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On construction of evolutionary operator for rectangular linear optical multiport

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The work of Knill et. al. (2001) established the possibility of nondeterministic realization of certain quantum logic operations using linear optical elements, ancilla photons and postselection techniques. It was also shown that any discrete unitary operator acting on N optical modes can be implemented by a triangular multiport device constructed from a series of beam splitters and phase shifters (see work of Reck, Zeilinger et. al., 1994). Here, we consider the rectangular linear optical multiport that is used for the probabilistic realization of unitary transformations on n qubits. This kind of linear optical scheme is suitable for probabilistic realization of unitary operators using ancilla photons and projective measurements. Qubits are encoded into the bosonic states of optical modes in two possible polarizations, and a number of ancilla photons and photodetectors are used for postselection of the qubits' state, based on the output of the detectors. We derive a procedure of evolutionary operator calculation for schemes of the considered type and present algorithms for their efficient computation on symmetric state space. We also provide complexities for different algorithms for the computation of evolutionary operator and estimate demands of resources in each case. A destructive Toffoli gate, acting on three qubits, using one ancilla photon and a photodetector, is implemented using schemes of the presented type.

Keywords: Quantum computing with linear optics, Projective measurements, Postselection, Photon detectors, Realization of unitary operator, Toffoli gate.

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1. Introduction

The usage of photons and linear optical elements, such as beam splitters, phase shifters, mirrors, presents the possibility of realizing controllable and scalable quantum computing [1]. Photons demonstrate easily observable, obvious quantum effects and are also able to maintain their coherent states for a long time. Construction of optical devices is comparably easy [1, 2]. For instance, these devices do not require low temperatures (except for realization of single-photon sources). In 2001, E. Knill and R. Lafflame published the work [2] which established the possibility of constructing quantum logic gates based on linear optics. Projective measurements and ancilla photons were used to introduce theinteraction between photons in order to implement non-deterministic realizations of certain quantum operations, like CNOT [3–5].

The experimental realization of unitary operators $U(N)$ transforming N optical modes using beam splitters and phase shifters was proposed in paper [6]. Here we present another type of optical multiport that transforms states of photon qubits and performs quantum teleportation by means of a number of ancilla photons and the detectors provided. The problem is to experimentally realize multiqubit unitary operators using this type of optical schemes. The main result, presented in this paper, is a procedure for calculating the evolutionary operator corresponding to mentioned optical schemes and efficient computational algorithms. This result introduces a step towards creating a method for the automatic construction of non-deterministic linear optical multiports corresponding to a given quantum circuit.

This paper has five sections. In section 2, we derive the evolutionary operator corresponding to a rectangular optical lattice having two-mode linear optical elements in its nodes. This can be implemented using beam splitters coupled by single-mode optical fibers. For the described network, we learn how to compute the corresponding single-particle evolutionary operator. In sections 3 and 5, we show how to efficiently compute multiparticle evolutionary operator. On the basis of the optical network considered in section 2 and inspired by CNOT gate, presented in [7], we propose a destructive Toffoli gate acting on three qubits (see Sec. 4).

2. Constructing evolutionary operator for rectangle linear optical multiport

Consider the rectangular grid having n rows and m columns (see Fig. 1) in which the nodes are polarizing beam splitters with no phase shift. Here, we compute the single-particle evolutionary operator for this optical grid. Taking into account only the transmitted and reflected modes, an evolutionary operator for this scheme can be implemented by scattering matrix that transforms the amplitudes of the $2(n+m)$-mode state. Each beam splitter is associated with a unitary transformation on four optical modes:

$$\begin{pmatrix} H_a' \\ V_a' \\ H_b' \\ V_b' \end{pmatrix} = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & \cos(\theta) & -\sin(\theta) \\ 0 & 0 & \sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} H_c' \\ V_d' \\ H_d' \\ V_c' \end{pmatrix}, \tag{1}$$

where (a,b) and (c,d) denote input and output spatial modes, H and V denote two possible polarizations and parameter θ describes reflectivity and transmittance of the beam splitter [6, 8].

