NANOSYSTEMS: PHYSICS, CHEMISTRY, MATHEMATICS, 2014, 5 (3), P. 363–373

COMPARISON OF WAVELET TRANSFORM AND FOURIER TRANSFORM APPLIED TO ANALYSIS OF NON-STATIONARY PROCESSES

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PACS 02.60.-x, 02.70.-c

This article contains a comparison of three data analysis methods' informativity: wavelet transform, Fourier transform and short-time Fourier transform. This work contains an attempt to find the most sensitive method for the detection of quasiharmonic components in experimental data that have pronounced non-stationary behavior.

Results of high-frequency near-field sounding, IR-spectroscopy and NMR analysis of water, and also model harmonic signal were used as non-stationary processes for analysis.

Keywords: wavelet transform, Fourier transform, short-time Fourier transform, non-stationary processes analysis, quasiharmonic signal.

Received: 12 April 2014
Revised: 26 May 2014

1. Introduction

There are several cases when real experimental data is non-harmonic, contains noise, and is non-stationary in time range. This leads to difficulties in using the most popular method — the Fourier transform. The finding of quasiharmonic components is a very important problem in data analysis because this information could quite precisely show the repeatability of processes that occur in a studied system.

The wavelet transform has been recognized for the analysis of experimental data as a method that gives information about process that is not available for the Fourier transform. But there still exists no criteria for the selection of which method should be used for the analysis of experimental data.

Here, we tried to compare results of applying three methods to model signal that (as we know) contains or does not contain (quasi)harmonic components — as the first part, and applying to experimental data — as the second part. Results of such comparison give us information about the 'harmonic sensitivity' of each method and could give some criteria for selecting the appropriate method for analysis.

Let's look at the formulation of a problem. We have some signal that changes over time:

$$x = f(t).$$

This signal could contain both harmonic components and random noise. The purpose of analysis is to find quasi-harmonic components independently of noise.

The first method, coming from classical spectral analysis, is the Fourier transform. This method could be defined just using the formula of the transform (1):

$$F\left(w\right)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}f\left(t\right)e^{-iwt}dt. \tag{1}$$

The result of this transform shows the spectral (frequency) content of the signal. We can get the best result for this transform if the signal is harmonious on all time axes. But, if we apply this method to the signal, which apart the harmonic components also has the noise, the result will be less unambiguous.

In addition, a more significant problem is the fact that the Fourier transform of two completely different signals can be very similar — for example, for a sum of two sine waves and the signal from two successive sine waves. This problem could be partly solved using the second method — the Short-time Fourier transform (STFT). It is defined by the following formula:

$$F\left(t,w\right)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{+\infty}f\left(\tau\right)W\left(\tau-t\right)e^{-iw\tau}d\tau. \tag{2}$$

There is function $W(t)$ that is called window function. A definite property of this function is that it has a norm that equals 1: $\int\limits_{-\infty}^{+\infty}W\left(\tau\right)d\tau=1.$

Insertion of the window function into the formula for the Fourier transform gives us a chance to explore signal in short-time area — the window function is defined at compact with fixed width of the support and product of the window function and the signal cuts a small part of the signal. Applying of the Fourier transform formula to this product then gives us the spectral components corresponding exactly to this part of signal. As soon as the window 'slides' over the signal, this gives us information about the spectral components of each such small part of the signal. But, as soon as this method is based on the first method, we still have a lot of difficulties with the detection of quasiharmonic components in a non-stationary signal.

The last method is the wavelet transform. This method is based on absolutely different premises, unlike the previous methods, and because of that, it has other properties and results of its application.

The wavelet transform is defined by the following formula:

$$T(a,b)=\frac{1}{\sqrt{|a|}}\int\limits_{-\infty}^{+\infty}f\left(t\right)\Psi^{*}\left(\frac{t-b}{a}\right)d\tau. \tag{3}$$

As the 'mother wavelet' can be different functions, the selection of this function will give different properties to the resulting transform. These functions could be complex-valued or real-valued, could be defined as a compact set or as the whole real axis. It may or may not be based on quasi-harmonic functions. In this article, we used Morlet wavelet as the 'mother wavelet' for the transform. [1–5].

2. Model signals

Let's apply these methods to model signal.

2.1. Analysis of 'pure' signal

'Pure' signal — the signal which has two harmonics with different frequencies. There is a plot of this signal on Fig. 1.

