

The effect of vertical throughflow in Rivlin–Ericksen elastico-viscous nanofluid in a non-Darcy porous medium

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The instability of convection in a Rivlin–Ericksen elastico-viscous nanofluid with vertical throughflow is investigated using the linear stability theory. A modified Brinkman model is employed and single-term Galerkin method is used to solve the conservation equations. Nine dominating parameters are extracted from the analysis. Due to the combined effect of vertical throughflow, Brownian motion, and thermophoresis, the Rayleigh number is reduced by a substantial amount. It is found that through flow delays the convection while other nanofluid parameter enhance the convection. The thermal capacity ratio, kinematics viscoelasticity, and Vadasz number do not govern stationary convection. Using the convective component of nanoparticle flux, the critical wave number is a function of nanofluid parameters as well as throughflow parameter. Major trends are investigated briefly by plotting the graphs.

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1. Introduction

Rivlin and Erickson [1] were the first to propose the theoretical and rheological model for elastic-viscous fluid. Rudraiah et al. [2] studied the overstable convection of an Oldroyd fluid and they found that effect of elasticity is to elongate the cells. Kim et al. [3] found that elasticity parameters have a destabilizing effect on the onset of convection. The effect of permeability, magnetic field, and rotation on Rivlin–Ericksin elastico-viscous fluid was studied by [4, 5] and they found that rotation, and magnetic field diminish the instability, while permeability enhances the instability. Sheu [6, 7] examined the linear stability in Visco-Elastic nanofluids and studied the combined effects of Brownian diffusion, thermophoresis diffusion, and viscoelasticity. Viscoelastic fluids are widely used in paper coating, chemical industries, and manufacturing processes.

A nanofluid contains nano-sized particles (diameter less than 100 nm) which are suspended in the base fluid. The term, “nanofluid” was first coined, by Choi [8] and Buongiorno [9] to describe the importance of the various transport mechanisms in nanofluids. Using the model given by [9], Nield and Kuznetsov [10, 11] analyzed the thermal stability problem and found that nanoparticles have a destabilizing effect.

The effect of throughflow was generally stabilizing, however, for a small amount of throughflow, it shows the destabilizing effect [12, 13]. Nield [14] found that destabilization occurs when the throughflow is away from impermeable boundaries. In the absence of internal heat generation, throughflow destabilized the system in a very small amount of throughflow at the different boundary conditions (top and bottom layer are not identical) [1]. Barletta et al. [15] studied the effect of vertical throughflow with viscous dissipation. Nield and Kuznetsov [16, 17] investigated the effect of vertical throughflow on both the boundaries (isoflux and isothermal). By using the Buongiorno model [16], we studied the effect of vertical throughflow on the onset of nanofluid convection, later it, this model was revised by using more realistic boundary conditions [18–20].

From the above brief review, it is clear that many researchers have carried out extensive studies on the onset of convection in Rivlin–Ericksin elastico-viscous nanofluids recently; however, the effect of vertical throughflow in Rivlin–Ericksin elastico-viscous nanofluid using the new boundary conditions (in which nanoparticle flux is the sum of diffusive, connective, and thermophoretic terms) has not been yet studied. Therefore, in this paper, the effects of viscoelastic parameters, nanofluid parameters, and throughflow parameters are investigated analytically and numerically for Rigid-rigid boundary conditions in a non-Darcy porous media.

2. Analysis

2.1. Mathematical formulation

An infinite horizontal Rivlin–Ericksen elastico-viscous nanofluid confined between rigid impermeable boundaries $z = 0$ and $z = H$ is considered. The pore sizes are large compared to nanoparticles. We use the modified Brinkman–Darcy model to define the nature of the porous medium and the Oberbeck–Boussinesq approximations are employed. We take temperatures T_0 and T_1 ($T_1 > T_0$) at the lower and the upper wall. The homogeneity and local thermal equilibrium in the porous medium is also assumed. The governing equations for mass, momentum, energy and nanoparticles for Rivlin–Ericksen elastico-viscous nanofluid in porous medium are written below (details of following equations can be seen in the articles given by [6,7,11]):