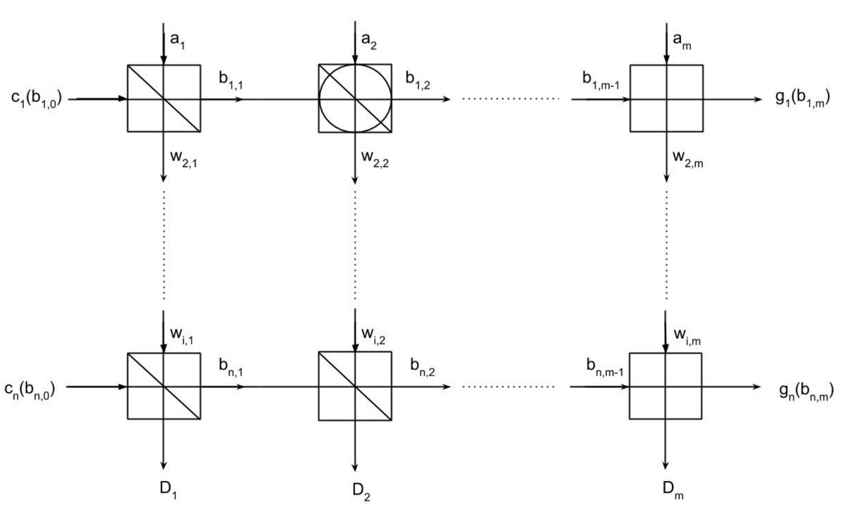

FIG. 1. Grid network with input ports $c_1 \dots c_n$, $a_1 \dots a_m$ and outputs $g_1 \dots g_n$, $D_1 \dots D_m$, for convenience of further calculations some ports are labeled twice

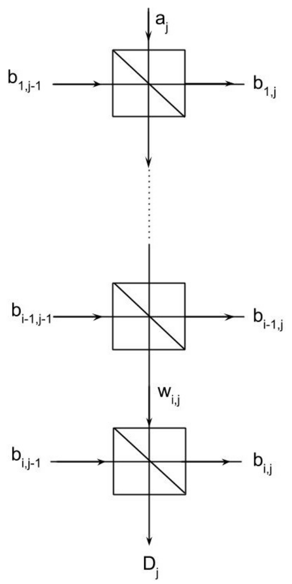

FIG. 2. Subscheme $N(1\dots i, j)$

First, we consider separately j-th column of length i (see Fig. 2). We denote this subscheme as $N(1\dots i, j)$ and find out how it affects the state of the photon. We imply that initially photon is in the modes $(Ha_j, Va_j; Hc_{1,j}, Vc_{1,j}, \dots Hc_{i,j}, Vc_{i,j})$, so the input state for $N(1\dots i, j)$ is the following $2(i+1)$-mode state (order of basis elements is preserved):

$$|\varphi_{1\dots i,j}\rangle_{in} = \varphi_{Ha_j}\left|H_{a_j}\right\rangle + \varphi_{Va_j}\left|V_{a_j}\right\rangle + \varphi_{Hb_{1,j}}\left|H_{b_{1,j}}\right\rangle + \varphi_{Vb_{1,j}}\left|V_{b_{1,j}}\right\rangle + \dots + \varphi_{Hb_{i,j-1}}\left|H_{b_{i,j-1}}\right\rangle + \varphi_{Vb_{i,j-1}}\left|V_{b_{i,j-1}}\right\rangle,$$

which is transformed into an output state of the form:

$$|\varphi_{1\dots i,j}\rangle_{out} = \varphi_{HD_j}\left|H_{D_j}\right\rangle + \varphi_{VD_j}\left|V_{D_j}\right\rangle + \varphi_{Hb_{1,j}}\left|H_{b_{1,j}}\right\rangle + \varphi_{Vb_{1,j}}\left|V_{b_{1,j}}\right\rangle + \dots + \varphi_{Hb_{i,j}}\left|H_{b_{i,j}}\right\rangle + \varphi_{Vb_{i,j}}\left|V_{b_{i,j}}\right\rangle,$$

assuming that finally the photon can be found in the modes $(HD_j, VD_j; Hb_{1,j}, Vb_{1,j} \dots Hb_{i,j}, Vb_{i,j})$. Input and output states of the particle corresponding to the transformation on $2i$ modes given by the subscheme $N(1\dots i-1, j)$ are respectively of the following form:

$$|\varphi_{1\dots i-1,j}\rangle_{in} = \varphi_{Ha_j}\left|H_{a_j}\right\rangle + \varphi_{Va_j}\left|V_{a_j}\right\rangle + \varphi_{Hb_{1,j-1}}\left|H_{b_{1,j-1}}\right\rangle + \varphi_{Vb_{1,j}}\left|V_{b_{1,j}}\right\rangle + \dots + \varphi_{Hb_{i-1,j-1}}\left|H_{b_{i-1,j-1}}\right\rangle + \varphi_{Vb_{i-1,j-1}}\left|V_{b_{i-1,j-1}}\right\rangle,$$

$$|\varphi_{1\dots i-1,j}\rangle_{out} = \varphi_{Hw_{i,j}}\left|Hw_{i,j}\right\rangle + \varphi_{Vw_{i,j}}\left|Vw_{i,j}\right\rangle + \varphi_{Hb_{1,j}}\left|H_{b_{1,j}}\right\rangle + \varphi_{Vb_{1,j}}\left|V_{b_{1,j}}\right\rangle + \dots + \varphi_{Hb_{i-1,j}}\left|H_{b_{i-1,j}}\right\rangle + \varphi_{Vb_{i-1,j}}\left|V_{b_{i-1,j}}\right\rangle.$$

For each subscheme, we write a transition matrix, that changes the basis of the input states to the basis of the output states and matches the corresponding scattering matrix. Let then $U^{(i-1,j)}$ be matrix of size $2(i-1)\times 2(i-1)$ corresponding to scattering operator of $N(1\dots i-1, j)$ so $|\psi_{1\dots i-1,j}\rangle = U^{(i-1,j)}\,|\varphi_{1\dots i-1,j}\rangle$. If we compare the $|\varphi_{1\dots i,j}\rangle_{in}$ and $|\varphi_{1\dots i-,j}\rangle_{in}$ states, then the latter doesn't have a spatial mode $b_{i,j-1}$ and has $w_{i,j}$ instead of D_j. Using this remark and assuming that element in the node (i,j) is associated with scattering matrix $T^{(i,j)}$ (see (1)) with matrix elements $\|t_{k,l}\|_{k,l=1}^{4}$, we get the following system:

$$\begin{pmatrix} \varphi_{Hw_{i,j}} \\ \varphi_{Vw_{i,j}} \\ \varphi_{Hb_{1,j}} \\ \varphi_{Vb_{1,j}} \\ \vdots \\ \varphi_{Hb_{i-1,j}} \\ \varphi_{Vb_{i-1,j}} \end{pmatrix} = U^{(i-1,j)} \begin{pmatrix} \varphi_{Ha_j} \\ \varphi_{Va_j} \\ \varphi_{Hb_{1,j-1}} \\ \varphi_{Vb_{1,j-1}} \\ \vdots \\ \varphi_{Hc_{i-1,j-1}} \\ \varphi_{Vc_{i-1,j-1}} \end{pmatrix}, \quad \begin{pmatrix} \varphi_{HD_j} \\ \varphi_{VD_j} \\ \varphi_{Hb_{i,j}} \\ \varphi_{Vb_{i,j}} \end{pmatrix} = T^{(i,j)} \begin{pmatrix} \varphi_{Hw_{i,j}} \\ \varphi_{Vw_{i,j}} \\ \varphi_{Hb_{i,j-1}} \\ \varphi_{Vb_{i,j-1}} \end{pmatrix},$$

from which the recurrent relation for the scattering matrix $U^{(i,j)}$ of size $2(i+1)\times 2(i+1)$, that transforms $|\varphi_{1\dots i,j}\rangle_{in}$ into $|\varphi_{1\dots i,j}\rangle_{out}$ is:

$$U^{(i,j)} = \begin{pmatrix} t_{1,1} & t_{1,2} & 0 & \dots & 0 & t_{1,3} & t_{1,4} \\ t_{2,1} & t_{2,2} & 0 & \dots & 0 & t_{2,3} & t_{2,4} \\ 0 & 0 & 1 & & \mathbf{0} & 0 & 0 \\ \vdots & \vdots & & \ddots & & \vdots & \vdots \\ 0 & 0 & \mathbf{0} & & 1 & 0 & 0 \\ t_{3,1} & t_{3,2} & 0 & \dots & 0 & t_{3,3} & t_{3,4} \\ t_{4,1} & t_{4,2} & 0 & \dots & 0 & t_{4,3} & t_{4,4} \end{pmatrix} \left(\begin{array}{ccc|cc} & & & 0 & 0 \\ & U^{(i-1,j)} & & \vdots & \vdots \\ & & & 0 & 0 \\ \hline 0 & \dots & 0 & 1 & 0 \\ 0 & \dots & 0 & 0 & 1 \end{array}\right), \tag{2}$$

where $U^{(1,j)} = T^{(1,j)}$. Obviously, each factor of expression (2) can be transformed to block-diagonal form, where each block is unitary. Then, we conclude that $U^{(i,j)}$ is also unitary, as expected.

