problem.

Acknowledgments

The work was supported by Federal Targeted Program "Scientific and Educational Human Resources for Innovation-Driven Russia" (contract 16.740.11.0030), grant 11-08-00267 of Russian Foundation for Basic Researches, Federal Targeted Program "Researches and Development in the Priority Directions Developments of a Scientific and Technological Complex of Russia 2007-2013" (state contract 07.514.11.4146) and grants of the President of Russia (state contract 14.124.13.2045-MK and grant MK-1493.2013.1).

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