$$\nabla \cdot \mathbf{V}_D = 0, \tag{1}$$

$$\frac{\rho_f}{\varepsilon} \frac{d\mathbf{V}_D}{dt} = -\nabla \mathbf{p} - \frac{1}{k} \left(\mu + \mu' \frac{\partial}{\partial t} \right) \mathbf{V}_D + \left[\varphi \rho_p + (1 - \varphi) \{ \rho (1 - \beta (T - T_0)) \} \right] \mathbf{g}, \tag{2}$$

$$\frac{\partial T}{\partial t} + \frac{(\rho c)_f}{(\rho c)_m} \mathbf{V}_D \cdot \nabla T = k_m \nabla^2 T + \frac{\varepsilon (\rho c)_p}{(\rho c)_m} \left(D_B \nabla \varphi \cdot \nabla T - \frac{(\varphi - \varphi_0) \mathbf{V}_D}{\varepsilon} + \frac{D_T}{T_1} \nabla T \cdot \nabla T \right), \tag{3}$$

$$\frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon} \mathbf{V}_D \cdot \nabla \varphi = D_B \nabla^2 \varphi + \frac{D_T}{T_1} \nabla^2 T, \tag{4}$$

where \mathbf{V}_D is the Darcy velocity, μ and μ' are viscosity and kinematic viscoelasticity, ε is the porosity of the porous medium, p , t and \mathbf{g} are the pressure, time and gravity vector, T is the nanofluid temperature, φ is the nanoparticles volume fraction, ρ is the density of the nanofluid, ρ_p is the density of nanoparticles, $(\rho c)_p$ is the volumetric heat capacity for the nanoparticles, $(\rho c)_f$ is the volumetric heat capacity for the nanofluid, $(\rho c)_m$ is the effective heat capacity of the porous medium, (k_m) is the effective thermal conductivity of porous media, (β) is the volumetric thermal expansion coefficient, (D_B) is the Brownian diffusion coefficient, (D_T) is the thermophoresis diffusion coefficient.

On the boundaries, it is assumed that the temperature and throughflow velocity are constant. The total nanoparticle flux (sum of diffusive, thermophoretic, and connective terms) is assumed to be zero at the boundaries.

The boundary conditions are:

$$\begin{aligned} w = W_c, \quad T = T_0, \quad D_B \frac{\partial \varphi}{\partial z} - (\varphi - \varphi_0) W_c + \frac{D_B}{T_\infty} \frac{\partial T}{\partial z} &= 0 \quad \text{at } z = 0, \\ w = W_c, \quad T = T_1, \quad D_B \frac{\partial \varphi}{\partial z} - (\varphi - \varphi_0) W_c + \frac{D_B}{T_\infty} \frac{\partial T}{\partial z} &= 0 \quad \text{at } z = 1. \end{aligned} \tag{5}$$

Defining the dimensionless variables as follows:

$$\begin{aligned} (x', y', z') &= \frac{(x, y, z)}{H}, \quad \mathbf{V}(u', v', w') = \frac{\mathbf{V}_D(u, v, w) H}{\kappa}, \quad t' = \frac{t \kappa}{\sigma H^2}, \\ p' &= \frac{p K}{\mu \kappa}, \quad \varphi' = \frac{\varphi - \varphi_0}{\varphi_0}, \quad T' = \frac{T - T_1}{T_0 - T_1}, \end{aligned} \tag{6}$$

where $\kappa = \frac{k_m}{(\rho c_p)_f}$.

Eqs. (1)–(4) in non-dimensionless form (after dropping the primes) can be written as:

$$\nabla \cdot \mathbf{V} = 0, \tag{7}$$

$$\frac{1}{V_a} \frac{d\mathbf{V}}{dt} = -\nabla \mathbf{p} - \left(1 + F \frac{\partial}{\partial t} \right) \mathbf{V} - R_m \hat{\mathbf{k}} + R_a T \hat{\mathbf{k}} - R_n \varphi \hat{\mathbf{k}}, \tag{8}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) T = \nabla^2 T + \frac{N_B}{L_e} \nabla \varphi \cdot \nabla T - N_B \varphi \mathbf{V} + \frac{N_A N_B}{L_e} \nabla T \cdot \nabla T \tag{9}$$

$$\frac{1}{\sigma} \frac{\partial \varphi}{\partial t} + \frac{1}{\varepsilon} \mathbf{V} \cdot \nabla \varphi = \frac{1}{L_e} \nabla^2 \varphi + \frac{N_A}{L_e} \nabla^2 T. \tag{10}$$