To finish the construction of the grid scattering matrix, we consider a rectangular sub-scheme having i rows and j columns with top-left angle matching the one of the large scheme. Let it be denoted as $N(1\ldots i, 1\ldots j)$ (see Fig. 1). The corresponding input and output states are respectively of the form:

$$
\begin{gathered}
|\varphi_{1\ldots i,1\ldots j}\rangle_{in} = \varphi_{Ha_j}|Ha_j\rangle + \varphi_{Va_j}|Va_j\rangle + \ldots + \varphi_{Ha_1}|Ha_1\rangle + \varphi_{Va_1}|Va_1\rangle + \varphi_{Hc_1}|\rangle + \varphi_{Vc_1}|\rangle + \\
\ldots + \varphi_{Hc_i}|Hc_i\rangle + \varphi_{Vc_i})|Vc_i\rangle, \\
|\varphi_{1\ldots i,1\ldots j}\rangle_{out} = \varphi_{HD_j}|HD_j\rangle + \varphi_{VD_j}|VD_j\rangle + \ldots + \varphi_{VD_1}|VD_1\rangle + \varphi_{Hb_{1,j}}|Hb_{1,j}\rangle + \\
\varphi_{Vb_{1,j}}|Vb_{1,j}\rangle + \ldots + \varphi_{Hb_{i,j}}|Hb_{i,j}\rangle + \varphi_{Vb_{i,j}}|Vb_{i,j}\rangle.
\end{gathered}
$$

If $Y^{(i,j-1)}$ is the matrix of the operator corresponding to $N(1\ldots i, 1\ldots j-1)$, then using the recurrence for calculating $U^{(i,j)}$, we get the following system:

$$
\begin{pmatrix} \varphi_{HD_{j-1}} \\ \varphi_{VD_{j-1}} \\ \cdots \\ \varphi_{HD_1} \\ \varphi_{VD_1} \\ \varphi_{Hb_{1,j-1}} \\ \varphi_{Vb_{1,j-1}} \\ \cdots \\ \varphi_{Hb_{i,j-1}} \\ \varphi_{Vb_{i,j-1}} \end{pmatrix} = Y^{(i,j-1)} \begin{pmatrix} \varphi_{Ha_{j-1}} \\ \varphi_{Va_{j-1}} \\ \cdots \\ \varphi_{Va_1} \\ \varphi_{Hc_1} \\ \varphi_{Vc_1} \\ \cdots \\ \varphi_{Hc_i} \\ \varphi_{Vc_i} \end{pmatrix}, \quad \begin{pmatrix} \varphi_{HD_j} \\ \varphi_{VD_j} \\ \varphi_{Hb_{1,j}} \\ \varphi_{Vb_{1,j}} \\ \vdots \\ \varphi_{Hb_{i,j}} \\ \varphi_{Vb_{i,j}} \end{pmatrix} = U^{(i,j)} \begin{pmatrix} \varphi_{Ha_j} \\ \varphi_{Va_j} \\ \varphi_{Hb_{1,j-1}} \\ \varphi_{Vb_{1,j-1}} \\ \vdots \\ \varphi_{Hb_{i,j-1}} \\ \varphi_{Vb_{i,j-1}} \end{pmatrix}
$$

from which, we eventually derive the following recurrent relation for matrix of operator $Y^{(i,j)}$ of size $2(i+j) \times 2(i+j)$:

$$
Y^{(i,j)} = \left(\begin{array}{c|c|c} \mathbf{O} & \begin{matrix} 1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & 1 \end{matrix} & \mathbf{O} \\ \hline U_1^{(i,j)} & \mathbf{O} & U_2^{(i,j)} \end{array}\right) \left(\begin{array}{c|c} \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \mathbf{O} \\ \hline \mathbf{O} & Y^{(i,j-1)} \end{array}\right), \tag{4}
$$

where $U_1^{(i,j)} = U^{(i,j)}[1\ldots 2(i+1); 1, 2]$ and $U_2^{(i,j)} = U^{(i,j)}[1\ldots 2(i+1); 3\ldots 2(i+1)]$ (here we denote submatrix of matrix M with rows $r_1, \ldots, r_n$ and columns $c_1 \ldots, c_m$ by $[r_1, \ldots, r_n; c_1 \ldots, c_m]$). Following the same reasoning as for $U^{(i,j)}$, it is also obvious that $Y^{(i,j)}$ is unitary. Hereby, for scattering matrix of the large scheme (see Fig. 1), it is enough to compute $Y(m, n)$.

