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Original article

A numerical investigation of modified Burgers' equation in dusty plasmas with non-thermal ions and trapped electrons

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ABSTRACT In this paper, one-dimensional lower order modified Burgers' equation (MBE) in dusty plasmas having non-thermal ions and trapped electrons is investigated numerically by finite difference explicit method. The numerical results obtained by the finite difference explicit method for various values of the nonlinear and dissipative coefficients have been compared with the analytical solutions. The obtained numerical results are found to have good agreement with the analytical solutions. It is found that the nonlinear and dissipative coefficients have very important effect on the dust acoustic waves in the system. The absolute error between the analytical and the numerical solutions of the MBE is demonstrated. The stability condition is derived in terms of the equation parameters and the discretization using the von Neumann stability analysis. It has been observed that the waves become flatten and steeper when the dissipative coefficient decreases. It can be concluded that the finite difference explicit method is suitable and efficient method for solving the modified Burgers' equation.

KEYWORDS plasma, dusty plasmas, non-thermal ions, reductive perturbation method, modified Burgers' equation, finite difference explicit method, von Neumann stability analysis

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1. Introduction

In the last few decades, many researchers studied the electrostatic and electromagnetic waves propagation in dusty plasmas in various environments such as in the upper part of the Earth atmosphere, planetary rings, comet tails, interstellar space, the solar atmosphere and low temperature plasmas in laboratory [1–3]. Many authors [4, 5] investigated the effect of higher order nonlinearity for dusty plasma considering the negative ions and hot isothermal as well as non-isothermal electrons. Asgari et al. [6] derived the nonlinear Burgers' equation with a non-thermal ion in dusty plasma environment. Many authors [7–9] also investigated the nonlinear behaviors of electrostatic waves in a dusty plasma with trapped particles as well as in unmagnetized and magnetized plasmas. Dev et al. [10] have investigated the wave propagation in a non-magnetized and warm dusty plasma containing trapped electrons as well as non-thermal positive and negative ions under the influence of lower order nonlinearity. In this paper, we deal with the nonlinear modified Burgers' equation in dusty plasmas having negative and positive non-thermal ions with trapped electrons. The finite difference method was first developed by Thomas in 1920 to solve nonlinear hydrodynamic equations [11]. Finite difference methods are the first techniques for numerical solving of nonlinear partial differential equations [12]. The most commonly used finite difference methods for the solution of partial differential equations are as follows: Explicit method, Implicit method and Crank Nicolson method. Many authors applied finite difference explicit method to obtain numerical solutions of nonlinear partial differential equations. The modified Burgers' equation is a nonlinear expansion of the Burgers' equation. There are several methods proposed for solving the modified Burgers' equation that can be briefly presented as follows.

Bratsos et al. [13] applied a finite difference scheme for calculating the numerical solution of the modified Burgers' equation. A collocation method based on quantic splines was proposed by Ramadan and El-Danaf [14]. Irk [15] also proposed the sextic B-spline collocation method for numerical solution of the modified Burgers' equation. Aswin and Awasthi [16] have solved the modified Burgers' equation using iterative differential quadrature algorithms. Roshan and Bharma [17] solved the modified Burgers' equation by the Petrov–Galerkin method. Grienwank and El-Danaf [18] have employed a non-polynomial spline based method to obtain numerical solutions of the modified Burgers' equation. Duan et al. [19] proposed Lattice Boltzmann method to obtain numerical solution of modified Burgers' equation. Bashan et al. [20] used quintic B-spline Differential Quadrature method to solve modified Burgers' equation. Ucar et al. [21] proposed the finite difference method for numerical solution of the modified Burgers' equation. Karakoc et al. [22] have proposed the quartic B-spline subdomain finite element method (SFEM) for finding the numerical solution of the Burgers' equation and modified Burgers' equation. In the paper, we have used the finite difference explicit method to obtain the numerical solution of the modified Burgers' equation in dusty plasmas having non-thermal ions and trapped electrons. The finite difference approximation to partial derivatives can be obtained from Taylor series expansion using either the backward, forward or central difference approximations. The paper is organized in the following manner. In section 2, we discuss the basic equations governing the proposed dusty plasma system, modified Burgers' equation with trapped electrons and reductive perturbation technique. In section 3, the stability analysis of the finite difference explicit method is presented. We include the solutions and discussion of numerical results in section 4. Conclusion is given in section 5.

