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Temperature dependence of the optical fiber cable parameters in subcarrier wave quantum communication systems

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A common approach to establishing long-distance synchronization links in quantum communication (QC) systems is based on using optical signals transmitted in cables, where they decay and are distorted. It is necessary to evaluate the transformation of the signal parameters during propagation and their influence on the QC systems. We investigate the temperature dependence of the synchronization signal phase of a subcarrier wave quantum communication system (SCWQC) in optical fiber cables. A temperature model was created in order to determine the signal phase delay in the cable. We estimate the influence of daily temperature fluctuations on the phase delay in ground- and air-based cables. For systems operating with ground-based cables, they do not have any significant impact on the synchronization of the signal phase. However, for systems operating through air-based cables, phase adjustment is required every 158 ms for stable operation. This allowed us to optimize the parameters for a calibration procedure of a previously-developed SCWQC system, increasing the overall sifted key generation rate.

Keywords: quantum communications, clock synchronization, temperature dependence of the signal.

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1. Introduction

During the last two decades, significant progress has been made in the development of experimental quantum communication systems (QC) [1, 2] which could have potential future use in protecting confidential data by quantum key distribution technique (QKD) [1]. This has led to increased interest in their integration into telecommunication infrastructure [1,2]. One of the promising approaches to QC in optical fibers is subcarrier wave quantum communication system (SCWQC) [1].

In practical QC, systems several technological challenges still remain, including synchronization of their receiver and transmitter devices by precisely controlling the phase of high-frequency electrical modulating signals. A common approach to establishing long-distance synchronization links in QC is based on using optical signals transmitted in a separate channel [3, 4]. However, these synchronization signals decay and are distorted during transmission through the fibers, therefore it is necessary to evaluate transformation of their parameters, its influence on the QC system operation, and to develop methods of compensating for the negative effects.

This paper investigates the temperature dependence of the synchronization signal in optical fiber in ground- and air-based cables. A temperature model is created in order to determine the signal phase delay in the cable. We estimate the influence of daily temperature fluctuations on the phase delay in ground- and air-based cables. Finally, we apply the calculation results to a synchronization system of a previously developed experimental SCWQC device [5–8] in order to optimize its sifted key [1] generation rate.

2. Synchronization system

2.1. Subcarrier wave quantum communication system clock synchronization problem

The main parameter that determines the characteristics of QC and QKD systems is QBER. It is defined as the ratio between the erroneous bits and the total number of received bits. QBER is the main factor that limits the maximal possible distance of secure quantum key distribution [1]. For the widely-used BB84 protocol family, the limiting QBER value is about 11 % [1].

Two main factors contribute to the QBER value: the signal visibility in the quantum channel and the dark count of the detector:

$$\mathrm{QBER} = \frac{1-V}{2} + \frac{p}{4\mu\eta \cdot 10^{(-\alpha L-\beta)/10}},$$

where V is interference pattern visibility, β represents the losses in the receiver module, p is the dark count probability per bit, α is the optical fiber attenuation coefficient at the central wavelength of the photon source, L is the optical fiber length and η is the detection efficiency. Visibility value represents the probability of photon out-of-phase detection and therefore depends on the quality of optical phase matching of the transmitter and receiver phase modulators used for encoding qubits. Clearly, it is always beneficial to maximize the quantum channel visibility. In modern practical QC systems [3, 4, 6–8] the achieved visibility value is usually around 98 – 99 %. As is shown on Fig. 1, such visibility values in SCWQC system are correspond to maximum modulating signal phase mismatch $\Delta\varphi = 0.043$.