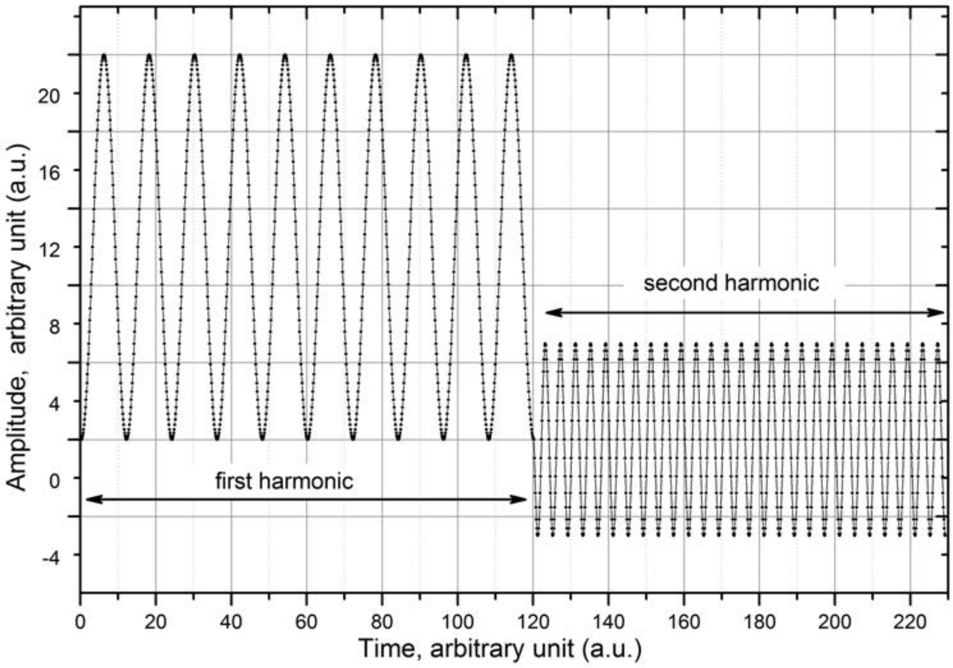


FIG. 1. Model 'pure' signal – the signal which has two harmonics with different frequencies

As described before, our purpose is to select method that gives us the most information about the quasiharmonic components of a signal.

The Fourier transform is shown in Fig. 2 as frequency characteristics. There are two peaks at two frequencies that correspond to two harmonic components in the signal. But even having information about the harmonic components, we have no information about true form of the signal.

Short-time Fourier transform with window height equals 200 ticks is shown in Fig. 3.

STFT of model 'pure' signal detects two harmonics with frequencies equal $\sim$ 0.09 arbitrary unit (a.u.) and $\sim$ 0.245 a.u.

The wavelet transform and its cross-sections are shown in Fig. 4(a–c). Dashed lines in Fig. 4a show positions of the cross sections.

Analysis of the obtained results shows us that the wavelet transform allows us to precisely detect harmonic components of the signal and gives us good representation of analyzed signal.

2.2. Analysis of 'pure' signal with a noise

In this section we add 'white noise' with an amplitude that almost equals the amplitude of the 'pure' signal. Our purpose is also to find two harmonic components using three methods. The plot of this signal is available in Fig. 5, its Fourier transform is in Fig. 6, short-time Fourier transform is given in Fig. 7 and its wavelet transform is in Fig. 7(a–c).

The Fourier transform of the 'noised' signal gives us precise information about the harmonic components in the signal, as for the 'pure' signal.