2. Basic equations and Modified Burgers' Equation

The basic equations, governing the dynamical system, of the dust particles in a one-dimensional dust acoustic wave for the dusty plasma is as follows [10]:

$$\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x}\left(n_d v_d\right) = 0, \tag{1}$$

$$\frac{\partial v_d}{\partial t} + v_d\frac{\partial v_d}{\partial x} + \frac{1}{m_d n_d}\frac{\partial p_d}{\partial x} = -\frac{q_d}{m_d}\frac{\partial \psi}{\partial x} + \mu\frac{\partial^2 v_d}{\partial x^2}, \tag{2}$$

$$\frac{\partial p_d}{\partial t} + v_d\frac{\partial p_d}{\partial x} + \gamma p_d\frac{\partial v_d}{\partial x} = 0, \tag{3}$$

$$\frac{\partial^2 \psi}{\partial x^2} = 4\pi e\left(n_e + n_n - n_p + z_d n_d\right), \tag{4}$$

where n_d is the number density of the negatively charged dust particles in the plasma, v_d is the dust fluid velocity, ψ is the electrostatic potential, z_d is the number of electrons residing on the dust surface at equilibrium, p_d is the pressure of the dust fluid, e is the electron charge and m_d is the mass of the dust particle and $\gamma = 3$ is the adiabatic index. The non-thermal number densities of positive ions n_p and negative ions n_n can be expressed by the using the following relations [23]:

$$n_p = n_{p0}\left(1 + \alpha\phi + \alpha\phi^2\right) exp\left(-p\phi\right), \tag{5}$$

$$n_n = n_{n0}\left\{1 + \alpha\sigma_p\phi + \alpha\left(\sigma_p\right)^2\right\} exp\left(z_n\sigma_p\phi\right) \tag{6}$$

with $\alpha = \frac{4\gamma_1}{1+3\gamma_1}$, where γ_1 represents the population of the non-thermal ions, and $\sigma_p = \frac{T_p}{T_n}$. Also, $z_p(z_n)$ is the positive (negative) ion's charge state, T_e is the electron temperature and $T_p(T_n)$ is the positive ion (negative ion) temperatures.

The electron density in the presence of trapped electron can be expressed by using the following relations [7, 24] :

$$n_e = n_{e0}\left\{1 + (\beta\phi) - b\left(\beta\phi\right)^{3/2} + \frac{1}{2}\left(\beta\phi\right)^2 - \ldots\right\}, \tag{7}$$

where $\beta = \frac{T_p}{T_e}$, $b = \frac{4\left(1-\gamma_2\right)}{3\sqrt{\pi}}$ and the parameter γ_2 is as follows $\gamma_2 = \frac{T_{ef}}{T_{et}}$, with T_{ef} and T_{et} being the temperatures of free electrons and trapped electrons in plasma, respectively.

The parameter γ_2 determines the nature of the distribution function, giving a plateau if $\gamma_2 = 0$ and a dip if $\gamma_2 < 0$ and a hump shape if $\gamma_2 > 0$. However, $\gamma_2 = 1$ corresponds to the Maxwellian distribution of the electrons. In the present plasma system, the range of γ_2 will be considered as $0 < \gamma_2 < 1$ for non-isothermal (trapped) electrons.