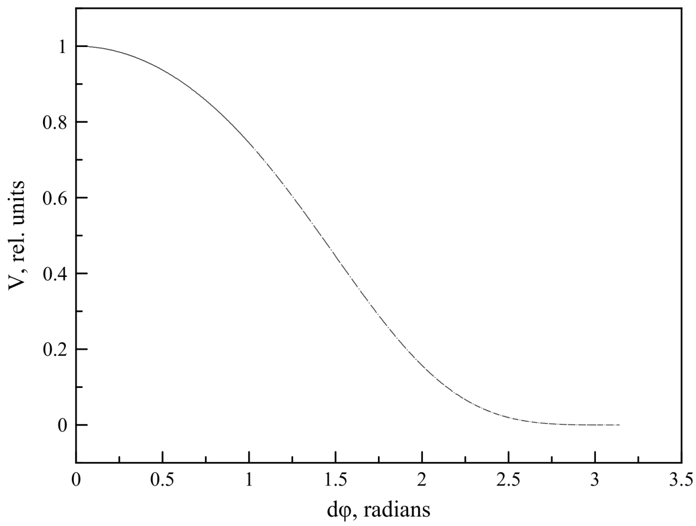

FIG. 1. Signal phase mismatch dependence of visibility

2.2. Subcarrier wave quantum communication system clock synchronization

Since phase matching between the transmitter (Alice) and receiver (Bob) is defined by clock signals from respective voltage controlled oscillators (VCO), preservation of the optical synchronization signal phase appears crucial to achieving high quantum signal visibility. The uncontrollable synchronization signal phase shift appears under influence of different external

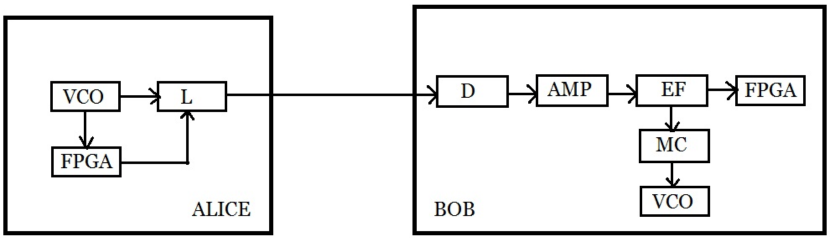

FIG. 2. Block diagram of the synchronization subsystem in SCWQC system

effects. One of them is daily temperature fluctuation that affects the refractive index of the fiber, thus inducing optical delays. For the SCWQC system described in [4, 5], this problem is solved by the calibration procedure described below.

The synchronization pulse is sent through a separate optical fiber to avoid its influence on QBER [3,4]. Synchronization channel laser L generates a sinusoidal signal with a frequency defined by the VCO that acts as a generator (Fig. 2). This signal is also modulated by the transmitter logic board (FPGA). After passing through the communication channel optical signal is recorded at the photodetector (D) in receiver block, where it amplified (AMP) and filtered in the electric filter (EF). The sinusoidal component flows through the microcontroller (MC) to the receiver VCO and used to adjust the oscillator frequency. The component inserted by the transmitter logic is used as the start and reset signal. A subsystem composed of the VCO and a phase-locked lope device then generates the driving signal of the optical modulator with frequency Ω. A more detailed description of phase modulating in the system is provided in [6].

As the synchronization subsystem performs two functions, synchronization of transmitter and receiver generators and transmitting a start/reset signal, two main problems caused by signal phase difference appear. First, the generators in the transmitter and receiver modules have to be precisely phase-locked, so that the phase delay of the synchronization signal should not exceed $\Delta\varphi$. Second, the delay between the synchronization and quantum signals in different fibers should not exceed 10 % of the gate interval for correct system operation. Thus, for the described system with 100 MHz clock rate, the limiting delay value is 1 ns.

In order to solve these problems in the SCWQC system, a novel calibration procedure was developed. After a certain time interval of quantum bit transmission (t_{qc}), the procedure switches the system into calibration mode (time interval t_{cal}), during the course of which it resets the gate starting time and redefines the optical phase values induced by different modulator driving voltages ("modulation tables") in Alice and Bob units. During SCWQC experiments in optical fibers, the values of t_{qc} and t_{cal} were chosen to be 50 ms and 10 ms, respectively. Therefore, the effective sifted key generation rate is reduced by approximately 15 %.