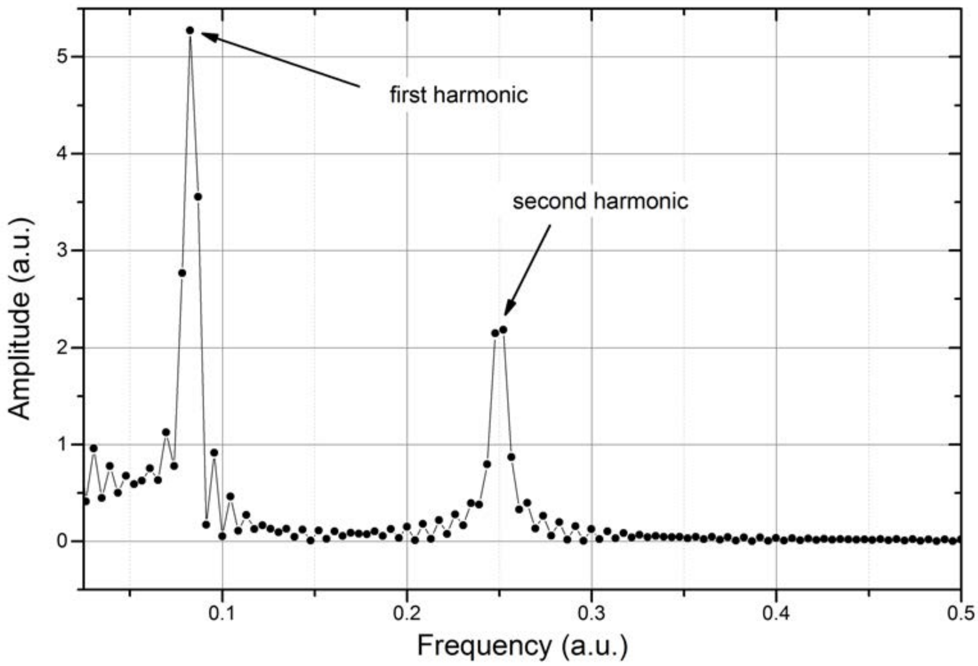


FIG. 2. Fourier transform of model 'pure' signal

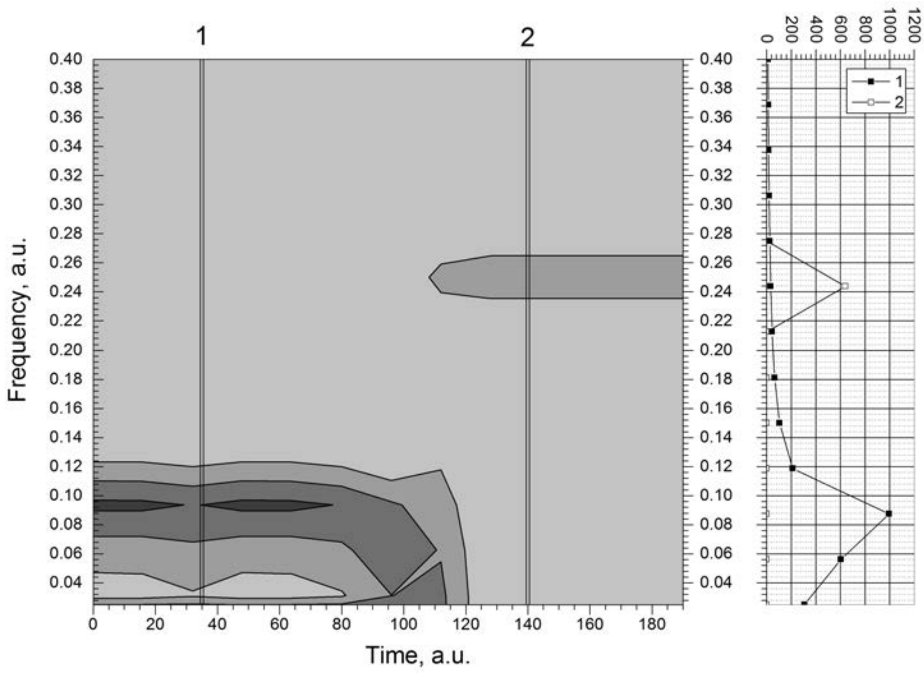
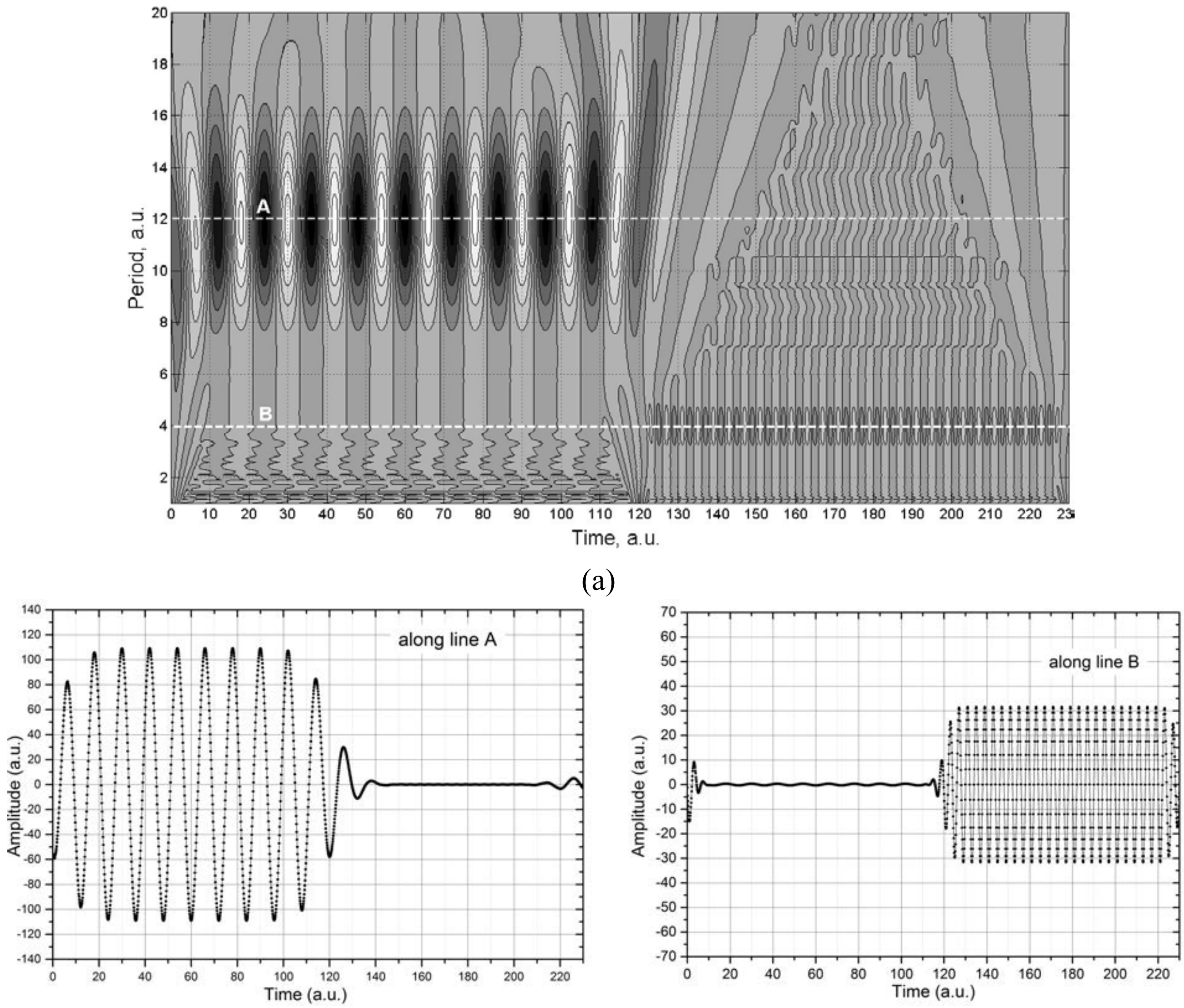