3. Multiparticle systems

Consider the scheme on Fig. 1 and an arbitrary state of n photons incident to ports $c_1, ...c_n$:

$$|\varphi_0\rangle = \sum_{\rho\in\Pi_l} \varphi(\rho)\left|\rho^1_{c_1}\dots\rho^n_{c_n}\right\rangle, \quad \sum_{\rho\in\Pi_n} |\varphi(\rho)|^2 = 1,$$

$$\Pi_k = \{H, V\}^{\times k}. \tag{5}$$

which is a state of n qubits encoded into photon polarization modes H and V. Suppose we define a state of m ancilla photons incident to ports $a_1, ..., a_m$ as:

$$|\varphi_A\rangle = \bigotimes_1^m \varphi_{H_{a_j}}\left|H_{a_j}\right\rangle + \varphi_{V_{a_j}}\left|V_{a_j}\right\rangle,$$

and input state as $|\varphi_{in}\rangle = Sym(\varphi_0 \otimes \varphi_A)$. Then, we can write the output in the following form (see (5)) :

$$|\psi_{out}\rangle = \sum_{\substack{\tau\in T_{m'}\\ P\in\Pi_{m'}}} \left(\mu(\tau)\left|P^1\dots P^{m'}\right\rangle_\tau \sum_{\rho\in\Pi_{n'}} \psi(\rho,P,\tau)\left|\rho^1\dots\rho^{n'}\right\rangle\right) + c_\perp\left|\psi_\perp\right\rangle,$$

$$T_k = \{\{D_{s_i}\}_1^k|\ s_i\in 1,\dots,k;\ s_{i+1}\geq s_i\} \tag{6}$$

where $|\psi_\perp\rangle$ is a state orthogonal to those having only one photon in each of modes $g_1,\dots,g_{n'}$. The product of state vectors is defined as their symmetrization:

$$\left|P^1\dots P^{m'}\right\rangle_\tau\left|\rho^1\dots\rho^{n'}\right\rangle = \frac{\sum_{\sigma\in\Sigma_{\rho,P,\tau}}|\sigma_1\rangle\dots|\sigma_{n'+m'}\rangle}{\sqrt{|\sigma|!}},$$

$$\Sigma_{\rho,P,\tau} = \{\pi(P^1_{D\tau_1},\dots,P^{m'}_{D\tau_{m'}},\rho^1_{g_1},\dots,\rho^{n'}_{g_{n'}})\}.$$

Evolutions generated by linear optical elements preserve the photon total number so $n' = n + m - m'$. Suppose that we detect photons in modes $D_1,\dots,D_{m'}$. As seen from (6), with the appropriate choice of ancilla photon states and postselection based on polarization and quantity of photons on each detector, we can project the output in modes $g_1,\dots,g_{n'}$ into the state having $\sum_{\rho\in\Pi_{n'}}|\psi(\rho)|^2 = 1$, thus meaning that we received a state of n' qubits. The scheme is considered non-destructive in case $n = n'$ and destructive if $n' < n$ as we collapsed the state of one or more qubits.

Let scheme with evolutionary operator U transform state of $q = n + m$ input photons. In order to obtain the transformation for bosonic states, we compute the restriction of the multiparticle evolutionary operator $U^{\otimes q}$ on its invariant subspace Sp_i. Then, the input state space is given by:

$$Sp_0 = \operatorname{span}\left(\{|X1_{m_1}\rangle\otimes|X2_{m_2}\rangle\otimes\dots\otimes\left|Xq_{m_q}\right\rangle|Xi\in\{H,V\},\right.$$
$$\left. m_j\in\{c_1,\dots,c_n,a_1,\dots,a_m\}\,,\, i,j = 1\dots q\}\right), \tag{7.1}$$

where c_j are spatial modes corresponding to input ports of the scheme. The restriction operator can be computed as follows, assuming that matrix of $U^{\otimes q}$ is of size $2^q\times 2^q$ and $\dim(Sp_i) = k$:

$$U_i^{\otimes q} = R_i^\dagger U^{\otimes n} R_i, \tag{7.2}$$

where R_i is a $2^q\times k$ matrix which columns represent basis vectors of Sp_i. It can also easily be shown that for invariant subspaces, the restriction operator is unitary.