In the following sections, we find the temperature dependence of synchronization signal phase. Then, we develop a temperature model of optical fiber cables, which will be employed during the scheduled SCWQC experiments, and calculate the temperature-induced phase delay. This allows us to optimize the value of t_{cal} and therefore the sifted key rate in practical SCWQC systems.

2.3. Phase temperature dependence

To determine the maximum interval between two following calibration procedures, we need to know the temporal delay Δt of the signal, which corresponds to the maximum tolerated phase delay, $\Delta\varphi$. The Δt value depends on the cable length L and refractive index changes in the fiber Δn:

$$\Delta t = \frac{L\Delta n}{c}. \tag{1}$$

In turn, the fiber's refractive index is linearly dependent upon the temperature T and two constants, representing the change of n due to temperature (T-component) and bending (R-component) [9]:

$$\Delta n = \left[\left(\frac{dn}{dT}\right)_T + \left(\frac{dn}{dT}\right)_R\right]\Delta T.$$

As shown in [9], for the Corning SMF-28 fiber widely used in telecommunication cables, these coefficients are:

$$\left(\frac{dn}{dT}\right)_T = 6.8\cdot 10^{-5}\ ^\circ\mathrm{C}^{-1},$$

$$\left(\frac{dn}{dT}\right)_R = 0.8\cdot 10^{-5}\ ^\circ\mathrm{C}^{-1}.$$

We chose the optical cable length $L = 100$ km, since it is a typical distance for metropolitan area local networks. For this L value, the signal delay is:

$$\Delta t = 2.53\cdot 10^{-8}\Delta T\ [\mathrm{s}].$$

The relation between the phase and time delays of a signal with frequency Ω is given by:

$$\Delta t = \Delta\varphi/\Omega.$$

The Ω value is typically several gigahertz for SCWQC systems [4, 5]; we used a value of 4 GHz in our calculations.

It can be seen that the phase delay is in direct ratio with the temperature T of the fiber core. A temperature model of the cable was therefore created in order to determine it.

3. Process modeling

3.1. Cable temperature model

To estimate the temperature fluctuations of the fiber optical cable core we have developed the following temperature model. We solve the heat equation and choose appropriate boundary conditions for the two cases: the air- and ground-based cables.

The two-dimensional heat equation for the cable is:

$$c\rho\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right), \tag{2}$$

where λ is thermal conductivity, ρ represents density and is heat capacity. We write the air temperature in the form $T_a(t) = \overline{T}_a + \delta T_a\cos(\omega t)$, where $\overline{T}_a$ is the average daily temperature, δT_a is the temperature amplitude.

The first boundary condition corresponds to the cable in the air:

$$\lambda\left.\frac{\partial T}{\partial r}\right|_{r=R} = \alpha(T_a(t) - T|_{r=R}), \tag{3}$$

where α is the coefficient of convective heat exchange, r and R are the radial coordinate and radius of the cable.

The temperature $T_g(t)$ in the ground at depth z has reduced amplitude and phase shift [10].The second boundary condition corresponds to the cable located in the ground:

TABLE 1. Cable element parameters

Element	R, mm	Thermal diffusivity, m^2/s, $\cdot 10^{-6}$
Outer jacket	3	0.11
Strength member	2.3	13.3/0.09*
Coating	2	0.11
Water blocking material	1.35	0.174
Optical fiber	0.45	1.1
Central member	0.45	0.15

*for the air /ground cables

$$T|_{r=R} = T_g(t) \equiv \overline{T}_a + \delta T_a \exp\left(-z\sqrt{\frac{\omega}{2a}}\right) \cos\left(\omega t - z\sqrt{\frac{\omega}{2a}}\right), \tag{4}$$

where a is the thermal diffusivity of ground.

Figure 3a shows the model of the cable with chosen configuration. The cable consists of an outer jacket, a strength member, a coating, water blocking material, optical fibers and a central member.