FIG. 3. Short time Fourier transform of model 'pure' signal and its cross-sections at $t = 30$ ticks and $t = 140$ ticks

STFT of model 'noised' signal (Fig. 7) allows us to detect two harmonic components with frequencies $\sim$ 0.09 a.u. and $\sim$ 0.245 a.u. that are very close to true values of these components.

Figures 8(a–c) shows the wavelet transform of the model 'noised' signal and its harmonic components (along dashed lines) that the signal contains.



(a)

(b) (c)

FIG. 4. (a) – Wavelet transform of model 'pure' signal. Dashed line shows harmonic components that form the signal; (b, c) – Cross-section of harmonic component along a dashed line A (b) and B (c) (fig. 4(a))

So, the wavelet transform of the 'noised' signal gives precise information about values of components' frequencies and common image of signal.

3. Experimental signal

This section refers to the application of these methods to the experimental result by analogy with signals from the first paragraph.

The experimental result is dynamics of the integrated intensity of Nuclear Magnetic Resonance (NMR) from water protons in magnetic field of Earth. There is a plot of this signal on Fig. 9.

Results of data fromy the Fourier transform analysis show us that the detection of the harmonic components in this signal is ambiguous (Fig. 10).

Using STFT (Fig. 11), we could suppose that the signal contains only one harmonic component with frequency equals ~ 0.02 a.u.

