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Original article

Tunneling current of contact of fractal object with metal and superlattice

Natalia N. Konobeeva[a], Mikhail B. Belonenko[b]

Volgograd State University, University avenue, Volgograd, Russia

[a]yana_nn@volsu.ru, [b]belonenko@volsu.ru

Corresponding author: Natalia N. Konobeeva, yana_nn@volsu.ru

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ABSTRACT In this paper, we study the features of the electric current under conditions of the tunnel effect in fractal structures. Based on the electron dispersion law for fractal objects, an expression for finding the tunneling current is obtained. Current-voltage characteristics are constructed for the following contacts: fractal-fractal, fractal-metal, fractal-superlattice. The influence of the fractal dimension on the characteristics of the tunneling current is revealed.

KEYWORDS fractals, fractions, tunneling, current-voltage characteristic.

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1. Introduction

Recently, more and more attention of researchers has been attracted by sets of fractional dimensions [1] – fractals, which allow one not only to describe physical, biological and other phenomena, but also to obtain new theoretical and practical results in various fields of human activity, including medicine [2,3], biology [4], astrophysics [5], geography [6]. Fractals also made a great contribution to the development of modern electrical engineering and electronics. As an example, we can note: a fractal antenna [7,8], a fractal capacitor [9], fractal coding [10], fractal analysis of power system failures [11], elastic electronics devices [12]. A wide range of use of fractals is possible due to their properties such as self-similarity, space filling and fractal dimension, different from the topological one.

It is also known, that elastic vibrations occur in materials with a fractal structure. In this case, the concept of localized vibrational states, called fractons, is introduced [13]. Interest in this class of quantum states of matter is increasing every year, which is associated with the search for new states of matter, which is the main direction in condensed matter physics [14–18]. Similarly, electronic states in fractal structures will be localized, by analogy with Anderson localization [19] and, accordingly, give a nontrivial contribution, for example, to the tunneling current.

Although the tunneling current measurement method itself has a number of disadvantages (the distance between the samples determines the current, the degree of "roughness" of the samples, etc.), in our opinion, it has the advantage that it allows optimizing the choice of the second sample to achieve the highest sensitivity (for example, see [20]). In this paper, fractals are considered on the basis of two-dimensional lattices, and the formalism allows generalization to the case of a higher dimension.

2. Basic equations

Our system is a contact of two different materials: the first of which is a fractal object, and the second is a metal, a superlattice or a fractal lattice. Thus, a metal probe, superlattice (SR) or other fractal is brought to the fractal at an angle of 90 °C. The aim of this work is to calculate the tunneling current in the considered system of two contacting materials.

The electron dispersion law for a fractal object can be written as:

$$\varepsilon_1 (p_x, p_y) = V \left(p_x^2 + p_y^2\right)^{0.5\sigma}, \tag{1}$$

here (p_x, p_y) are the electron quasi-momentum components, V is the analogue of the Fermi velocity for fractals, σ is the dimension that is used to describe the fracton states [21]:

$$\sigma = \frac{2d_f}{d_w}, \tag{2}$$

d_f is the fractal dimension of an object, d_w is the diffusion index.

The dimension of a fractal is one of its most important characteristics [22]. In the general case, there are several definitions of this quantity. In particular, fractal dimensions can be divided into two classes, which are called "metric" and "probabilistic". The former describes only the geometry of a metric space. The latter takes into account both the geometry of the given set and the probability distribution supported by this set [23]. In this paper, we will be interested in the geometric fractals. Therefore, by fractal dimension, we mean the degree of space filling by it or a measure of the degree of geometric irregularity of an object. Note that the fractal dimension is not an integer. For example, for the Koch curve it is ≈ 1.26, for the Cesaro fractal it is about 1.78.

An important point is the choice of indicator σ. For the case of the simplest lattices without defects (hexagonal, square, triangular, similar three-dimensional ones), σ is calculated, as well as all other exponents and dimensions, analytically. Here, by defects, we mean the absence of certain bonds in the lattice. In this case, it coincides with the exponent in the electron dispersion law (1), which is also considered analytically. It is natural to assume that similar calculations can be performed for decorated gratings, which corresponds to the analogy with fracton modes.

