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PLASMON POLARITONS EXCITATION AT RAPIDLY GENERATED PLASMA INTERFACE

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In this paper we studied the possibility of the appearance of surface plasmon polaritons at the plasma/dielectric interface with rapidly generated plasma in the right half- space, when the field is generated by a plane source, was studied. The source was located parallel to the interface, and at an angle to it. It was shown that the frequency-converted wave outgoing from plasma boundary corresponds to the plasmon polariton in the case when the initial field is generated by the plane source located at the angle α to the plasma boundary for the following condition $\varepsilon_1 \sin^2(\alpha) - \varepsilon - \frac{{w_e}^2}{w^2} > 0$.

Keywords: plasmon polariton, integral equations, plasma, electromagnetic field, non-stationary phenomena, Riemann's surface.

1. Introduction

Recent advances in nano-fabrication enable one to carry numerous nano-photonic experiments including subwavelength metal structures fabrication. In turn,this flurry of activity has, reawakened interest in theoretical research of surface plasmon polaritons, despite the fact that the fundamental properties of surface plasmon polaritons have been known for nearly five decades [1, 2]. By definition, surface plasmons are the quanta of surface-charge-density oscillations, but the same terminology is commonly used for collective oscillations in the electron density at the surface of metal. The surface charge oscillations are naturally coupled to electromagnetic waves, which explains their designation as polaritons [3]. Plasmon polaritons are used in near-field microscopy, optical imaging systems with nanometer resolution, hybrid photonic-plasmonic devices and metamaterials with negative refractive index, environment sensing, surface plasmon sensors for the analysis of biological bonds, etc. Surface plasmon polaritons are electromagnetic waves propagating at the interface between two different media. Surface plasmons have been utilized almost exclusively at optical frequencies because it needs the lossless negative permittivity medium to excite them, which is typical for metal at these frequencies. The negative permittivity of metals is provided by plasma which has a large electron density. Conversely, plasma is a medium which can easily change its own parameters, among which is its electron density, and the plasma can simply be generated in the initial dielectric medium [2]. Therefore, it is of intense interest to investigate the interaction of electromagnetic waves with the non-stationary plasma surface whose density varies over time and its permittivity becomes negative. The initial time of the non-stationary beginning becomes an important factor. The introduction of this initial time moment allows to distinguish the "switching on" of field and the beginning of non-stationarity.

The radiation of the plane source in a homogeneous stationary medium is well known, but in the case of an inhomogeneous layered medium, it is more complicated. If the medium is non-stationary, the radiation of the plane source takes on a less trivial form.

In papers [4,5], the novel mechanism of frequency upshifting of p-polarized electromagnetic wave, which is obliquely incident one on a thin plasma layer with slowly growing electron density, was presented. In this paper the transformation of external field radiation of the plane source from rapidly generated plasma was considered.

2. The radiation of the plane source (initial field is parallel to the plasma boundary)

We consider the medium with dielectric permittivity ε, where the electromagnetic field is radiated by a plane source $\mathbf{j} = \mathbf{q}\delta(x-a)e^{i\omega t}$, where $\mathbf{j}$ is a current describing extrinsic source, $\mathbf{q}$ is the vector directed along a source. The plane source is parallel to the plane YOZ, see Fig. 1

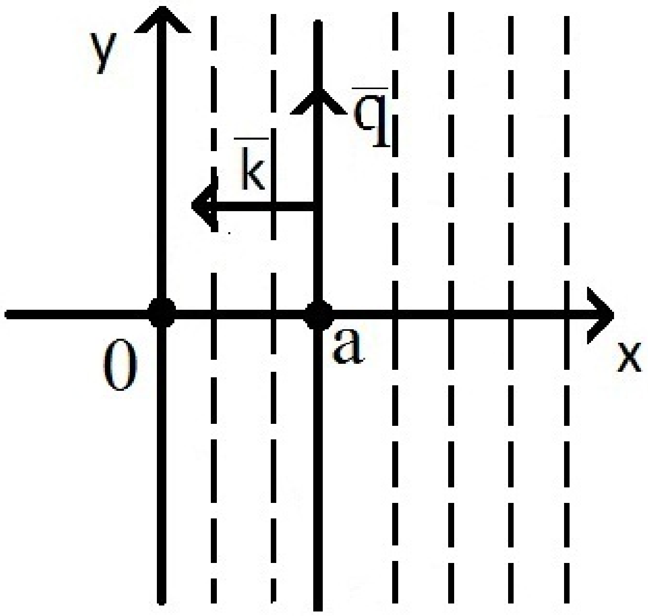

Fig. 1. The plane source is parallel to the plasma boundary and it is located at a distance a from the interface in plasma half-space, where $\mathbf{q}$ is the vector directed along a source, $\mathbf{k}$ is the wave vector. The rapidly generated plasma in the right half-space is designed the vertical dashed lines.