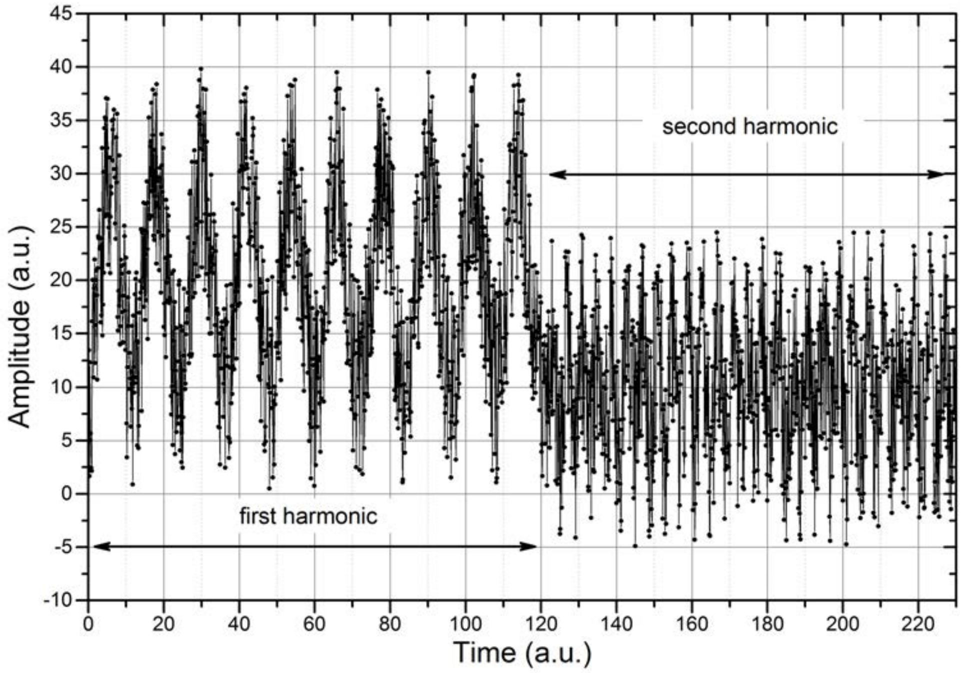


FIG. 5. Model 'noised' signal

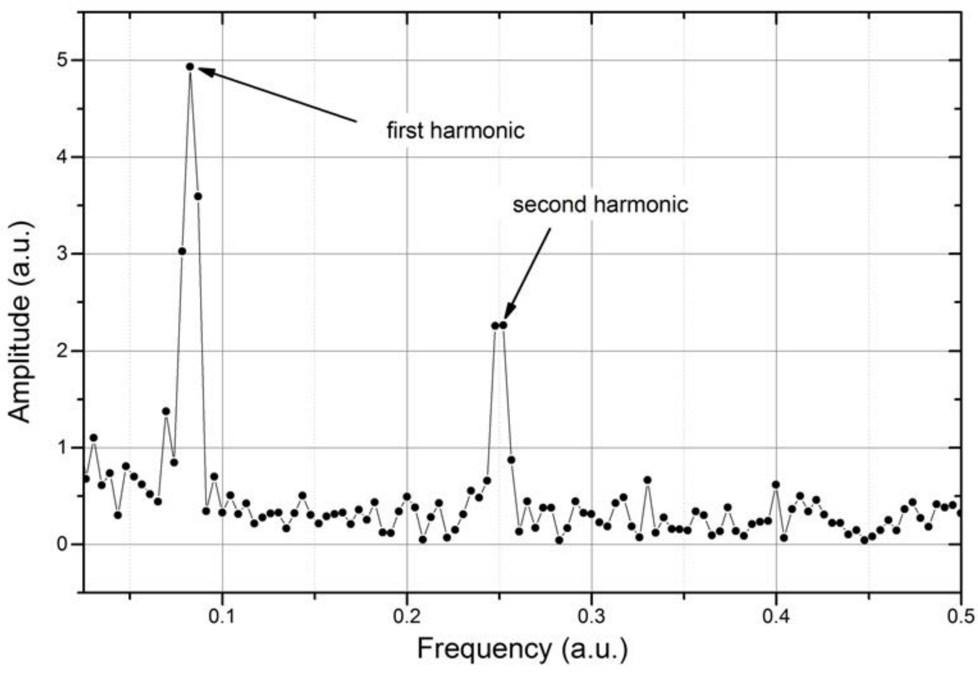


FIG. 6. Fourier transform of model 'noised' signal

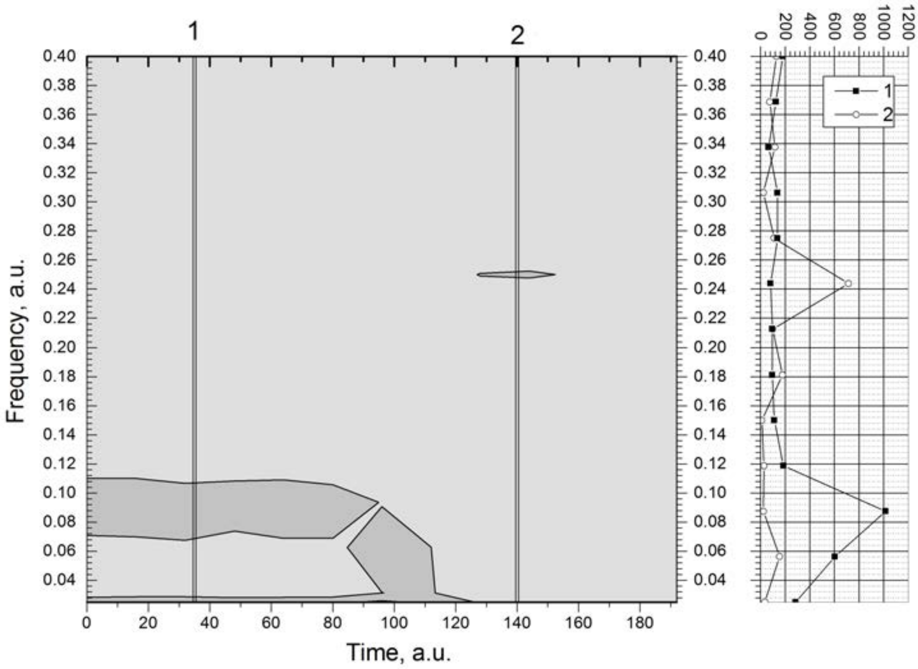


FIG. 7. Short-time Fourier transform with cross-sections of 'noised' signal

In contrast with the Fourier methods, the Wavelet transform allows us to detect the existence of quasi-harmonic components in the signal (Fig. 12(b,c)) with the periods equal ~ 20 a.u. and ~ 50 a.u. (frequencies ~ 0.05 and ~ 0.025 a.u. respectively). Wavelet analysis in this case gives us adequate description of the dynamics of quasi-harmonic processes that take place in the system.

The two-dimensional scan of one-dimensional process, where frequency and time are considered as two independent variables, allows us to analyze the properties of the studied process simultaneously in frequency- and in time-fields, which is very important for the analysis of many experiments.

4. Conclusion

As a result of the comparison of these three methods, we can say that the wavelet transform at least gives us some information that could be compared with the results of conventional Fourier methods. The application of this method to the experimental data shows us that the Wavelet transform:

- allows us to claim the hypothesis about existence of quasi-harmonic components in non-stationary (in time-field) signals with some frequencies (periods);
- gives us a full and precise image of the quasi-harmonic components' dynamics in signal.

So, almost all fields of science where the Fourier transform is a conventional method for analyzing experimental data, the Wavelet transform can be used as a higher quality method for finding quasi-harmonic components in any signals [6–9].