4. Constructing Toffoli gate using linear optics

Inspired by non-destructive CNOT gate presented in work [7] and with use of algorithms given in section 5, it became possible to construct a scheme implementing quantum Toffoli transformation. Although it is known that at least five two-qubit gates are necessary for a non-destructive Toffoli gate [9]. It turns out that for the destructive Toffoli gate on three bosonic qubits only eight polarization modes are needed (see Fig. 3). We use an ancilla photon in equal superposition state and an arbitrary three-qubit state on left input ports of the following scheme:

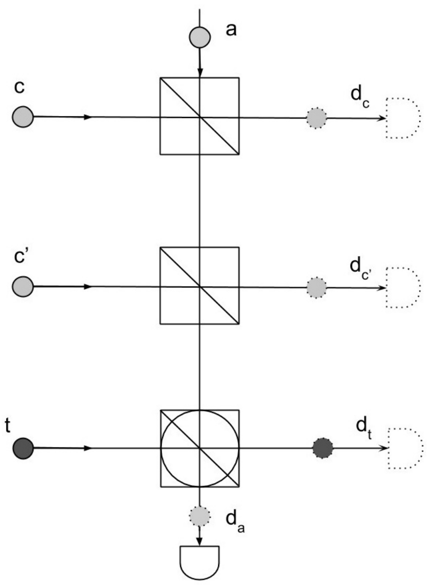

FIG. 3. Destructive Toffoli gate

$$|\psi_a\rangle = \frac{1}{\sqrt{2}}|H_a\rangle + \frac{1}{\sqrt{2}}|V_a\rangle,$$
$$|\psi_0\rangle = \alpha_1|H_c H_{c'} H_t\rangle + \alpha_2|H_c H_{c'} V_t\rangle + \ldots + \alpha_8|V_c V_{c'} V_t\rangle.$$

The initial state $|\psi_{in}\rangle = Sym(\psi_0 \otimes \psi_A)$ is transformed into the output state, which can be written in the form of equation 6 (see section 3):

$$|\psi_{out}\rangle = \frac{1}{2\sqrt{2}}|H_{d_a}\rangle\left(\alpha_1\left|H_{d_c}H_{d_{c'}}H_t\right\rangle + \alpha_2\left|H_{d_c}H_{d_{c'}}V_{d_t}\right\rangle\right) +$$
$$\frac{1}{2\sqrt{2}}|V_{d_a}\rangle\left(\alpha_3|H_{d_c}V_{d_c}H_{d_t}\rangle + \alpha_4|H_{d_c}V_{d_c}V_{d_t}\rangle\right) +$$
$$\frac{1}{2\sqrt{2}}|H_{d_a}\rangle\left(\alpha_5\left|H_{d_{c'}}V_{d_{c'}}H_{d_t}\right\rangle + \alpha_6\left|H_{d_{c'}}V_{d_{c'}}V_{d_t}\right\rangle\right) +$$
$$\frac{1}{2\sqrt{2}}|H_{d_a}\rangle\left(\alpha_7\left|V_{d_c}V_{d_{c'}}V_{d_t}\right\rangle + \alpha_8\left|V_{d_c}V_{d_{c'}}H_{d_t}\right\rangle\right) + \frac{\sqrt{2}}{2}|\psi_\perp\rangle.$$

In the output state, the probability amplitudes of vectors from the following sets are equal to 0 :