At zero moment of time the half-space $x > 0$ is ionized and the plasma appears in this half-space. The plasma permittivity is given by the known expression $\overline{\varepsilon}(\omega_e, \omega) = \varepsilon_1 - \frac{\omega_e{}^2}{\omega^2}$, where ε_1 describes the dispersionless part of the new medium in the half-space $x > 0$ after the zero moment, ω_e is the plasma frequency [3].The initial field of source radiation is a plane wave propagating perpendicularly to the plane of the source. By using the Green's function G [6] let's find the initial field of the source, which is given by $\mathbf{j} = \mathbf{q}\delta(x-a)e^{i\omega t}$:

$$\mathbf{E_0} = G \cdot \frac{\partial \mathbf{j}}{\partial t} = -\frac{\partial G}{\partial t} \cdot \mathbf{j} = -\frac{v}{4\pi}\hat{D}\mathbf{q}\int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} d\mathbf{r}' \frac{\theta(t - t' - \frac{|\mathbf{r}-\mathbf{r}'|}{v})}{|\mathbf{r}-\mathbf{r}'|}\delta(x'-a)e^{i\omega t'} =$$
$$= -\frac{v}{2\pi}\hat{D}\mathbf{q}e^{i\omega t - i\frac{\omega}{v}|x-a|}\left((\mathbf{e_1}, \mathbf{q})\,\mathbf{e_1} - \mathbf{q}\right) = \frac{v}{2\pi}\mathbf{q}e^{i\omega t - i\frac{\omega}{v}|x-a|}, \quad (1)$$

where differential operator $\hat{D} = \left(\bigtriangledown \bigtriangledown - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)$, and vectors $\mathbf{q} = (0, q, 0)\, \mathbf{e_1} = (1, 0, 0)$.

Let's consider how the electromagnetic field changes after plasma formation. It is convenient to find the solution to this problem using the integral equations method in time domain [6,7]. It follows that the problem's solution in the half-space $x < 0$ (external field) can be represented by two terms:

$$\mathbf{E} = \mathbf{E_0} + \hat{N} * \mathbf{E_0}. \tag{2}$$

Here, $\mathbf{E_0}$ is the field of source and the second term is given by the operator $\hat{N} * \mathbf{E_0} = \int_0^\infty \left\langle \mathbf{x} \mid \hat{N} \mid \mathbf{x}' \right\rangle \mathbf{E_0}(\mathbf{x}') d\mathbf{x}'$. The symbol (*) designates the convolution,

$$(a * b)(\mathbf{x}) = \int_{-\infty}^{\infty} dt' \int_{\infty} d\mathbf{r}' a(\mathbf{x} - \mathbf{x}') b(\mathbf{x}')$$

and $\mathbf{x} = (t, \mathbf{r})$ is a 4D spatial-time vector. Integration is performed over the whole 4D space $-\infty < t' < \infty$, $-\infty < x', y', z' < \infty$. The term in the convolution is determined by the extrinsic current. The transition to the impulse representation (Fourier-Laplace representation)in a rectangular system of coordinates is performed by virtue of transformation functions:

$$< \mathbf{x}||\mathbf{p} >= \delta_{ij} e^{pt + i\mathbf{kr}},\ < \mathbf{p}||\mathbf{x} >= \delta_{ij} e^{-pt - i\mathbf{kr}},$$

where $\mathbf{p} = (p, \mathbf{k})$, p is a complex variable of the Laplace transformation, $\mathbf{k}$ is a real variable of the 3D Fourier transformation.

The kernel of external resolvent operator $\hat{N}$ (reflection operator) in the coordinate representation has the form:

$$\left\langle \mathbf{x} \mid \hat{N} \mid \mathbf{x}' \right\rangle = \theta(-x) \frac{{v_1}^2 - v^2}{v^2 v_1} \int d\mathbf{p}_\perp \frac{1}{2\varphi_1} \{v_1 v u_m P + p^2 u_e I_\perp\} e^{p(t-t') + \frac{\varphi}{v}x - \frac{\varphi_1}{v_1}x' + i\mathbf{k}_\perp(\mathbf{r}_\perp - \mathbf{r'}_\perp)} \theta(x'), \tag{3}$$

where the vector $\mathbf{r}_\perp = (y, z)$ is located in the plane $x = 0$, and $v = \frac{c}{\sqrt{\varepsilon\mu}}$, $v_1 = \frac{c}{\sqrt{\varepsilon_1 \mu_1}}$ are wave-phase velocities.