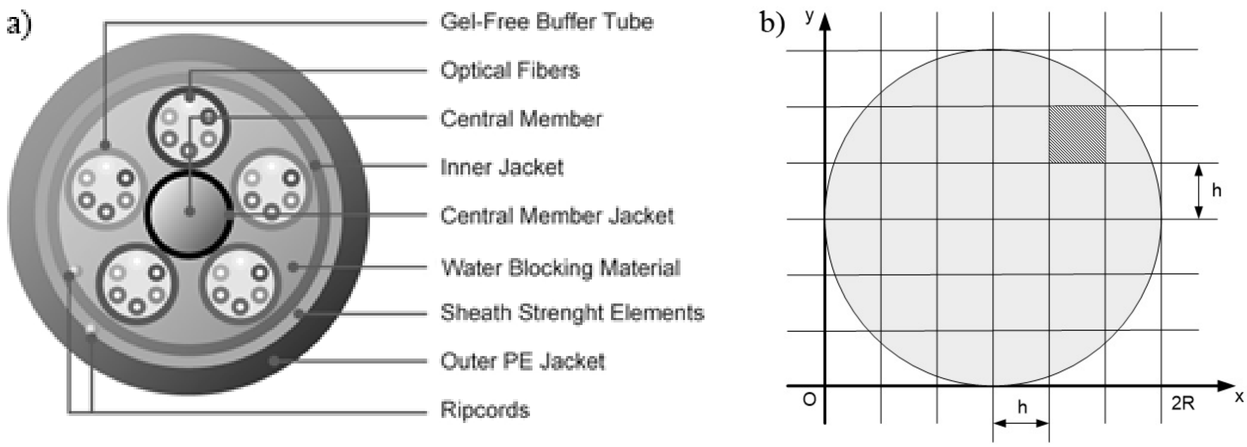

FIG. 3. a) Cable model with chosen configuration; b) The computational domain model

Let the radius of the cable be R. We then divide the area using a uniform grid consisting of $n + 1$ knot (Fig. 3b). We solve the eq. (2) using a difference Crank–Nicolson scheme with boundary conditions (3–4) with Thomas algorithm for $n = 1000$. Different elements of the cable have different thermal diffusivity coefficients, as indicated in Table 1.

3.2. Modeling results and discussion

The main difference between an air-based cable and one lying in the ground is the lack of steel armor, optional for this type of operation, which is replaced by an aramid yarn. Therefore in the case of air-based cable, the thermal diffusivity of the strength member will be lower. In the cable lying underground, the temperature is distributed in the form of a wave, depending on the depth. In our model, we have chosen a typical ground depth of 1 meter. With a small-sized cable, even after a while, the temperature and the difference between the coefficients of thermal diffusivity for different elements were insignificantly affected.

The initial temperature T_a of the cable is 15 degrees; the initial (mean) environmental temperature $\overline{T}_a$ is 15 degrees, with $\delta T_a = 10$ degrees oscillation amplitude over a period of 24 hours. Temperature fluctuations for the chosen fiber in the cable over 12 hours are shown on Fig. 4.