Now, N_d dust number density is normalized to its equilibrium value n_{d0}, V_d dust-fluid velocity is normalized to $C_{sd} = \left(\frac{Z_d k_B T_p}{m_d}\right)^{1/2}$, ϕ is the DA wave's potential normalized to $\frac{k_B T_p}{e}$, where k_B is the Boltzmann constant, the time variable T is normalized to $\omega_{pd}^{-1} = \left(\frac{m_d}{4\pi n_{d0} z_d^2 e^2}\right)^{1/2}$, the space variable X is normalized to $\lambda_{Dd}^{-1} = \left(\frac{4\pi n_{d0} z_d^2 e^2}{k_B T_p}\right)^{1/2}$, and pressure p_d is normalized to $p_d = n_{do} k_B T_d$.

The normalized forms of the basic equations (1 – 4) are as follows

$$\frac{\partial N_d}{\partial T} + \frac{\partial}{\partial X}\left(N_d V_d\right) = 0, \tag{8}$$

$$N_d\frac{\partial V_d}{\partial T} + N_d V_d\frac{\partial V_d}{\partial X} + \sigma_d\frac{\partial P_d}{\partial X} = N_d\frac{\partial \phi}{\partial X} + \eta\frac{\partial^2 V_d}{\partial X^2}, \tag{9}$$

$$\frac{\partial P_d}{\partial T} + V_d\frac{\partial P_d}{\partial X} + 3P_d\frac{\partial V_d}{\partial X} = 0, \tag{10}$$

$$\frac{\partial^2 \phi}{\partial X^2} = p_1\phi - p_2\phi^{3/2} + p_3\phi^2 - p_4\phi^{5/2} + \left(N_d - 1\right) \tag{11}$$

with the overall charge neutrality condition

$$n_{e0} = n_{p0} - Z_d n_{d0} - n_{n0} \tag{12}$$

and

$$\mu_p = \frac{n_{p0}}{Z_d n_{d0}}, \quad \sigma_d = \frac{T_d}{Z_d T_p}, \quad \mu_n = \frac{n_{n0}}{Z_d n_{d0}}, \quad \frac{n_{e0}}{Z_d n_{d0}} = \mu_p - \mu_n - 1, \quad \eta = \frac{\mu}{\omega_{pd}\lambda_D^2},$$

$$p_1 = \{(\mu_p - \mu_n - 1)\,\beta + \mu_n\,(z_n + \alpha\sigma) - \mu_n\,(\alpha - z_p)\},$$

$$p_2 = b\,(\mu_p - \mu_n - 1)\,\beta^{3/2},$$

$$p_3 = \left\{(\mu_p - \mu_n - 1)\,\frac{\beta}{2} + \mu_n\left(\frac{(z_n)^2}{2} + \alpha\sigma z_n + \alpha\,(\sigma)^2\right) - \mu_p\left(\frac{(z_p)^2}{2} - \alpha z_p + \alpha\right)\right\}$$

and

$$p_4 = (\mu_p - \mu_n - 1)\,b\beta^{5/2}.$$

The modified Burgers' equation for the propagation of small and finite amplitude dust-acoustic shock waves (DASWs) is derived. Here the independent variables ξ and τ are the new stretched variables given by $\xi = \epsilon^{1/2}\,(x - V_0 t)$ and $\tau = \epsilon\, t$, where V_0 is the phase speed (normalized to C_{sd}) and ϵ is a small parameter $(0 < \epsilon < 1)$ which measures the weakness of the dispersion. The dependent variables N_d, V_d, P_d and ϕ can be expanded in the power series as follows [10]:

$$N_d = 1 + \epsilon N_d^{(1)} + \epsilon^{3/2} N_d^{(2)} + \dots, \tag{13}$$

$$V_d = \epsilon V_d^{(1)} + \epsilon^{3/2} V_d^{(2)} + \dots, \tag{14}$$

$$P_d = 1 + \epsilon P_d^{(1)} + \epsilon^{3/2} P_d^{(2)} + \dots, \tag{15}$$

$$\phi = \epsilon\phi^{(1)} + \epsilon^{3/2}\phi^{(2)} + \epsilon^2\phi^{(3)} + \dots. \tag{16}$$