The dimensionless parameters in Eqs. (8)–(10) namely, Prandtl number. (Pr), modified Darcy number (\tilde{D}_a), Rayleigh number (R_a), basic density Rayleigh number (R_m), nanoparticle Rayleigh number (R_n), Lewis number (L_e), Péclet number Q , Modified diffusivity Ratio (N_A), modified particle-density increment (N_B), Vadasz

number (Va), Thermal capacity ratio(σ), kinematics viscoelasticity (F) are defined as follows:

$$\begin{aligned} Pr &= \frac{\mu}{\rho\kappa}, \quad \tilde{D}_a = \frac{K}{H^2}, \quad R_a = \frac{\rho g \alpha H (T_0 - T_1)}{\mu\kappa}, \quad R_m = \frac{[\rho_p \varphi_0 + \rho(1 - \varphi_0)]gH}{\mu\kappa}, \quad R_n = \frac{\{(\rho_p - \rho)\varphi_0\}gH}{\mu\kappa}, \\ L_e &= \frac{\alpha_m}{D_B}, \quad Q = \frac{HV}{k_m}, \quad N_A = \frac{D_T(T_0 - T_1)}{D_B T_1 \varphi_0}, \quad N_B = \frac{(\rho c)_p \varphi_0}{(\rho c)_f}, \quad Va = \frac{\varepsilon Pr}{Da}, \quad \sigma = \frac{(\rho c_p)_m}{(\rho c_p)_f}, \quad F = \frac{\mu' K}{\mu \sigma H^2}. \end{aligned} \tag{11}$$

The dimensionless form of boundary conditions are written as:

$$\begin{aligned} w = Q, \quad T = 1, \quad \frac{\partial \varphi}{\partial z} - QL_e \varphi + N_A \frac{\partial T}{\partial z} &= 0 \quad \text{at } z = 0, \\ w = Q, \quad T = 0, \quad \frac{\partial \varphi}{\partial z} - QL_e \varphi + N_A \frac{\partial T}{\partial z} &= 0 \quad \text{at } z = 1. \end{aligned} \tag{12}$$

2.2. Basic solutions

The basic state of nanofluid is assumed to be time independent and is described by:

$$\mathbf{V} = \mathbf{V}_b = (0, 0, Q), \quad p = p_b(z), \quad T = T_b(z), \varphi = \varphi_b(z). \tag{13}$$

Then, Eqs. (8)–(10) are simplified as:

$$\frac{d^2 T_b}{dz^2} + \frac{N_B}{L_e} \frac{d\varphi_b}{dz} \frac{dT_b}{dz} + \frac{N_A N_B}{L_e} \left(\frac{dT_b}{dz} \right)^2 - Q_v \frac{dT_b}{dz} - Q_v N_B \varphi_b \frac{dT_b}{dz} = 0, \tag{14}$$

$$\frac{d^2 \varphi_b}{dz^2} + N_A \frac{d^2 T_b}{dz^2} - Q_v L_e \frac{d\varphi_b}{dz} = 0. \tag{15}$$

Solving Eq. (15) and Eq. (16) with boundary condition gives:

$$T_b = \frac{e^{Q_v z} - e^{Q_v z}}{e^{Q_v} - 1}, \tag{16}$$

$$\varphi_b = \frac{(1 - L_e - N_A)(e^{Q_v L_e z} - 1)}{(1 - L_e)(e^{Q_v L_e} - 1)} + \frac{N_A(e^{Q_v z} - 1)}{(1 - L_e)(e^{Q_v} - 1)}. \tag{17}$$

We differentiate the T_b, φ_b with respect to z :

$$\frac{dT_b}{dz} = \frac{Q e^{Q_v z}}{(1 - e^Q)}, \tag{18}$$

$$\frac{d\varphi_b}{dz} = \frac{QL_e(\varepsilon - L_e - N_A)e^{\frac{QL_e z}{\varepsilon}}}{(1 - L_e)\varepsilon(e^{\frac{QL_e}{\varepsilon}} - 1)} + \frac{\varepsilon N_A Q e^{Q_v z}}{(\varepsilon - L_e)(e^Q - 1)}. \tag{19}$$

Using the limit $Q \rightarrow 0$, the basic solution becomes $T_b = 1 - z, \varphi_b = z$; this is same solution as obtained by [11].