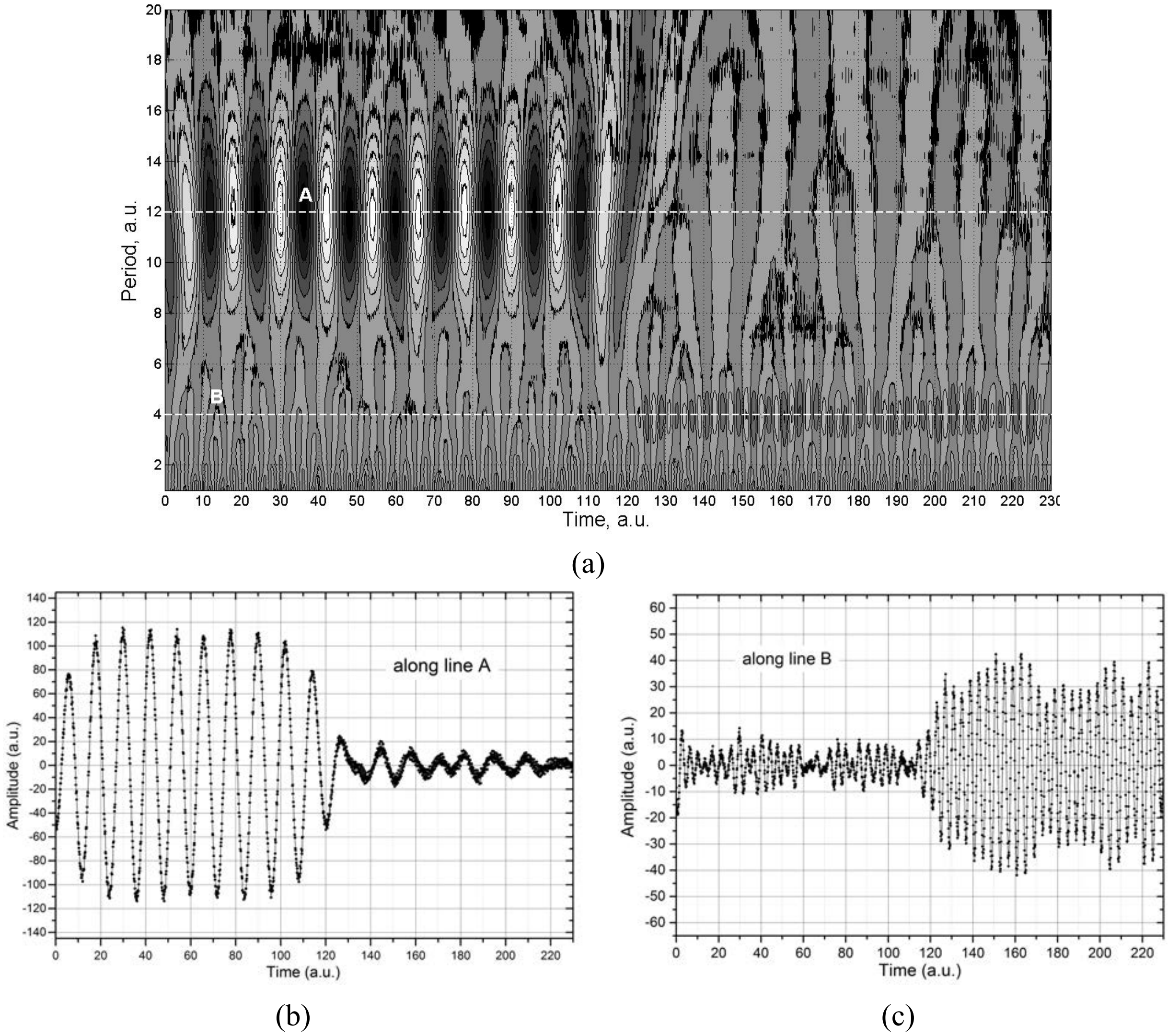


FIG. 8. (a) – Wavelet transform of 'noised' signal with dashed lines at harmonic components; (b, c) – Cross-section of harmonic component at line A (b) and B (c)

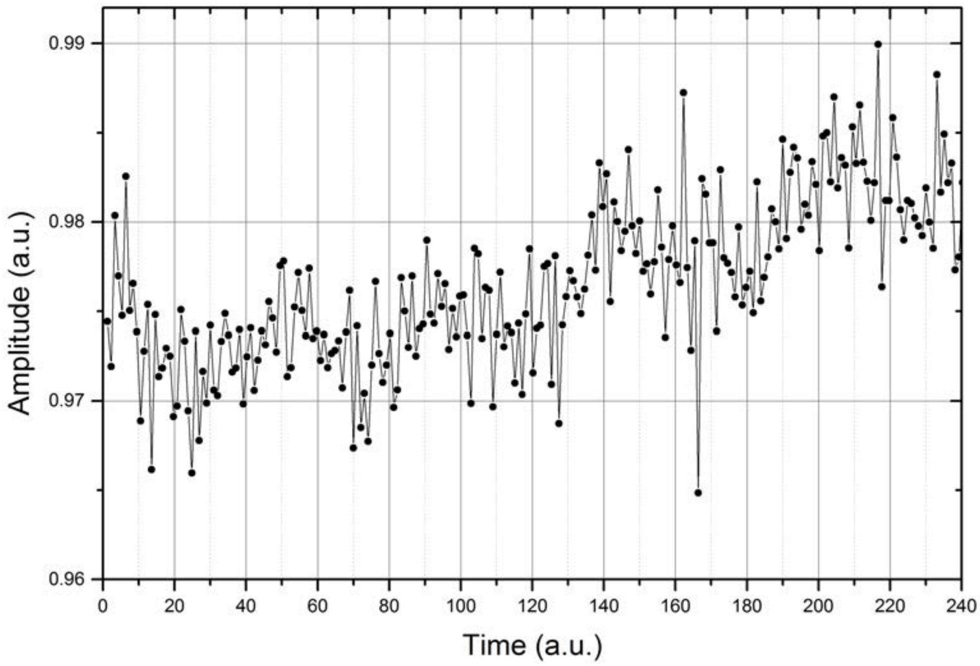


FIG. 9. Plot of experimental signal (the integrated intensity of Nuclear Magnetic Resonance signal from water protons in magnetic field of the Earth)