Substituting the stretched coordinates and the expression for N_d, V_d, P_d and ϕ into the normalized basic equations (8 – 11) and equating the coefficients of lower order in ϵ, the required lower order modified Burgers' equation is obtained

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\left(\phi^{(1)}\right)^{1/2}\frac{\partial\phi^{(1)}}{\partial\xi} = B\frac{\partial^2\phi^{(1)}}{\partial\xi^2}, \tag{17}$$

where the nonlinear coefficient A and dissipative coefficient B are given by the following expressions

$$A = \frac{3p_2\left(V_0^2 - 3\sigma_d\right)}{4p_1 V_0}, \quad B = \frac{\eta}{2}. \tag{18}$$

Equation (17) represents the well-known lower order modified Burgers' equation describing the nonlinear propagation of dust-acoustic shock waves in electronegative dusty plasma with non-thermal ions and trapped electrons.

The stationary shock wave solution of the modified Burgers' equation (17) is obtained by transforming independent variables ξ and τ to $\xi = \zeta - U_0\tau'$ and $\tau = \tau'$ where U_0 is the speed of the shock waves.

Now, the analytical solution of the modified Burgers' equation is given by the formula

$$\phi^{(1)} = \left\{\phi_{m1}\left\{1 - \tanh\left(\frac{\xi}{\delta_1}\right)\right\}\right\}^2, \tag{19}$$

where $\phi_{m1} = \dfrac{3M}{4A}$ and $\delta_1 = \dfrac{4B}{M}$ are the amplitude and the width of the shock waves, respectively, and M is the Mach number.

3. Stability analysis of the explicit finite difference method

In this section, the stability of the finite difference explicit method is investigated by using von Neumann stability analysis. The von Neumann stability theory in which the growth factor of a Fourier mode is defined as $u_{i,j} = \xi^j e^{Ikhi} = \xi^j e^{I\theta i}$ where $I = \sqrt{-1}$, ξ^j is the amplitude at time level k is the wave number and $h = \Delta x$.

To investigate the stability of the numerical scheme, the non-linear term $u^{1/2}u$ of the modified Burgers' equation has been linearized by putting $u^{1/2} = L$, where L is constant.

From (29):

$$u_{i,j+1} = u_{i,j} + \frac{kAL}{2h}\Big[u_{i-1,j} - u_{i+1,j}\Big] + \frac{kB}{h^2}\Big[u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\Big],$$

$$u_{i,j+1} = \left(1 - \frac{2kB}{h^2}\right)u_{i,j} + \left(\frac{kAL}{2h} + \frac{kB}{h^2}\right)u_{i-1,j} + \left(\frac{kB}{h^2} - \frac{kAL}{2h}\right)u_{i+1,j}.$$

Substitute $u^{1/2} = L$ in the above equation, we get

$$\xi^j e^{I\theta i}\xi = \xi^j e^{I\theta i}\left[\left(1 - \frac{2kB}{h^2}\right) + \left(\frac{kAL}{2h} + \frac{kB}{h^2}\right)e^{-I\theta} + \left(\frac{kB}{h^2} - \frac{kAL}{2h}\right)e^{I\theta}\right],$$

$$\xi = \left(1 - \frac{2kB}{h^2}\right) + \left(\frac{kAL}{2h} + \frac{kB}{h^2}\right)e^{-I\theta} + \left(\frac{kB}{h^2} - \frac{kAM}{2h}\right)e^{I\theta},$$

$$\xi = \left(1 - \frac{2kB}{h^2}\right) + \frac{kAL}{2h}\left(e^{-I\theta} - e^{I\theta}\right) + \frac{kB}{h^2}\left(e^{I\theta} + e^{-I\theta}\right),$$
$$\xi = \left(1 - \frac{2kB}{h^2}\right) + \frac{kAL}{2h}\left(-2I\sin\theta\right) + \frac{kB}{h^2}2\cos\theta,$$
$$\xi = \left(1 - \frac{2kB}{h^2}\right) - \frac{kAL}{h}\sin\theta + \frac{2kB}{h^2}\cos\theta.$$