2.3. Perturbed solutions

For perturbations of small disturbance onto the base fluid, we assume that:

$$\mathbf{V} = \mathbf{V}_b + \mathbf{V}', \quad T = T_b + T', \quad \varphi = \varphi_b + \varphi', \quad p = p_b + p'. \tag{20}$$

Substituting the perturbed solutions in Eqs. (7)–(10) and Eq. (12), we get:

$$\nabla \cdot \mathbf{V}' = 0, \tag{21}$$

$$\frac{1}{V_a} \frac{\partial \mathbf{V}'}{\partial t} + w' = -\frac{\partial p'}{\partial z} - \left(1 + F \frac{\partial}{\partial t} \right) + R_a T' \hat{\mathbf{k}} - R_n \varphi' \hat{\mathbf{k}}', \tag{22}$$

$$\begin{aligned} \frac{\partial T'}{\partial t} + \frac{dT_b}{dz} w' + Q \frac{\partial T'}{\partial z} &= \nabla^2 T' + \frac{N_B}{L_e} \left(\frac{d\varphi_b}{dz} \frac{\partial T'}{\partial z} - \frac{dT_b}{dz} \frac{\partial \varphi'}{\partial z} \right) + \frac{2N_A N_B}{L_e} \frac{dT_b}{dz} \frac{\partial T'}{\partial z} \\ &\quad - N_B \left(Q \frac{dT_b}{dz} \varphi' + \varphi_b \frac{dT'}{dz} w' + Q \varphi_b \frac{\partial T'}{\partial z} \right), \end{aligned} \tag{23}$$

$$\frac{\partial \varphi'}{\partial t} + Q \frac{\partial \varphi'}{\partial z} + \frac{d\varphi_b}{dz} w' = \frac{1}{L_e} \nabla^2 \varphi' + \frac{N_A}{L_e} \nabla^2 T'. \tag{24}$$

Eq. (22) is operated with $\hat{\mathbf{k}} \cdot \text{curl curl}$ and the use is made of the identity $\text{curl curl} = \text{grad div} - \nabla^2$ together with Eq. (21). As a result of this procedure, pressure is eliminated from Eq. (22) and we get:

$$\frac{1}{Pr} \frac{\partial}{\partial t} \nabla^2 w' + \left(1 + M + F \frac{\partial}{\partial t} \right) \nabla^2 w' = R_a \nabla_H^2 T' - R_n \nabla_H^2 \varphi'. \tag{25}$$

In Eq. (25) ∇_H^2 is the 2-D Laplacian operator in the horizontal plane.

Analyzing the disturbance into the normal modes, perturbation equations are introduced as the following form:

$$[w', T', \varphi'] = [W(z), \Theta(z), \Phi(z)]f(x, y) \exp(st). \tag{26}$$

Using the normal mode analysis, the following equations for the amplitudes Θ, Φ, W are obtained:

$$(D^2 - a^2) \left(1 + \frac{s}{V_a} + sF \right) W + R_a a^2 \Theta - R_n a^2 \Phi = 0, \tag{27}$$

$$\begin{aligned} (-1 + N_B Q) \frac{dT_b}{dz} W + \left(D^2 + \frac{N_B}{L_e} \frac{d\varphi_b}{dz} D + \frac{2N_A N_B}{L_e} \frac{dT_b}{dz} D - N_B Q \varphi_b D - Q_v D - a^2 - s \right) \Theta \\ + \left(\frac{N_B}{L_e} \frac{dT_b}{dz} D - N_B Q \frac{dT_b}{dz} \right) \Phi = 0, \end{aligned} \tag{28}$$

$$\frac{d\varphi_b}{dz} W - \frac{N_A}{L_e} (D^2 - a^2) \Theta - \frac{1}{L_e} \left((D^2 - a^2) - Q_v D - \frac{s}{\sigma} \Phi \right) = 0, \tag{29}$$

with boundary conditions as:

$$W = 0, \quad \Theta = 0, \quad D\Phi - \frac{QL_e}{\varepsilon} \Phi + N_A D\Theta = 0, \quad \text{at } z = 0, 1. \tag{30}$$

Here, a is a dimensionless wave number, and $s = i\omega$ is a complex growth factor, where ω is a dimensionless frequency (it is a real number).