In this formula, block matrices are defined as follows:

$$P = \begin{pmatrix} -{k_\perp}^2 & -i\frac{\varphi}{v}\mathbf{k}_\perp \\ -i\frac{\varphi}{v}{\mathbf{k}_\perp}^* & \hat{k}_\perp \end{pmatrix}, I_\perp = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, {\mathbf{k}_\perp}^* = \begin{pmatrix} k_2 \\ k_3 \end{pmatrix}, \hat{k}_\perp = \begin{pmatrix} {k_2}^2 & k_2 k_3 \\ k_3 k_2 & {k_3}^2 \end{pmatrix},$$

$\mathbf{k}_\perp = \begin{pmatrix} k_2 & k_3 \end{pmatrix}$ and ${k_\perp}^2 = {k_2}^2 + {k_3}^2$. The coefficients

$$u_m = \frac{2 v_1 \varphi}{v\varphi + v_1 \varphi_1}, u_e = \frac{2 v_1 \varphi}{v\varphi_1 + v_1 \varphi}$$

are similar to the Fresnel's formulas for parallel and perpendicular polarizations and

$$\varphi = \sqrt{p^2 + v^2 {k_\perp}^2}, \varphi_1 = \sqrt{p^2 + {v_1}^2 {k_\perp}^2}, v_1 = \frac{cp}{\sqrt{\varepsilon_1 p^2 + {\omega_e}^2}}.$$

Here $\mathbf{p}_\perp = (p, \mathbf{k}_\perp)$ - Fourier - Laplace transformation variables. Thus, substituting the expression for the initial field of source (1) and the reflection operator (3) in equation (2), we obtain:

$$\langle \mathbf{x} \mid \mathbf{E} \rangle = \langle \mathbf{x} \mid \mathbf{E_0} \rangle + \left\langle \mathbf{x} \mid \hat{N} \mid \mathbf{x}' \right\rangle * \langle \mathbf{x}' \mid \mathbf{E_0} \rangle = \frac{v}{2\pi}\mathbf{q} e^{i\omega t - i\frac{\omega}{v}|x-a|} +$$
$$+ \frac{v}{2\pi}\theta(-x)\int_0^\infty dt' \int_{-\infty}^{\infty} d\mathbf{r}' \int d\mathbf{p}_\perp \frac{{v_1}^2 - v^2}{v^2 v_1}\frac{1}{2\varphi_1}\{v_1 v u_m P + p^2 u_e I_\perp\}\mathbf{q}\cdot$$
$$\cdot\, e^{p(t-t') + \frac{\varphi}{v}x - \frac{\varphi_1}{v_1}x' + i\mathbf{k}_\perp(\mathbf{r}_\perp - \mathbf{r}'_\perp)}\theta(x')e^{i\omega t' - i\frac{\omega}{v}|x'-a|} \quad (4)$$

First, we calculate integrals over the spatial and temporal coordinates, and then over Fourier-Laplace variables. The integration over the variable x' gives the following expression:

$$\int_0^\infty e^{-i\frac{\omega}{v}|x'-a| - \psi_1 x'}dx' = \int_0^a e^{-i\frac{\omega}{v}(a-x') - \psi_1 x'}dx' + \int_a^\infty e^{-i\frac{\omega}{v}(x'-a) - \psi_1 x'}dx' = \frac{e^{-\psi_1 a} - e^{-ika}}{ik - \psi_1} + \frac{e^{-\psi_1 a}}{ik + \psi_1}$$

where $\psi_1 = \frac{\varphi_1}{v_1} = \frac{1}{c}\sqrt{\varepsilon_1 p^2 + {\omega_e}^2 + c^2 {k_\perp}^2})$ for the condition $Re(\psi_1) > 0$. Later, this condition will be taken into account for the calculation of residuals in the integral over the variable p (see the expression (4)).
After integration over the spatial and Fourier transform variables, the second term in (4) takes the following form:

$$\int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{p(t+\frac{x}{v})}}{p - i\omega} p \frac{v_1 - v}{v^2}\left(\frac{e^{-\frac{a}{c}\sqrt{\varepsilon_1 p^2 + {\omega_e}^2}} - e^{-ika}}{ik - \frac{1}{c}\sqrt{\varepsilon_1 p^2 + {\omega_e}^2}} + \frac{e^{-\frac{a}{c}\sqrt{\varepsilon_1 p^2 + {\omega_e}^2}}}{ik + \frac{1}{c}\sqrt{\varepsilon_1 p^2 + {\omega_e}^2}}\right)\frac{dp}{2\pi i} =$$
$$= \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{e^{p(t+\frac{x}{v})}}{p - i\omega}\frac{cp}{v^2}\left(\frac{cp}{z(p)} - v\right)\left(\frac{e^{-\frac{a}{c}z(p)} - e^{-ika}}{ikc - z(p)} + \frac{e^{-\frac{a}{c}z(p)}}{ikc + z(p)}\right)\frac{dp}{2\pi i},$$

where $z(p) = \sqrt{\varepsilon_1 p^2 + {\omega_e}^2}$. Note, that the obtained integral will be equal to zero at $t + \frac{x}{v} < 0$, since, in this case, the integration contour can be closed by circle of infinite radius in the right half-plane, where the integrand has no singularities. To calculate the integral in the interval $t + \frac{x}{v} < 0$ the integration contour can be closed only to the left of the line γ: 1) The equation $p - i\omega = 0$, gives a simple pole, $p_1 = i\omega$, which isn't contained in the integration path, as it is located on the section between the branch points. 2) The expressions $ikc - z(p) = 0$, and $ikc + z(p) = 0$ give two poles $p_{2,3} = \pm\frac{i}{\sqrt{\varepsilon_1}}\sqrt{\varepsilon\omega^2 + {\omega_e}^2} = \pm i\omega_2$. There is a removable singularity (a finite limit of the integrand) at the point $p_2 = i\omega_2$, therefore the residual at this point is equal to zero. The selection of root sign in the equations $ikc \pm z(p) = 0$ follows from the condition $Re\psi_1 > 0$.

The integration contour contains all the singularities of the integrand. To obtain the unambiguous integrand, let's choose a Riemann's surface. The integrand is a double-valued function, because $z(p) = \sqrt{\varepsilon_1 p^2 + {\omega_e}^2}$ has two branch points $\pm i\frac{\omega_e}{\sqrt{\varepsilon_1}}$. It is necessary to allocate the branch of $z(p)$, for which the condition $Re\psi_1 = Re\sqrt{z^2 + \varepsilon\omega^2} > 0$ is performed. To uniquely identify $z(p)$, it can be considered the complex plane $p = \xi + i\eta$, as a two-sheeted surface. The surface sheets are joined along the banks of the cuts.
On each sheet, $z(p)$ is uniquely defined as a function of the variable p. To satisfy the condition $Re\sqrt{z^2 + \varepsilon\omega^2} > 0$, it should be glued the sheets of the Riemann's surface along the curve given by the equation $Re\sqrt{z^2 + \varepsilon\omega^2} = 0$. This condition determines the required branch line. We make the cuts in the complex plane p. For this, we write ${\psi_1}^2$ as follows:

$${\psi_1}^2 = \varepsilon_1 p^2 + {\omega_e}^2 + \varepsilon\omega^2 = \varepsilon_1(\xi^2 - \eta^2 + 2i\xi\eta) + {\omega_e}^2 + \varepsilon\omega^2.$$

The correct procedure of the choice of the cut can be made for a dissipative dielectric. To carry out this procedure, we assume that the medium has small losses (μ) $\omega_e = \overline{\omega}_e + i\mu$ and $\overline{\omega}_e \gg \mu$, $\mu \to 0$ corresponds to the limiting case of a lossless medium. Thus we have ${\psi_1}^2 = [\varepsilon_1\xi^2 - \varepsilon_1\eta^2 + {\overline{\omega}_e}^2 - \mu^2 + \varepsilon\omega^2] + 2i\,[\varepsilon_1\xi\eta + \overline{\omega}_e\mu]$. We make cuts for the ψ_1 in the complex plane p, so that the condition $Re\psi_1 > 0$ is fulfilled on one of the sheets of the Riemann's surface, and $Re\psi_1 < 0$ on the second sheet.

To satisfy these two conditions, it should be glued the sheets of the Riemann's surface along the curve given by the equation $Re\psi_1 = 0$. This equation determines the required branch line. We then plot the real and imaginary parts ${\psi_1}^2$ which depend on ξ and η, as shown in Fig. 2.