$$S_1 = \{\left|H_{d_c}V_{d_{c'}}H_{d_t}\right\rangle, \left|H_{d_c}V_{d_{c'}}V_{d_t}\right\rangle, \left|V_{d_c}H_{d_{c'}}V_{d_t}\right\rangle, \left|V_{d_c}H_{d_{c'}}H_{d_t}\right\rangle\},$$
$$S_2 = \{|V_{d_c}H_{d_c}H_{d_t}\rangle, |V_{d_c}H_{d_c}V_{d_t}\rangle, |V_{d_c}V_{d_c}H_{d_t}\rangle, |V_{d_c}V_{d_c}V_{d_t}\rangle, |H_{d_c}H_{d_c}H_{d_t}\rangle, |H_{d_c}H_{d_c}V_{d_t}\rangle\},$$
$$S_3 = \{\left|V_{d_{c'}}H_{d_{c'}}H_{d_t}\right\rangle, \left|V_{d_{c'}}H_{d_{c'}}V_{d_t}\right\rangle, \left|V_{d_{c'}}V_{d_{c'}}H_{d_t}\right\rangle, \left|V_{d_{c'}}V_{d_{c'}}V_{d_t}\right\rangle, \left|H_{d_{c'}}H_{d_{c'}}H_{d_t}\right\rangle, \left|H_{d_{c'}}H_{d_{c'}}V_{d_t}\right\rangle\},$$

therefore, when the following events occur:

- A_1 = {Each spatial mode has only one photon, photodetector registers single H-polarized photon}, $P(A_1) = \frac{1}{4}$,
- A_2 = {2 photons in mode d_c, photodetector registers single V-polarized photon}, $P(A_2) = \frac{1}{8}$,
- A_3 = {2 photons in mode $d_{c'}$, photodetector registers single H-polarized photon}, $P(A_3) = \frac{1}{8}$,

we get a quantum Toffoli transformation preserving the state of the controlled qubit, which succeeds with probability $P(A_1) + P(A_2) + P(A_3) = \frac{1}{2}$. The main difficulty concerning implementation of this scheme lies in necessity to count photons using photodetectors, although such devices seem to be available now.

5. Algorithm realization

We present an algorithm for efficient calculation of the evolutionary operator corresponding to a system of p particles and rectangle linear optical multiport considered in section 2 (see Supplementary Materials). This is an optimization of the straightforward algorithm achieved by means of sparse matrices and lazy tensor product calculation. It should be noted that when running procedure 2 (see Supplementary Materials) in parallel, due to the fact that first three nested loops are independent, it becomes possible to obtain a linear performance increase that is proportional to the number of processors in use, whereas number of threads can grow up to $Size(Sp)^2(p!)$.

6. Conclusion

In this paper, a rectangular linear optical grid was considered. Similar to the triangular multiport presented in work [6], in order to implement unitary transformation of N optical modes a number of beam splitters proportional to N^2 is required. However, the optical networks considered in the current paper are comparably easy to construct and permit the usage of a sufficient number of ancilla photons and detectors to realize quantum teleportation. The formal procedure of evolutionary operator calculation, corresponding to a given rectangular optical network, introduces a step towards the efficient experimental realization of an arbitrary unitary operator. At present, there are also several other problems of interest: construction of quantum logic gates based on linear optics, the problem of simulation of quantum optical gates with noise and implementation flaws being taken into account. Substantial problems that prevent these schemes from being used in practice are qubit phase drifts and photon loss, which is why effective correcting codes for qubit states are necessary. Unsolved technical problems are the construction of an extremely sensitive and low-latency photodetector and reliable single-photon sources.

7. APPENDIX

Here we present procedures for calculation of evolutionary operator corresponding to system of p particles and rectangle linear optical multiport (see alg. 1, 2, 3).

- @Transition_Matrix_Sparsed (see alg. 1) calculates matrix R_i and stores it in a sparse matrix.
- @Kron_And_Mult_Sparsed (see alg. 2) calculates $U_i^{\otimes n}$ (see article, eq. 7.2).
- @Lazy_kron (see alg. 3) is an auxiliary function for @Kron_And_Mult_Sparsed that calculates tensor product matrix elements.

Complexities of simple and optimized procedures for calculation of evolutionary operator are summarized in table 1. In table 2 dimentions of most important subspaces of Sp_0 (see article, eq. (7.1)) are given. In table 3 we present mean time and space resource demands for calculation of evolutionary operator on symmetric subspace.

TABLE 1. Complexity of procedures depending on subspace size $Size(Sp)$ and number of particles p

Procedure	Time	Space
Transition_Matrix_Sparsed	$O\left(Size(Sp)(p+1)!\right)$	$O\left(3(2p)^p\right)$
Transition_Matrix_Simple	$O\left(Size(Sp)\left(1+(p+1)!\frac{(1-(2p)^p)}{1-2p}+p!(2p)^p\right)\right)$	$O(Size(Sp)(2p)^p)$
Kron_And_Mult_Sparsed	$O\left(Size(Sp)^2(p!)^2p\right)$	$O\left(Size(Sp)^2\right)$
Kron_And_Mult_Simple	$O\left(\frac{1-(2p)^{2p}}{1-(2p)^2}+Size(Sp)^4(2p)^{2p}\right)$	$O\left((2p)^{2p}+Size(Sp)(2p)^p\right)$

TABLE 2. Subspaces of Sp_0.