Single-term Galerkin-type weighted residuals method is used to obtain approximate solutions to the system of Eqs. (27)–(29). Accordingly Θ, W, Φ (satisfying the boundary conditions exactly) are chosen as:

$$\Theta_1 = z - z^2, \quad W_1 = z(1 - z)^2, \quad \Phi_1 = \frac{-N_A(QL_e - 2 - 2QL_e z)}{QL_e^2}.$$

We substitute this minimal polynomial into Eqs. (27)–(29) and using the Orthogonality of trial functions, we obtain a system of 3 linear algebraic equations in the 3 unknowns. We get:

$$\begin{bmatrix} G1 & \frac{R_a a^2}{140} & \frac{R_n N_A a^2}{15\eta^2} \\ (-1 + N_B Q) A_{TW} & G2 & \frac{N_B}{L_e} B_{T\Phi} - N_B Q A_{T\Phi} \\ A_{\varphi W} & -\frac{N_A^2(12 + a^2)}{3L_e \eta^2} & \frac{N_A^2 a^2(12 + \eta^2) - 3 - s(12 + \eta^2)}{3L_e \eta^4} \end{bmatrix} \begin{bmatrix} W \\ \Theta \\ \Phi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{31}$$

where

$$G1 = \left(\frac{S}{V_a} + 1 \right) (504 + 24a^2 + a^4 + \tilde{D}_a(12 + a^2)),$$

$$G2 = \frac{-(10 + a^2)}{30} + \frac{N_B}{L_e} B_{\varphi\Theta} + \frac{2N_A N_B}{L_e} B_{T\Theta} - \frac{s}{30},$$

$$A_{TW} = \int_0^1 \left(\frac{dT_b}{dz} W \right) \Theta dz = \frac{6(120 + 60Q + 12Q^2 + Q^3) + e^Q(-120 + Q(60 + (-12 + Q)Q))}{Q^6(1 - e^Q)},$$

$$\begin{aligned} B_{\varphi\Theta} = \int_0^1 \left(\frac{d\varphi_b}{dz} D\Theta \right) \Theta dz = -\frac{\eta^2 + 6\eta + 12 - e^\eta(\eta^2 - 6\eta + 12)(Q - \eta - N_A Q)}{\eta^3(1 - e^\eta)(Q - \eta)} \\ - \frac{N_A(Q^2 + 6Q + 12 - e^Q(Q^2 - 6Q + 12))}{Q^2(1 - e^Q)(Q - \eta)}, \end{aligned}$$

$$B_{T\Theta} = \int_0^1 \left(\frac{dT_b}{dz} D\Theta \right) \Theta dz = \frac{Q^2 + 6Q + 12 - e^Q(Q^2 - 6Q + 12)}{Q^3(1 - e^Q)},$$

$$\begin{aligned} A_{\varphi\Theta} = \int_0^1 (\varphi_b D\Theta) \Theta dz = \frac{\eta^2 + 6\eta + 12 - e^\eta(\eta^2 - 6\eta + 12)(Q - \eta - N_A Q)}{\eta^4(1 - e^\eta)(Q - \eta)} \\ + \frac{N_A(Q^2 + 6Q + 12 - e^Q(Q^2 - 6Q + 12))}{Q^3(1 - e^Q)(Q - \eta)}, \end{aligned}$$

$$\begin{aligned}
B_{T\Phi} &= \int_0^1 \left(\frac{dT_b}{dz} D\Phi \right) \Theta dz = \frac{2\eta(2 + e^Q(-2 + Q) + Q)}{Q^2(1 - e^Q)}, \\
A_{T\Phi} &= \int_0^1 \left(\frac{dT_b}{dz} \Phi \right) \Theta dz = \frac{-2Q(2 + e^Q(-2 + Q) + Q) + \eta(12 + 6Q + Q^2 - e^Q(12 - 6Q + Q^2))}{Q^3(1 - e^Q)}, \\
A_{\varphi\Phi} &= \int_0^1 \left(\frac{d\varphi_b}{dz} W \right) \Phi dz = \frac{2(Q - \eta - N_A Q)(\eta - 2(1 + \eta z))(-12 - \eta(6 + \eta) + e^\eta(12 + (-6 + \eta)\eta))}{\eta^4(Q - \eta)(e^\eta - 1)} \\
&\quad + \frac{2N_A(\eta - 2(1 + \eta z))(-12 - Q(6 + Q) + e^Q(12 + (-6 + Q)Q))}{Q^3(Q - \eta)(e^Q - 1)}, \\
\eta &= \frac{QL_e}{\varepsilon}.
\end{aligned}$$