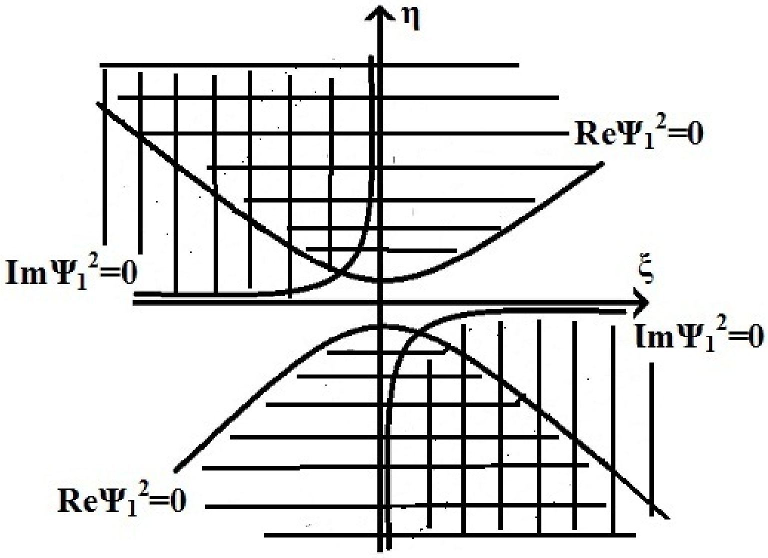

Fig. 2. The regions of the complex plane p ($Imp = \xi$, $Rep = \eta$) are bounded by the curves $Re{\psi_1}^2 = 0$ and $Im{\psi_1}^2 = 0$, for which the real and imaginary parts of ψ_1^2 maintain their signs: $Re{\psi_1}^2 < 0$ in the horizontal shading region, $Im{\psi_1}^2 < 0$ in the vertical shading region.

Then, we divide the plane p into regions by curves on which either $Re{\psi_1}^2 = \varepsilon_1\xi^2 - \varepsilon_1\eta^2 + {\overline{\omega}_e}^2 - \mu^2 + \varepsilon\omega^2 = 0$, or $Im{\psi_1}^2 = 2(\varepsilon_1\xi\eta + \overline{\omega}_e\mu)$. Thus, we obtained two regions formed by hyperboles intersection:

$$\frac{\varepsilon_1(\xi^2 - \eta^2)}{{\overline{\omega}_e}^2 - \mu^2 + \varepsilon\omega^2} = -1 \; and \; \xi = -\frac{\overline{\omega}_e\mu}{\varepsilon_1\eta},$$

for which the conditions $Re{\psi_1}^2 > 0$, $Im{\psi_1}^2 > 0$. To satisfy the condition $Re{\psi_1}^2 > 0$, it is necessary that the inequality $\mid Arg{\psi_1}^2 \mid < \pi$ is performed on the upper sheet of the Riemann surface. Hence, that it needs to choose a cut along the line defined by the equation $Arg{\psi_1}^2 = \pi$ or equivalent equations $Re\psi_1 < 0$ and $Im{\psi_1}^2 = 0$. As a result, the position of the cuts (shown in Fig. 3 by bold lines) was uniquely determined.

Finally, we obtain the following expression for the modified external field:

$$\langle \mathbf{x} \mid \mathbf{E} \rangle = \frac{v}{2\pi}\mathbf{q}e^{i\omega t - i\frac{\omega}{v}|x-a|} + \frac{v}{2\pi}\theta(-x)\mathbf{q}\frac{\varepsilon}{\varepsilon_1}\frac{\Omega - 1}{\Omega + 1}e^{-i\omega_2\left(t + \frac{x}{v}\right) + ika}\theta\left(t + \frac{x}{v}\right), \tag{5}$$