Subspace (Sp)	Size (Size(Sp))
Full	$(2p)^p$
Asym	$\binom{2p}{p}$
Sym	$\binom{3p-1}{p}$
Coinc	2^p

TABLE 3. Mean time and space for evolutionary operator calculation on symmetric subspace using an approx. 50 GFLOPS processor

	Time (s)		Space (MB)	
p	simple	sparsed	simple	sparsed
3	0.46	0.034	0.224	0.014
4	$2\cdot10^4$	25.09	69.16	0.4623
5	$1.6\cdot10^{10}$	$2.9\cdot10^4$	$38.9\cdot10^3$	16.43
6	$2.09\cdot10^{16}$	$4.8\cdot10^7$	$34.15\cdot10^6$	618.45

Algorithm 1 Transition_Matrix_Sparsed

Require: $p \geq 3,\ Sp$ {args: p for number of particles, matrix Sp which rows represent basis vectors of a given subspace. In MATLAB environment function that generates Sp for symmetric subspace may look like [a]

$$Sp\ =\ combinator(2*p,\ p,\ 'c',\ 'r').$$

}
$idx, jdx, vals \leftarrow array((2p)^p)$
$cnt \leftarrow 0$
for $i = 1$ **to** $len(Sp)$ **do**
  $Perms \leftarrow uniqueperms(Sp(i))$
  for $j = 1$ **to** $len(Perms)$ **do**
    $pos \leftarrow 1$
    for $k = 1$ **to** p **do**
      $pos \leftarrow l*(pos-1)+Perms(j,k)$
    end for
    $cnt \leftarrow cnt + 1$
    $idx(cnt) \leftarrow i$
    $jdx(cnt) \leftarrow pos$
    $vals(cnt) \leftarrow 1/sqrt(len(Perms))$
  end for
end for
$T = make\_sparse\_matrix(idx, jdx, vals)$

[a] `http://www.mathworks.com/matlabcentral/fileexchange/24325-combinator-combinations-and-permutations`

Algorithm 2 Kron_And_Mult_Sparsed

Require: $p \geq 3,\ U,\ Sp$ {args: p for number of particles, U for single particle evolutionary operator (see article, eq. 4), and matrix Sp for basis vectors of given subspace (see 1)}
$T \leftarrow Transition\_Matrix\_Sparsed(Sp, p)$
$m \leftarrow len(Sp)$
$Ur \leftarrow matrix(m, m)$
for $i = 1$ **to** m **do**
  $tmp\_norm\_i \leftarrow T(i, 1)$
  for $j = 1$ **to** m **do**
    $tmp\_norm\_j \leftarrow T(j, 1)$
    for $r = 1$ **to** $len(T(i))$ **do**
      $crj \leftarrow 0$
      for $r = 1$ **to** $len(T(j))$ **do**
        $up\_elem \leftarrow Lazy\_kron(T(i,r), T(j,l), p, U)$
        $crj \leftarrow crj + up\_elem * tmp\_norm$
      end for
      $Ur(i,j) = Ur(i,j) + tmp\_norm\_j * crj$
    end for
  end for
end for
return Ur

Algorithm 3 Lazy_kron

Require: $1 \leq i, j \leq (2p)^p, p \geq 3,\ U$ {args: i, j for row and column of element to be calculates, p for tensor product order and U for single-particle evolutionary operator (see article, eq. 4)}
$res \leftarrow 1$
while $p > 0$ **do**
  $ic, jc \leftarrow i\ \%\ rows(U), j\ \%\ cols(U)$
  if ic is 0 **then**
    $ic \leftarrow rows(U)$
  end if
  if jc is 0 **then**
    $jc \leftarrow cols(U)$
  end if
  $res \leftarrow res * U(ic, jc)$
  $i \leftarrow ceil(i/rows(U))$
  $j \leftarrow ceil(j/cols(U))$
  $p \leftarrow p - 1;$
end while
return res

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