The stability condition for the numerical scheme is as follows

$$|\xi| \leq 1,$$
$$|\xi| = \left|\left(1 - \frac{2kB}{h^2}\right) - \frac{kAL}{h}\sin\theta + \frac{2kB}{h^2}\cos\theta\right| \leq 1.$$

So the stability condition is $\frac{2kB}{h^2} \leq 1$ or $k \leq \frac{h^2}{2B}$. And thus restricts the allowable temporal step size. Since not all choices of spatial and temporal steps lead to convergent, the explicit scheme (29) is called conditionally stable.

4. Numerical results and discussion

It has been shown in Section 2 that the modified Burgers' equation has an analytical solution in the form (19). In this paper, the modified Burgers' equation (17) in dusty plasmas with non-thermal ions and trapped electrons is solved by the explicit finite difference method and the numerical results are compared with the analytical solutions.

For simplicity, we consider $\phi^{(1)}(\zeta,\tau) = u(x,t) \cong u(i\Delta x, j\Delta t) \cong u_{i,j}$.

Equation (17) can be expressed as

$$\frac{\partial u}{\partial t} + Au^{1/2}\frac{\partial u}{\partial x} = B\frac{\partial^2 u}{\partial x^2}. \tag{20}$$

For convenience, we put $M = \frac{1}{2}$, $U_0 = \frac{1}{2}$, $\phi_{m1} = \frac{3}{8A}$ and $\delta_1 = \frac{4B}{M} = 8B$ in (19).

The analytical solution of the modified Burgers' equation is given by the following expression

$$u(x,t) = \left\{\frac{3}{8A}\left\{1 - \tanh\frac{1}{8B}\left(x - \frac{t}{2}\right)\right\}\right\}^2. \tag{21}$$

The boundary and the initial conditions are taken from the exact solution. In this paper, the numerical solutions of (20) will be sought for the following initial and boundary conditions. In the interval $0 \leq x \leq 1$, $t \geq 0$, and with the initial condition

$$u(x,0) = \left\{\frac{3}{8A}\left\{1 - \tanh\frac{x}{8B}\right\}\right\}^2 \tag{22}$$

and the boundary conditions

$$u(0,t) = \left\{\frac{3}{8A}\left\{1 + \tanh\frac{t}{16B}\right\}\right\}^2, \tag{23}$$
$$u(1,t) = \left\{\frac{3}{8A}\left\{1 - \tanh\frac{1}{8B}\left(1 - \frac{t}{2}\right)\right\}\right\}^2. \tag{24}$$

We discretize the modified Burgers' equation by replacing $\frac{\partial u}{\partial t}$ by the forward difference and $\frac{\partial u}{\partial x}$ and $\frac{\partial^2 u}{\partial x^2}$ by the central difference approximation, i.e. as follows

$$\frac{\partial u}{\partial t} \approx \frac{u_{i,j+1} - u_{i,j}}{k}, \tag{25}$$
$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}, \tag{26}$$
$$\frac{\partial u}{\partial x} \approx \frac{u_{i+1,j} - u_{i-1,j}}{2h}. \tag{27}$$

Thus, (20) becomes as follows

$$\frac{u_{i,j+1} - u_{i,j}}{k} + Au_{i,j}{}^{1/2}\left[\frac{u_{i+1,j} - u_{i-1,j}}{2h}\right] = B\left[\frac{u_{i+1,j} - u_{i,j} + u_{i-1,j}}{h^2}\right], \tag{28}$$

which can be simplified

$$u_{i,j+1} = u_{i,j} + \frac{kA}{2h}u_{i,j}{}^{1/2}\Big[u_{i-1,j} - u_{i+1,j}\Big] + \frac{kB}{h^2}\Big[u_{i+1,j} - 2u_{i,j} + u_{i-1,j}\Big]. \tag{29}$$