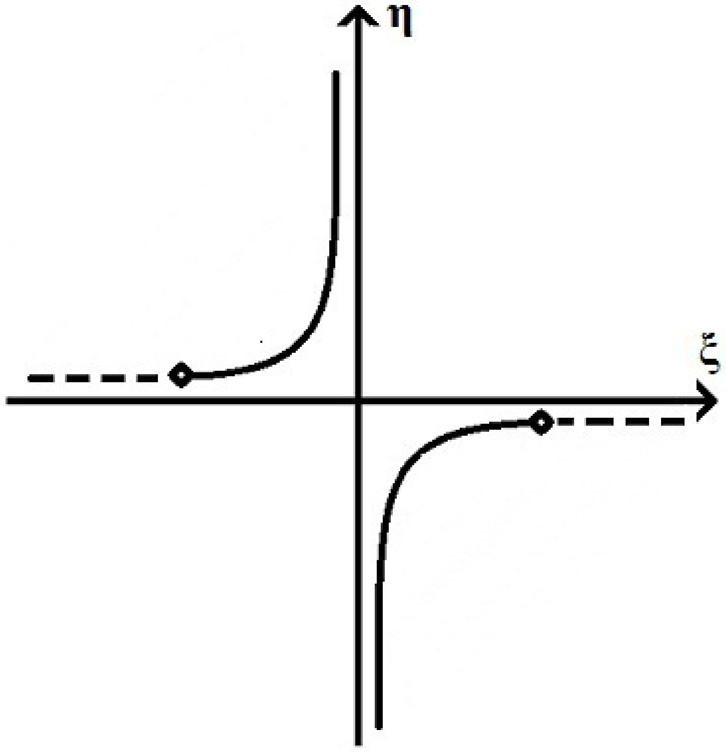

Fig. 3. Cuts in the complex plane p ($Imp = \xi$, $Rep = \eta$), satisfying the condition $Re\psi_1 > 0$ are designated the solid curves.

where $\Omega = \sqrt{\varepsilon + \frac{\omega_e{}^2}{\omega^2}}$. Consequently after the medium parameters jump in the left half-space, the moving boundary $x = -vt$ appears, which moves with the velocity v from the media interface. In the product band, $-vt < x < 0$, the wave propagates with a new frequency, $\omega_2 = \frac{1}{\sqrt{\varepsilon_1}}\sqrt{\varepsilon\omega^2 + \omega_e{}^2}$ and a new wave number, $\frac{\omega_2}{v}$. The external transformed field consists of monochromatic waves with frequencies ω and $\omega_2 = \frac{1}{\sqrt{\varepsilon_1}}\sqrt{\varepsilon\omega^2 + \omega_e{}^2}$. The waves with both frequencies propagate without attenuation in the external half-space.

3. The radiation of the plane source (the initial field is at the angle α to the plasma boundary)

Next, we consider the case when the electromagnetic field is radiated by a plane source $\mathbf{j} = \mathbf{q}\delta(s)e^{i\omega t}$, which is located at an angle α to the YOZ plane boundary, see Fig. 4. Similarly to the above case, at the zero moment of time, the half-space $x > 0$ is ionized and the plasma appears in this half-space. The plasma permittivity is given by the known expression $\overline{\varepsilon}(\omega_e, \omega) = \varepsilon_1 - \frac{\omega_e{}^2}{\omega^2}$, where ε_1 describes the dispersionless part of the new medium in the half-space $x > 0$ after the zero moment, ω_e is the plasma frequency.

Let's consider the transformation of the source field outside the plasma ($x < 0$), after the plasma's appearance. As with the previous case, we find the solution to this problem using the integral equations method in time domain [6, 7]. From this, it follows that the solution in the half-space $x < 0$ (external field) can be represented by formula (2). The problem is to study the field due to the sudden formation of plasma. At first, by using Green's function G [4], we find the initial field of source radiation before the plasma formation in the case when the plane source is located at an angle to the media interface,

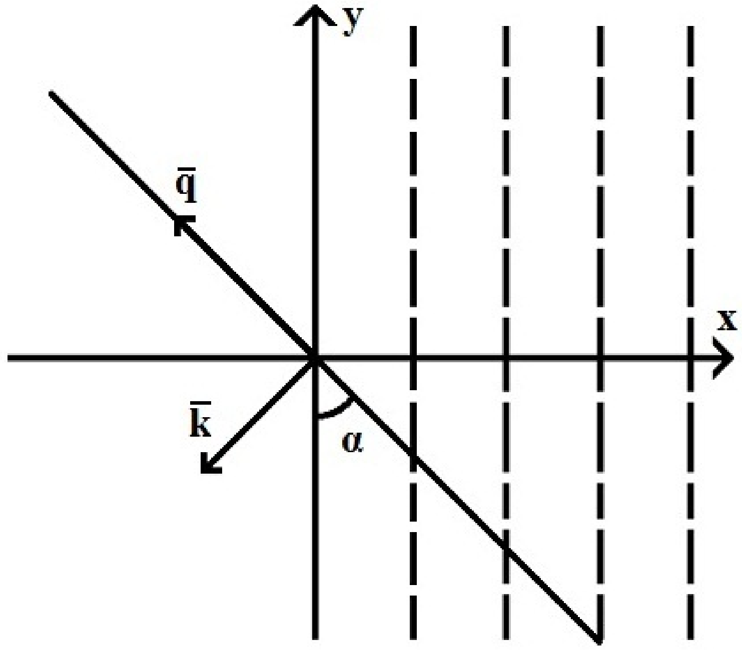

Fig. 4. The plane source is at an angle α to the plasma boundary. The rapidly generated plasma in the right half-space is designed the vertical dashed lines. Here $\mathbf{q}$ is the vector directed along a source, $\mathbf{k}$ is the wave vector.