The expression for the contact current density is given by the following formula [24]:

$$\begin{aligned} J(U) &= \frac{4\pi e\,|T|^2}{\hbar}\int_{-\infty}^{\infty} d\varepsilon\,\cdot\nu_1(\varepsilon+eU)\cdot\nu_2(\varepsilon)\left(N_F(\varepsilon)-N_F(\varepsilon+eU)\right), \\ \nu_i(\varepsilon) &= \iint_{px,py}\delta(\varepsilon-\varepsilon_i)\cdot dp_x\cdot dp_y, \end{aligned} \tag{3}$$

where $\delta(x)$ is the Dirac delta function, $\nu_i(\varepsilon)$ is the tunneling density of states for the i-th contact; $N_F(\varepsilon)$ is the equilibrium number of fermions with energy ε, T is the matrix element of the tunneling operator. Here and below, we use the "rough" contact approximation. That is, the surface of the fractal object is perpendicular to the surface of the contact material. This limitation is not fundamental and corresponds to the conditions of the experiment. Note that, for definiteness, we will apply stress to the fractal.

Here we use the Kubo approach. Within the framework of this approach, the tunneling current is determined only by the equilibrium number of electrons and the density of states of the contacting objects. The matrix element T is determined by the properties of the contact itself, i.e. the distance between objects, the angle of inclination (if one of the objects is made in the form of a probe), etc. Note that this approach makes it possible to do without solving Schrödinger-type equations on fractals [25–28] and use only an assumption about the form of the density of states. In addition to the proposed approach, the density of states can also be obtained by directly diagonalizing the Hamiltonian, which takes into account the electron energy at the fractal node and jumps between nodes. This approach has its limitations related to the size of the fractal and the computational resources. Therefore, in this paper, we chose the assumption of the density of states.

Let us choose a fractal, a metal (4) and a superlattice (5) as the materials with which the fractal object comes into contact. The electronic spectrum for them can be written in the following form:

$$\varepsilon_2(p_x,p_y)=\frac{p_x^2+p_y^2}{2m}, \tag{4}$$

where m is the effective electron mass.

$$\varepsilon_2(\mathbf{p})=\varepsilon_0-\Delta\cdot\cos(\mathbf{k}\cdot\mathbf{p}), \tag{5}$$

ε_0 is the quantum well electron energy, Δ is the tunneling integral determined by the overlap of the electronic wave functions in neighboring wells, $p=(p_x,p_y)$, k is the superlattice reciprocal lattice vector.

Next, we calculate the tunneling density of states for all types of contact materials using the properties of the Dirac delta function:

$$\nu_1(\varepsilon)=\frac{\varepsilon^{(3-\sigma)/\sigma}}{2\pi^2\sigma V^{4-\sigma}}, \tag{6}$$

$$\nu_2(\varepsilon)=\frac{\sqrt{2m^3\varepsilon}}{\pi^2}, \tag{7}$$

$$\nu_2(\varepsilon)=\frac{1}{\Delta\cdot\sqrt{1-\left(\frac{\varepsilon_0-\varepsilon}{\Delta}\right)^2}}. \tag{8}$$

Formulas (6), (7) and (8) correspond to the fractal, metal and superlattice.

3. Main results and discussion

For definiteness, we choose the Sierpinski carpet (Fig. 1) as a fractal object, for which the value of d_w can be calculated using the effective volume resistance [29].

Note that the fractal generator is defined by two numbers (a, b), where a is the size of the generator, b is the size of the holes. The fractal dimension of the carpet can be calculated as:

$$d_f=\frac{\log\left(a^2-b^2\right)}{\log a}. \tag{9}$$

Note that at first glance, one can assume that the dimension is taken into account very roughly and the properties are the same for different quantum graphs. But, as shown in [29] and formula (9), the quantities a and b affect the fractal dimension. And for graphs with different geometry, the dimension also changes.

Equation (3) is solved using numerical integration.

The dependence of the tunneling current for different materials in the contact on the voltage is shown in Fig. 2.