By the von Neumann stability condition, we consider the space steps $h = 0.01$, the time steps $k = 0.005$, 0.001 and 0.0005 and $B = 0.01$, 0.05 and 0.1. The nonlinear coefficient A is a function of p_1, p_2, σ_d and V_0. During the solution process, we consider $A = 1$, 1.5 and 2. Numerical solutions are obtained for different values of B and A. The obtained figures of the numerical results are compared with the figures obtained from the analytical solutions which are displayed in Figs. 1–3.

The accuracy of the present method is measured using the absolute error which is defined as $\left|u_i^{\text{Analytical}} - u_i^{\text{Numerical}}\right|$.

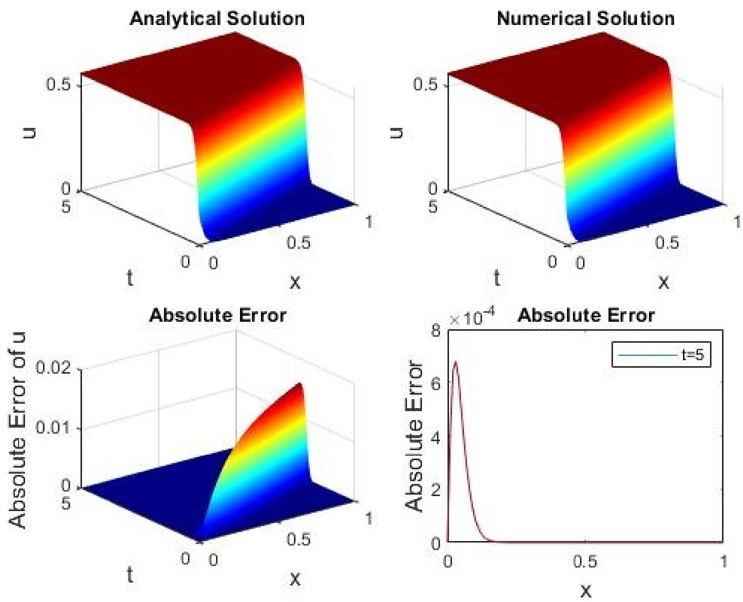

FIG. 1. Graph of analytical and numerical solutions with absolute error at $A = 1$, $B = 0.01$, $h = 0.01$, $k = 0.005$

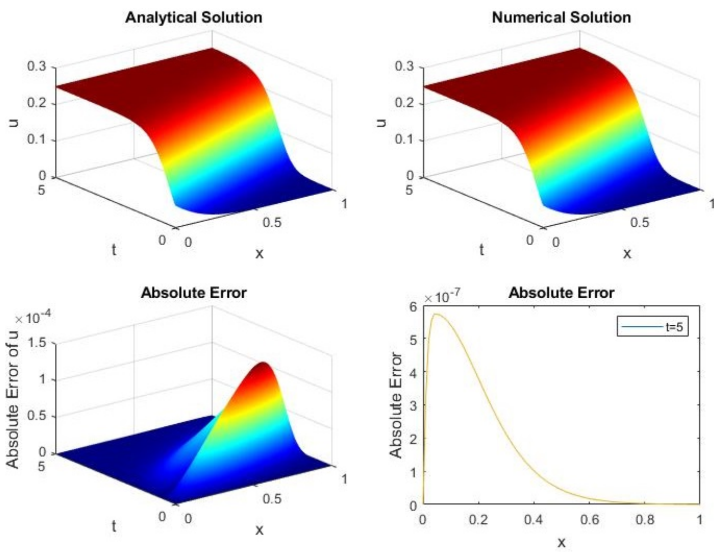

FIG. 2. Graph of analytical and numerical solutions with absolute error at $A = 1.5$, $B = 0.05$, $h = 0.01$, $k = 0.001$