$$\mathbf{E_0} = G \cdot \frac{\partial \mathbf{j}}{\partial t} = -\frac{\partial G}{\partial t} \cdot \mathbf{j} = -\frac{v^2}{4\pi} \hat{D}\mathbf{q} \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} d\mathbf{r}' \frac{\theta(t - t' - \frac{|\mathbf{r}-\mathbf{r}'|}{v})}{|\ \mathbf{r} - \mathbf{r}'\ |} \delta(s) e^{i\omega t'}. \tag{6}$$

It is more convenient to calculate the radiation of current for the case when the source is located parallel to the interface at the surface $x = 0$, when $a = 0$. That is, setting $\delta(s) = \delta(x)$, it is simple to make the rotation of the coordinate system by a corresponding angle. Then, according to this formula (1), the initial field will have the form:

$$\mathbf{E_0} = \frac{v}{2} \mathbf{q} e^{i\omega t - i\frac{\omega}{v}|x|}, \tag{7}$$

where $\mathbf{q} = (0, q_2, 0)$. Let's make rotation of the coordinate system by the angle α. The coordinate transformation with angle rotation has the following form:

$$x = x' \cos\alpha + y' \sin\alpha,\ y = -x' \sin\alpha + y' \cos\alpha.$$

Substituting the initial field, we obtain:

$$\mathbf{E_0} = \frac{v}{2} \mathbf{q} e^{i\omega t - i\frac{\omega}{v}|x' \cos\alpha + y' \sin\alpha|}, \tag{8}$$

where $\mathbf{q} = (q_2 \sin\alpha, q_2 \cos\alpha, 0)$.
Thus, it can immediately set the initial field of a plane wave which propagates at an angle to the plasma plane. Thereby, substituting the expression for the initial field of source (8) and the reflection operator (3) in (2), we obtain:

$$\langle \mathbf{x} \mid \mathbf{E} \rangle = \langle \mathbf{x} \mid \mathbf{E_0} \rangle + \left\langle \mathbf{x} \mid \hat{N} \mid \mathbf{x'} \right\rangle * \langle \mathbf{x'} \mid \mathbf{E_0} \rangle = \frac{v}{2} \mathbf{q} e^{i\omega t - i\frac{\omega}{v}|x\cos\alpha + y\sin\alpha|} + \\ + \frac{v}{2}\theta(-x) \int_0^\infty dt' \int_{-\infty}^\infty d\mathbf{r'} \int d\mathbf{p}_\perp \frac{1}{2\varphi_1} \frac{{v_1}^2 - v^2}{v^2 v_1} \{v_1 v u_m P + \\ + p^2 u_e I_\perp\} e^{p(t-t') + \frac{\varphi}{v}x - \frac{\varphi_1}{v_1}x' + i\mathbf{k}_\perp(\mathbf{r}_\perp - \mathbf{r'}_\perp)} \theta(x') e^{i\omega t' - i\frac{\omega}{v}|x'\cos\alpha + y'\sin\alpha|} \quad (9)$$

The integration over the spatial variables and Fourier transform variable is similar to the previous case for the condition $Re\sqrt{\varepsilon_1 p^2 + {\omega_e}^2 + c^2 {k_2}^2} > 0$. Then, the second term in (9) takes the following form:

$$\int_{-\infty}^\infty \frac{dk_2}{2\pi} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{dp}{2\pi i} \left((\varepsilon - \varepsilon_1)p^2 - {\omega_e}^2\right) \frac{c^2\varphi}{\psi} \mathbf{F}(p, k_2) \frac{2i\omega \sin\alpha e^{p(t + \frac{\phi}{v}x + ik_2 y)}}{(p - i\omega)(\psi - ick_2\cot\alpha)\left({k_2}^2 - \frac{\omega^2}{v^2}\sin^2\alpha\right)} \quad (10)$$

The function $\mathbf{F}(p, k_2) = \left(\frac{\mathbf{A_1}}{\varphi(\varepsilon_1 p^2 + {\omega_e}^2) + \sqrt{\varepsilon} p^2 \psi} + \frac{\mathbf{A_2}}{c(v\psi + c\varphi)}\right)$ has no singular points on the integration variables.