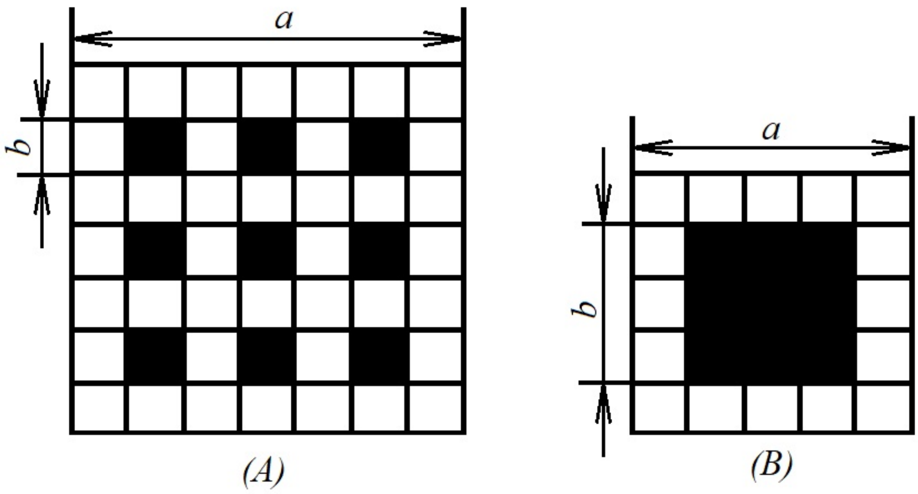

FIG. 1. Sierpinski carpet: (A) generator (7, 3); (B) generator (5,3)

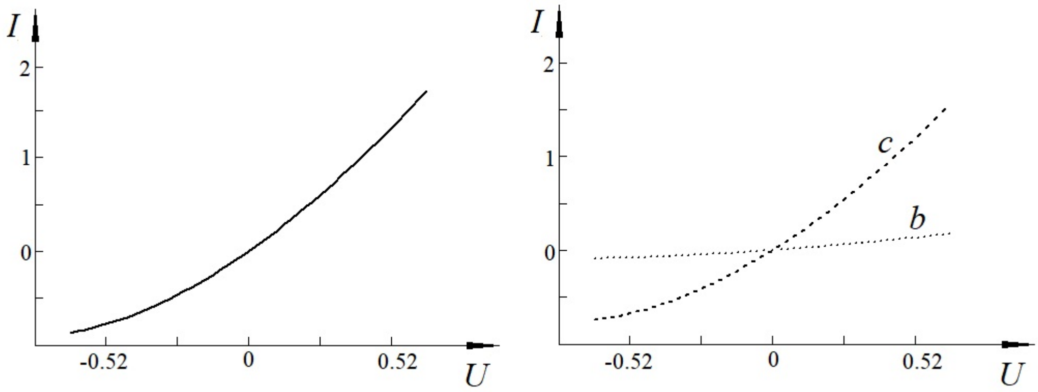

FIG. 2. Dependence of the tunneling current on voltage ($\sigma = 1.801$) for the contact of the fractal Sierpinski carpet (7,3): (a) with the Sierpinski carpet fractal (5,3); (b) with metal; (c) with superlattice. The non-dimensional unit of I-axis corresponds to μA (for curve a), mA (for curves b-c). The non-dimensional unit of U-axis corresponds to 1 V

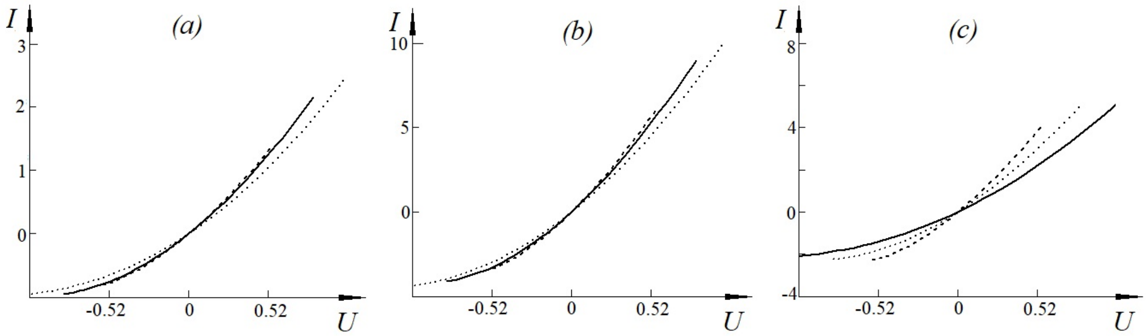

FIG. 3. Tunnel current versus voltage for different values of σ: (a) with Serpinsky carpet (5,3); (b) with metal; (c) with a superlattice. The solid line corresponds to $\sigma = 1.2$, the dotted line corresponds to $\sigma = 1.6$, the dotted line corresponds to $\sigma = 1.9$. The non-dimensional unit of I-axis corresponds to μA (for figure a), 0.1 mA (for figure b), mA (for figure c). The non-dimensional unit of U-axis corresponds to 1 V

Figure 2 shows the asymmetric nature of the dependence of the current on the voltage applied to the contact. This is explained by the features of the electronic structure (density of states) of the fractal object. We also note that the CVC for all three cases corresponds to the diode type. We can consider our system as a tunnel diode, since it is asymmetrical. Since on one side of the contact there is a fractal object, and on the other side there is a metal/SR/another fractal. The situation here is similar to a conventional tunnel contact, for example, a metal with a semiconductor. Fig. 2a is built separately due to the lower current value (by 3 orders of magnitude).