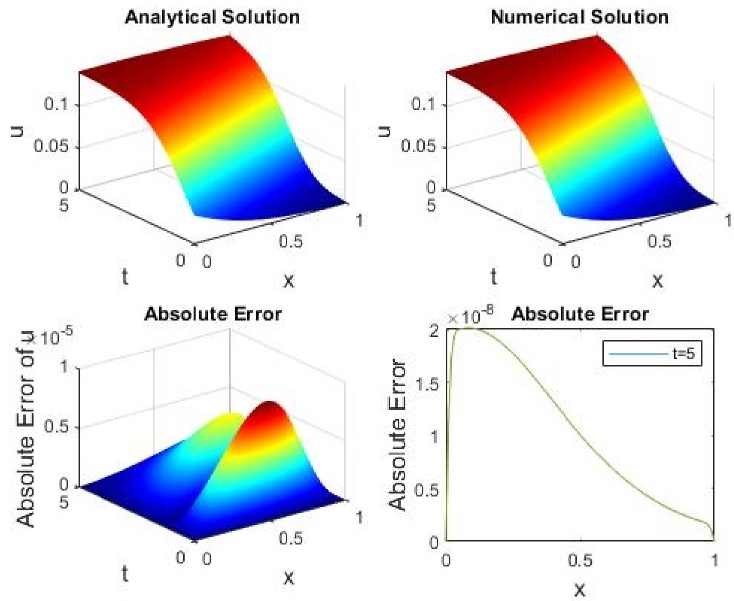

FIG. 3. Graph of analytical and numerical solutions with absolute error at $A = 2$, $B = 0.1$, $h = 0.01$, $k = 0.0005$

The analytical solutions and the computed numerical results together with their errors are plotted in Figs. 1 to 3 for various values of the nonlinear coefficient and the dissipative coefficient. But the graphs of the errors have been drawn at time $t = 5$. It can be seen that the maximal error occurs at the left hand boundary when the greater value of the dissipative coefficient $B = 0.1$ is considered and the maximal error is found around the location where the shock wave has the highest amplitude with the smaller value of the dissipative coefficient $B = 0.01$. It can be concluded that as the value of x increases, the errors gradually decreases and it is also seen that as the value of the dissipation coefficient increase, the error will increase.

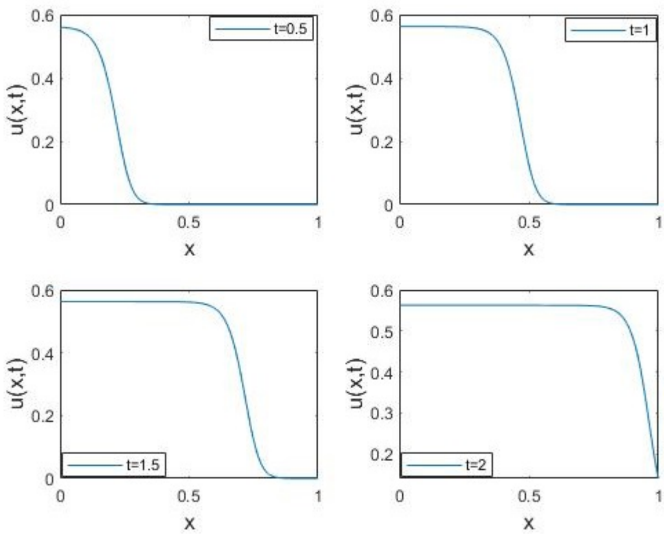

FIG. 4. The numerical solutions with $A = 1$, $B = 0.01$, $h = 0.01$, $k = 0.005$ at $t = 0.5, 1, 1.5, 2$