For all considered set of boundary conditions, oscillatory convection cannot be occurring. In the absence of oscillatory convection ($s = 0$), we get the following expression for R_a :

$$R_a = \frac{14((504 + 24a^2 + a^4 + \tilde{D}_a(12 + a^2))\alpha - N_A R_n \beta)}{\gamma}, \quad (32)$$

where:

$$\begin{aligned}
\alpha &= \frac{30\eta^2 N_A^2 N_B(12 + a^2)(B_{T\Phi} - QA_{T\Theta}L_e) + A_{\varphi W}\eta^2(N_a^2 a^2(12 + \eta^2) - 3)(N_B(B_{\Phi\Theta} + 2B_{T\Theta}N_A) - (10 + a^2)L_e)}{9a^2 L_e^2 \eta^4}, \\
\beta &= \frac{A_{\varphi W}\eta^2(30N_B(B_{\Phi\Theta} + 2B_{T\Theta}N_A) - (10 + a^2)L_e)A_{\varphi W}\eta^2 + 10N_A^2 A_{TW}(-1 + N_B Q)(12 + a^2)}{45L_e \eta^4}, \\
\gamma &= \frac{3L_e \eta^4 N_B(B_{T\Phi} - QA_{T\Theta}L_e) - A_{TW}(N_a^2 a^2(12 + \eta^2) - 3)(-1 + N_B Q)}{3L_e \eta^4}.
\end{aligned}$$

The Rayleigh number (R_a) is a function of $a\varepsilon$, \tilde{D}_a , Q , L_e , N_A , a , R_n , and N_B and from Eq. (32), it is also clear that the Rayleigh number (R_a) is independent of viscoelastic parameters (V_a, F, σ). Thus, the effect of throughflow in Rivlin–Ericksin elastico-viscous nanofluid is independent of viscoelastic parameters. To simplify the above expression, we have assumed that $Q \ll 1$, so that first order Galerkin approximation leads to a useful result:

$$R_a = \frac{140(G3)(504 + 24a^2 + a^4 + \tilde{D}_a(12 + a^2)) - (105(12 + a^2)(1 + N_B Q)N_A - 98(G3)L_e)R_n}{135a^2(1 + N_B Q)}, \quad (33)$$

where $G3 = 10 + a^2 - \frac{N_A N_B Q}{L_e}$.

In Eq. (33), for the admissible range of parameters the Rayleigh number decreases as a nanoparticle Rayleigh number increases. The corresponding critical wave number can be obtained from Eq. (33), which in the presence of throughflow, critical wave number is a function of the Péclet number, modified Darcy number as well as nanofluid parameters.

In the absence of vertical throughflow, Eq. (33) reduces to the following form:

$$R_a = \frac{140(10 + a^2)(504 + 24a^2 + a^4 + \tilde{D}_a(12 + a^2)) - 105(12 + a^2)N_A R_n - 98(10 + a^2)L_e R_n}{135a^2}. \quad (34)$$

In the absence of vertical throughflow ($Q = 0$) the Rayleigh number is not a function of the modified particle density increment (N_B).

3. Results and discussions

To study the effect of various parameters on the stability, Eq. (33) is analyzed analytically and numerically. According to data given by [1, 11], the following threshold values for alumina/water nanofluids of 1–100 nm are utilized: $L_e = 500$, $Q = 0.5$, $R_n = 0.1$, $N_A = 5$, $N_b = 0.01$. Fig. 1 shows the plot of critical Rayleigh number ($R_{a,c}$) versus nanoparticle Rayleigh number R_n for vertical throughflow $Q = 0.1, 0.3, 0.7, 1$. It can be seen that R_n destabilizes the onset of convection. Fig. 1 also shows that $R_{a,c}$ increases with increasing value of Q . Vertical through flow delays the onset of nanofluid convection and this effect is larger for higher nanoparticle Rayleigh number values.