Here, vectors $\mathbf{A_1} = \begin{pmatrix} k_2 q_2 \left(k_2 \sin(\alpha) - i\frac{\cos\alpha}{c}\psi\right) \\ k_2 q_2 \left(i\frac{\sin\alpha}{v} + k_2\cos\alpha\right) \\ 0 \end{pmatrix}$, $\mathbf{A_2} = \begin{pmatrix} 0 \\ q_2 \cos\alpha \\ 0 \end{pmatrix}$, and

$$\varphi = \sqrt{p^2 + v^2 {k_2}^2},\ \psi = \sqrt{\epsilon_1 p^2 + {\omega_e}^2 + c^2 {k_2}^2}.$$

Let's consider the peculiarities of the integrand in (10) over the variable p. The integrand has two simple poles at the points $p_1 = i\omega$, $p_2 = \frac{i}{\sqrt{\varepsilon_1}}\sqrt{{\omega_e}^2 + \frac{c^2 {k_2}^2}{\sin^2\alpha}}$. After calculation of the integral over the variable p in the expression (6) we obtain:

$$\int_\infty^\infty \frac{dk_2}{2\pi} \mathbf{F}(p_1, k_2) e^{i\omega t + x\sqrt{{k_2}^2 - k^2}} \frac{e^{ik_2 y}}{{k_2}^2 - k^2 \sin^2\alpha} + \\ + \int_\infty^\infty \frac{dk_2}{2\pi} \mathbf{F}(p_2, k_2) e^{\frac{it}{\sqrt{\varepsilon_1}}\sqrt{{\omega_e}^2 + \frac{v^2 {k_2}^2 \varepsilon}{\sin^2\alpha}} + \frac{x}{v\sqrt{\varepsilon_1}}\sqrt{-{\omega_e}^2 + v^2 {k_2}^2\left(\varepsilon_1 - \frac{\varepsilon}{\sin^2\alpha}\right)}} \frac{e^{ik_2 y}}{{k_2}^2 - k^2 \sin^2\alpha} \quad (11)$$

Finally, we obtain the following expression for the modified external field:

$$\langle \mathbf{x} \mid \mathbf{E} \rangle = \frac{v}{2} \mathbf{q} e^{i\omega t - i\frac{\omega}{v}|x'\cos\alpha + y'\sin\alpha|} + \\ + \frac{v}{2}\theta(-x) \hat{\mathbf{F}}(p_1, -k\sin\alpha) e^{i\left(\omega t - ikx\sqrt{\sin^2\alpha - 1} - ky\sin\alpha\right)} \theta\left(t + \frac{x}{v}\right) + \\ + \frac{v}{2}\theta(-x) \hat{\mathbf{F}}(p_2, -k\sin\alpha) e^{i\left(\omega_1 t - i\frac{kx}{\sqrt{\varepsilon_1}} M(\alpha) - k\, y\, \sin\alpha\right)} \theta\left(t + \frac{x}{v}\right), \quad (12)$$

where frequency $\omega_1 = \frac{\omega}{\sqrt{\varepsilon_1}}\sqrt{\varepsilon + \frac{{\omega_e}^2}{\omega^2}}$ and $M(\alpha) = \sqrt{\varepsilon_1 \sin^2\alpha - \varepsilon - \frac{{w_e}^2}{w^2}}$.

Plasmon polaritons can occur only if the projection of the wave vector on the propagation direction of the plasmon polaritons is real and the normal component of the wave vector is purely imaginary in both media [3]. In the second term (12), the projection of the wave vector on the propagation direction (x axis) is always imaginary, but in the third term this projection of the wave vector (then $x < 0$) can be real if $M^2(\alpha) = \varepsilon_1 \sin^2 \alpha - \varepsilon - \frac{w_e{}^2}{w^2} > 0$. From this, one can make the conclusion that the transformed wave frequency can decay with distance from the plasma boundary when $M^2(\alpha) = \varepsilon_1 \sin^2 \alpha - \varepsilon - \frac{w_e{}^2}{w^2} > 0$ and $\sin \alpha > 0$.

When the value of $M^2(\alpha)$ is positive, it is possible for the plasmon polariton to appear. One can see that the surface plasmon polaritons appearance is impossible for some media for any angle, and it's possible for the second ones at the certain angle α, and for the third ones for any value of the angle.

4. Conclusions

In this paper, the transformation of plane source radiation after medium ionization was studied by using Volterra's integral equations method. The plane source was considered at an angle to the interface with a sharp ionization of the medium, i.e. when the problem becomes non-stationary. It was shown that the wave with transformed frequency outgoing from plasma is similar to the plasmon polariton in the case when the initial field is generated by the plane source located at the angle α to the plasma boundary for the following condition $\varepsilon_1 \sin^2 \alpha - \varepsilon - \frac{w_e{}^2}{w^2} > 0$. The dependence of the wave vector projection on the source angle for various media was discussed and analyzed.

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