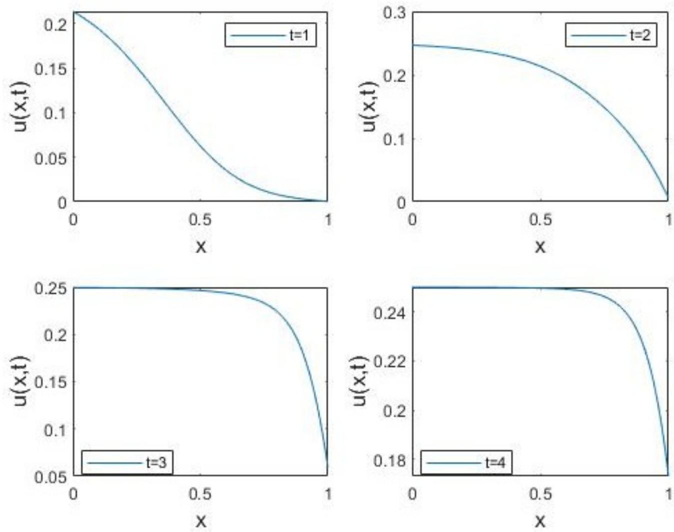

FIG. 5. The numerical solutions with $A = 1.5$, $B = 0.05$, $h = 0.01$, $k = 0.001$ at $t = 1, 2, 3, 4$

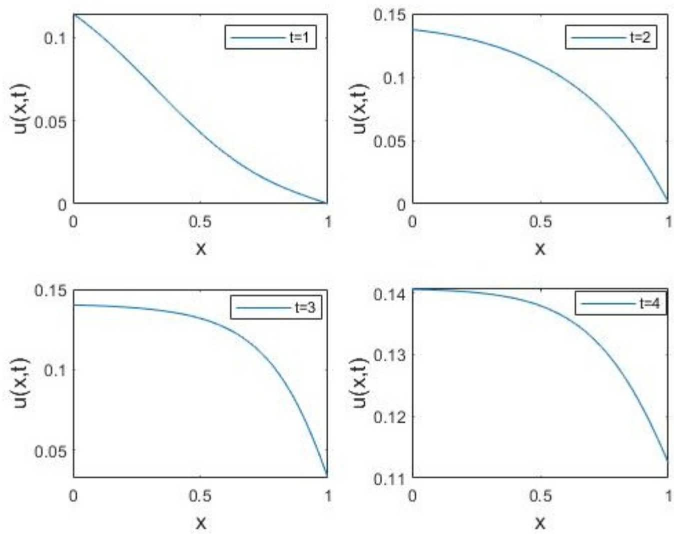

FIG. 6. The numerical solutions with $A = 2$, $B = 0.1$, $h = 0.01$, $k = 0.0005$ at $t = 1, 2, 3, 4$

The computed numerical results for various values of the nonlinear coefficient and the dissipative coefficient at different times are plotted in Figs. 4 to 6. From the figures, it has been observed that as the time increases, the curve of the numerical solution decays.

5. Conclusion

In this paper, the finite difference explicit method has been successfully used for obtaining the numerical solution of the modified Burgers' equation in dusty plasmas having non-thermal ions with trapped electrons. It is obtained using finite difference explicit method. Graphs have been plotted to show a comparison between the analytical and the numerical solutions for various values of the dissipative coefficient. The obtained numerical results show good accuracy when comparing it with the analytical results for various values of the dissipative coefficient. The absolute error has been computed and presented in graphical form. The effects of the nonlinear coefficient and the dissipative coefficient on the shock strength and steepness are investigated. It is found that the shock wave steepness depends more on the dissipative coefficient than on the nonlinear coefficient. In this study, it has been seen that when the dissipative coefficient decreases, the shock waves become flatten and the propagation front becomes steeper. In conclusion, it has been observed that the dissipative coefficient in dusty plasmas having non-thermal ions and trapped electrons plays an important role to dissipate the acoustic shock wave while propagating through the system.

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