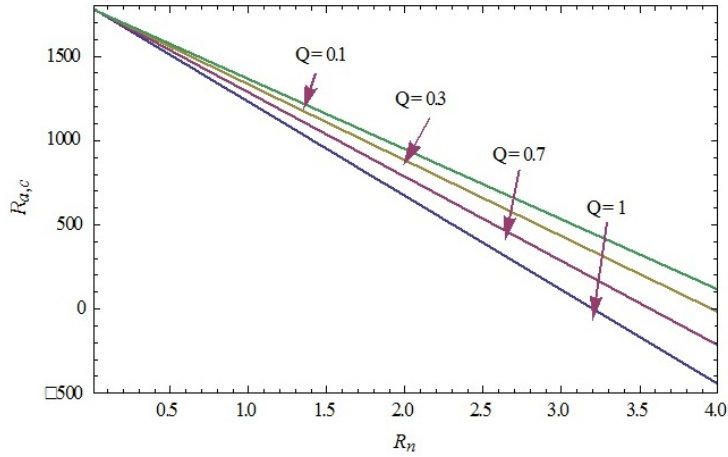


FIG. 1. Variation of $R_{a,c}$ with R_n for different values of Q

Figure 2 shows that the modified particle density increment (N_b) has no significant effect on the stability of Rivlin–Ericksin elasto-viscous nanofluid.

Figure 3 shows the variation of Rayleigh number R_a when $L_e = 500, 700$ and $N_A = 1.5$ respectively. It is seen that as the value of the Lewis number (L_e) increases, the Rayleigh number decreases. Thus, the Lewis number promotes nanofluid convection. The modified diffusivity ratio has a destabilizing effect on the onset of convection. This is because thermophoresis pushes the lighter nanoparticles upwards, which enhances the destabilizing effects on particle distributions.

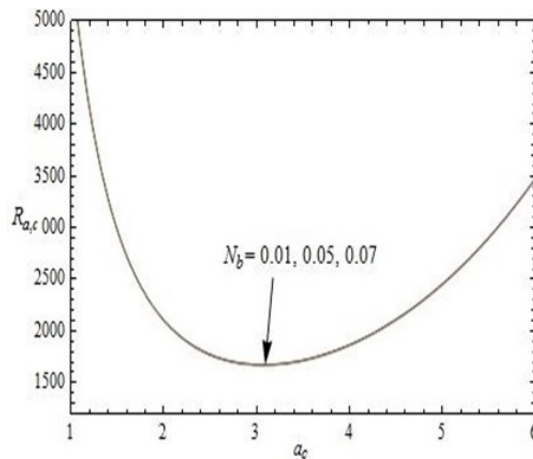


FIG. 2. Variation of $R_{a,c}$ with a_c for different values of N_b

4. Conclusions

In this paper, we investigate the effect of vertical throughflow in Rivlin–Ericksin elasto-viscous nanofluid for Rigid-rigid boundaries in a non-Darcy porous medium. It was observed that when using the connective term

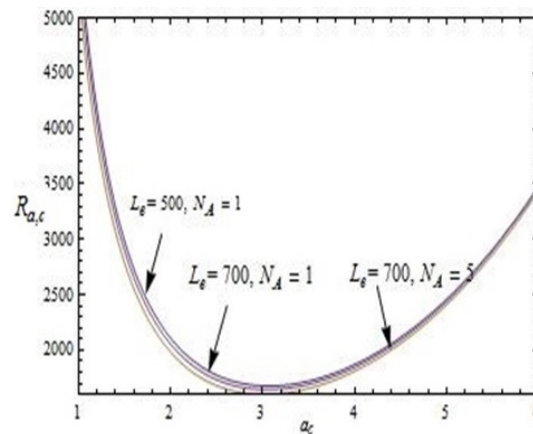


FIG. 3. Variation of $R_{a,c}$ with a_c for different values of N_b , L_e

of nanoparticle flux at the boundary, the Rayleigh number was independent of thermal capacity ratio, kinematics viscoelasticity parameter and Vadsaz number. The critical wave number was a function of the nanofluid parameters as well as Péclet number. Porosity, Lewis number, and nanoparticle Rayleigh number promotes instability while the modified particle-density increment had no significant effect on the stability. Vertical through flow delays the onset of nanofluid convection and this effect is larger for higher nanoparticle Rayleigh number values. New boundary conditions in terms of zero nanoparticle flux (the sum of the diffusive, thermophoretic, and connective terms) is more favourable for convection; however there is no drastic change in the magnitude of Rayleigh number.

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