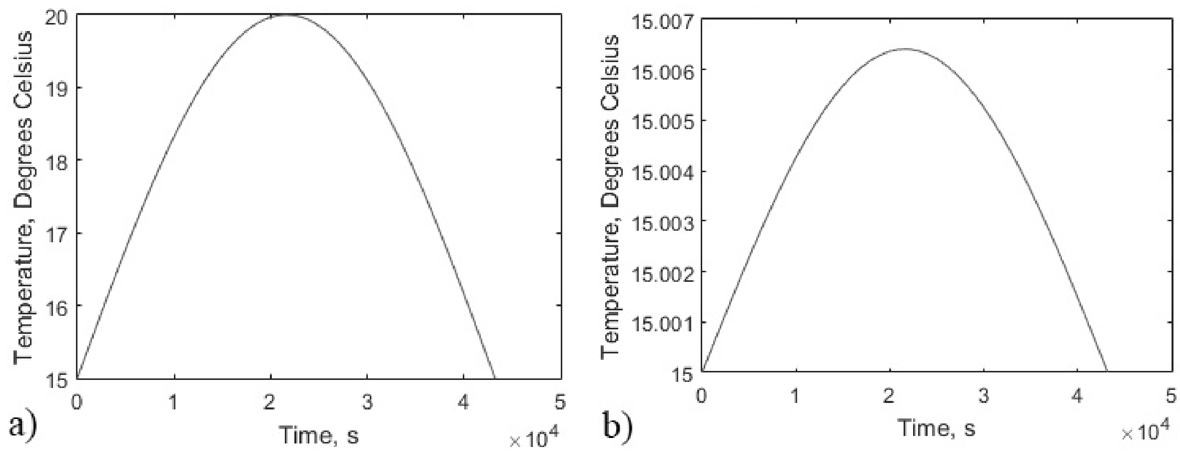

FIG. 4. Change of cable temperature during daytime in air (a) and in the ground (b)

As discussed previously, the signal phase delay in the system is in direct ratio with the temperature of the fiber optic core.

Figure 5 shows the dependence of the synchronization signal temporal delay induced by cable heating. Eq. (1) allows us to calculate the delay Δt, which leads to maximum tolerated phase difference $\Delta\varphi = 0.043$, as 0.0017 ns. According to our model (Fig. 5) for air-based cables, such Δt accumulates every 158 ms. Therefore, for SCWQC systems operating in air, the modulation signal adjustment may be performed every 158 ms instead of every 50 ms, as was previously implemented. This would allow for longer t_{qc} and about 15 % higher effective sifted bitrate.

At the same time, for ground-based cables, the temperature-induced signal delay never reaches the critical value due to the small daily temperature fluctuations at 1 meter depth underground (Fig. 6). Therefore, the temperature (under normal conditions) does not affect the course of SCWQC system operation.

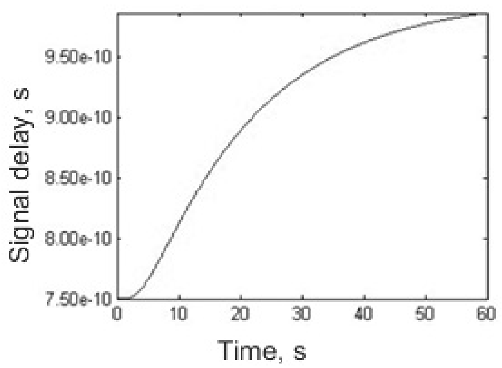

FIG. 5. Dependence of the signal delay time upon cable heating

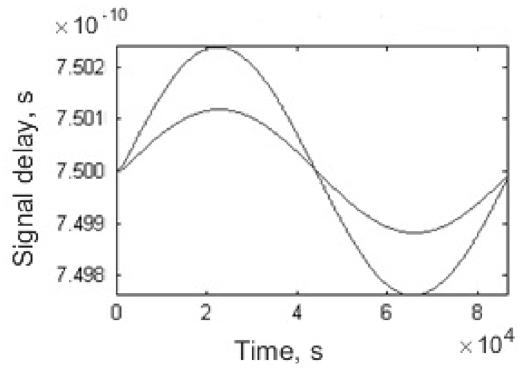

FIG. 6. The dependence of the signal delay time upon the heating cable at different points in a 24 hour period

4. Conclusion

Optical signals transmitted in cables for quantum communication systems are transformed during their propagation. In this paper, the temperature dependencies of the synchronization signals in optical fiber cables in the ground and in air were investigated. In order to determine the signal delay in the cable, a temperature model was created. The influence of daily temperature fluctuations on the phase delay was estimated for the air- and ground-based cables. The calculated results can be used to optimize calibration procedures in SCWQC systems.

For systems operating in ground-based cables, daily temperature fluctuations do not significantly impact the synchronization of the signal phase. However, it has been shown that for systems operating through air-based cables, phase adjustment is required every 158 ms for stable operation. This allowed us to optimize the parameters for the calibration procedure of a previously developed SCWQC system. Therefore, this allowed an increase in the overall sifted key generation rate.

Acknowledgements

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