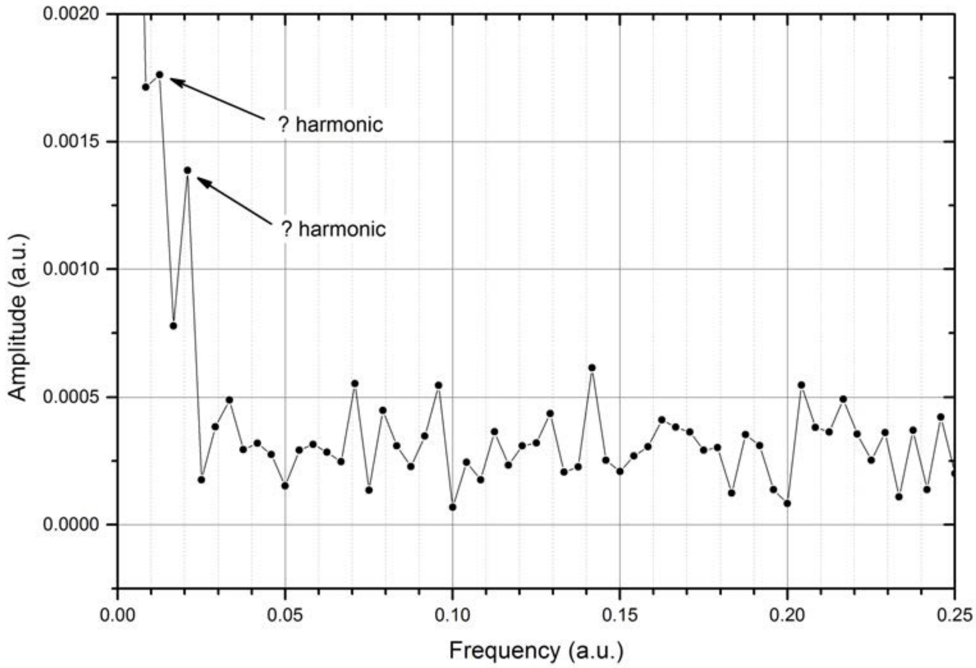


FIG. 10. Fourier transform of the experimental signal

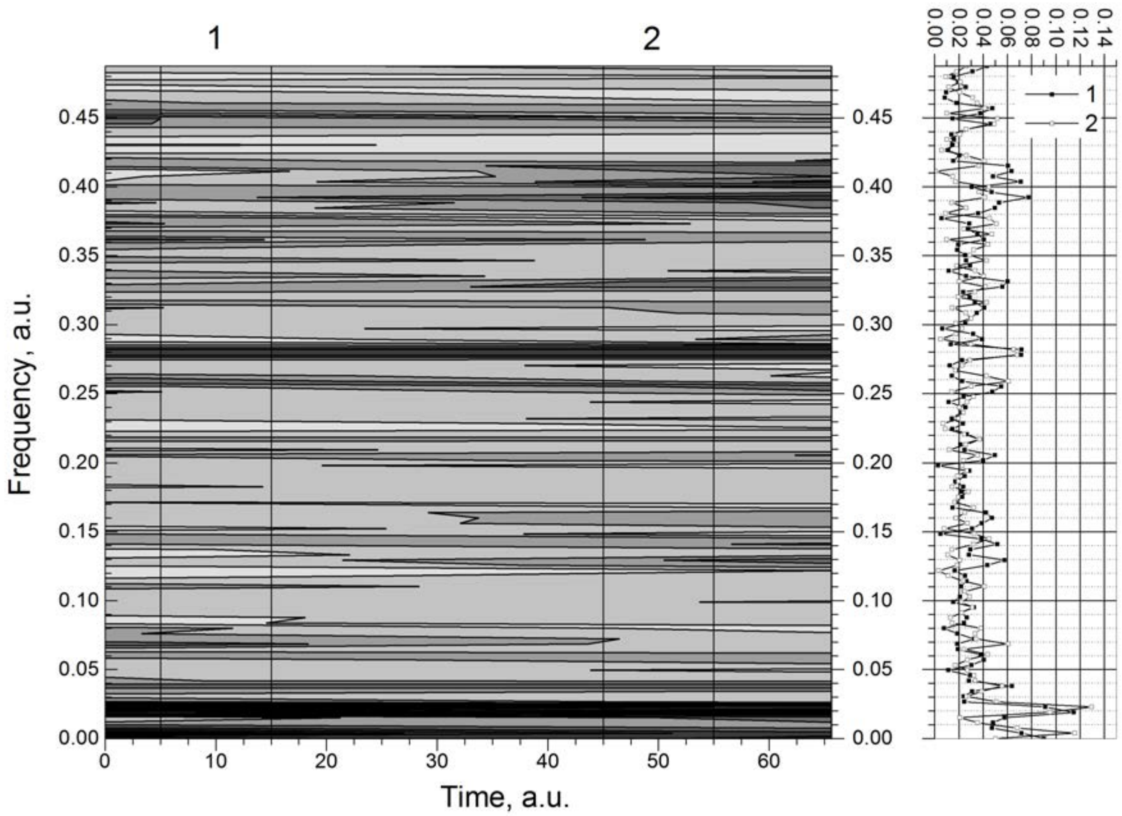
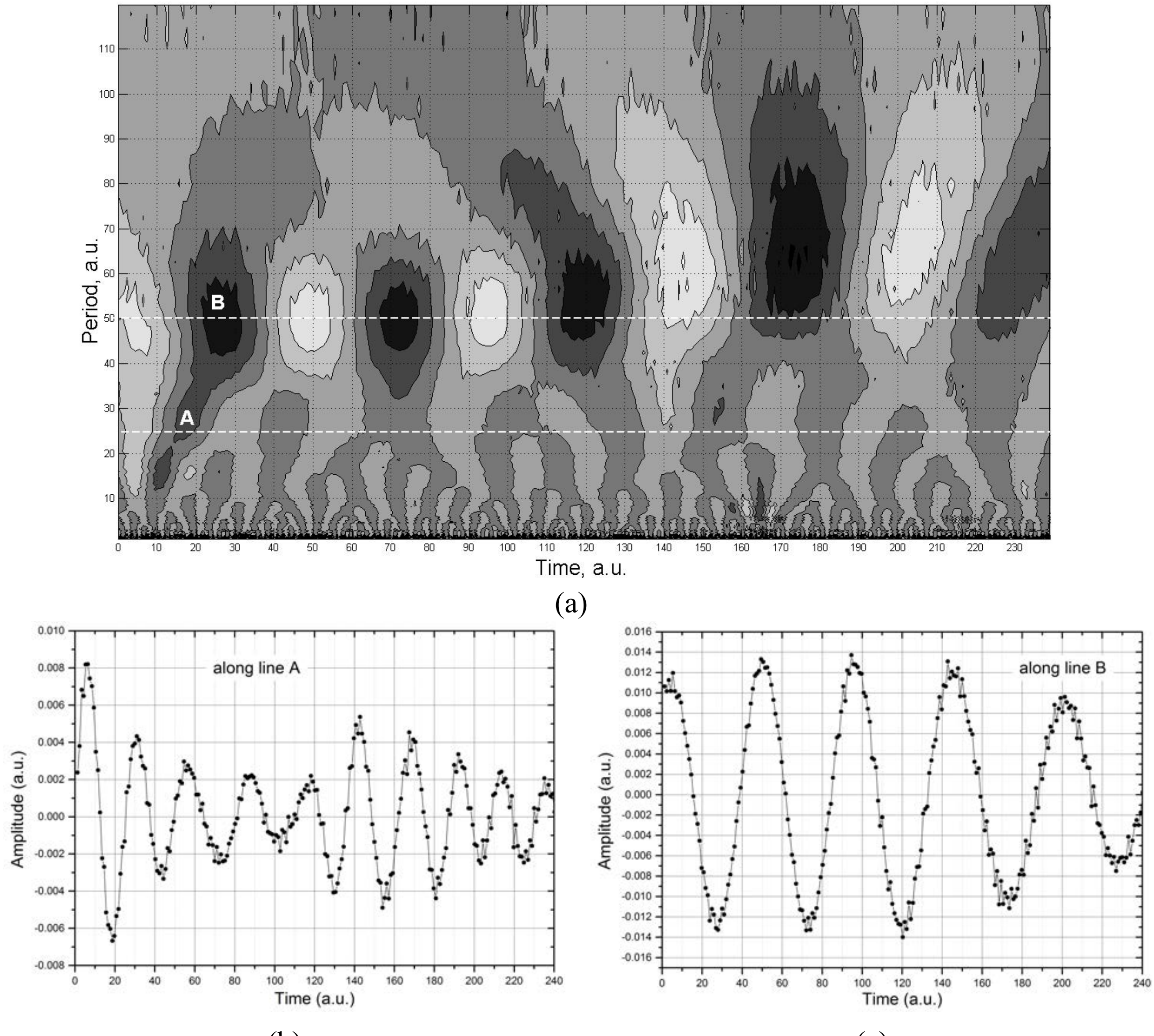


FIG. 11. Short-time Fourier transform of the experimental signal with its cross-sections



(a)

(b) (c)

FIG. 12. (a) – Wavelet transform of the experimental signal. Dashed lines show harmonic components that have been found in the experimental signal; (b, c) – Cross-section of harmonic component along dashed line A (b) and B (c)

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