The influence of the dimension σ on the tunneling current in different contacts is shown in Fig. 3.

As can be seen from Fig. 3, the fracton dimension, and, consequently, the form of the fractal object, has a significant effect on the CVC of the tunnel contact. This is especially pronounced for the case with a superlattice in contact. We note an important practical application. By preparing a fractal from one material (for example, using a laser-based procedure [30]), it is possible to achieve different current-voltage characteristics in tunneling contacts.

4. Conclusion

Here we formulate the main results:

(1) The expression for the tunneling current density of a two-dimensional material with a fractal structure is obtained.
(2) The current-voltage characteristics of a fractal object with a metal and a superlattice are constructed. Its asymmetric nature is found, which indicates the possibility of using fractal elements in diodes and transistors.
(3) Possibility to control the magnitude of the tunneling current using the fractal dimension determined by the fractal structure of the material is shown.

References

[1] Hausdorff F. Dimension und äuβeres Maβ. *Math. Ann.*, 1919, **79**, P. 157–179.
[2] Heymans O., Fissette J., Vico P., Blacher S., Masset D., Brouers F. Is fractal geometry useful in medicine and biomedical sciences? *Med Hypotheses*, 2000, **54**(3), P. 360-6.
[3] Uahabi K.L., Atounti M. Applications of fractals in medicine. *Annals of the University of Craiova, Mathematics and Computer Science Series*, 2015, **42**(1), P. 167–174.
[4] Buldyrev S.V. *Fractals in Biology. In: Meyers, R. (eds) Encyclopedia of Complexity and Systems Science*. Springer, New York, 2009, P. 3779–3802.
[5] Criscuoli S., Rast M.P., Ermolli I., Centrone M. On the reliability of the fractal dimension measure of solar magnetic features and on its variation with solar activity. *Astronomy and Astrophysics*, 2007, **461**(1), P. 331–338.
[6] Husain A., Reddy J., Bisht D., Sajid M. Fractal dimension of coastline of Australia. *Scientific Reports*, 2021, **11** Article number 6304.
[7] Abed A.T., Abu-AlShaer M.J., Jawad A.M.Fractal Antennas for Wireless Communications. In book:*Modern Printed-Circuit Antennas* Ed. H. Al-Rizzo. IntechOpen, London, 2020, 176 p.
[8] De Nicola F., Puthiya Purayil N.S., Spirito D., Miscuglio M., Tantussi F., Tomadin A., De Angelis F., Polini M., Krahne R., Pellegrini V. Multiband plasmonic Sierpinski carpet fractal antennas. *ACS Photonics*, 2018, **5**, P. 2418.
[9] Liu W., Wang Z., Wang G., Zeng Q., He W., Liu L., Wang X., Xi Y., Guo H., Hu C., Wang Z.L. Switched-capacitor-convertors based on fractal design for output power management of triboelectric nanogenerator. *Nature Communications*, 2020, **11** Article number 1883.
[10] Menassel R. Optimization of fractal image compression. In book: *Fractal Analysis* Ed. R. Koprowski, IntechOpen, London, 2020, 128 p.
[11] Tiwari A. Fractal applications in electrical and electronics engineering. *International Journal of Engineering Science and Advanced Technology*, 2012, **2**(3), P. 406–411.
[12] Fan J.A., Yeo W.-H., Su Y., Hattori Y., Lee W., Jung S.-Y., Zhang Y., Liu Z., Cheng H., Falgout L., Bajema M., Coleman T., Gregoire D., Larsen R.J., Huang Y., Rogers J.A. Fractal design concepts for stretchable electronics. *Nature Communications*, 2014, **5**, Article number 3266.
[13] Alexander S., Laerman C., Orbach R., Rosenberg H.M. Fracton interpretation of vibrational properties of crosslinked polimers, glasses and irradiated quartz. *Phys. Rev. B.*, 1983, **28**(8), P. 4615–4619.
[14] Yoshida B. Exotic topological order in fractal spin liquids. *Phys. Rev. B*, 2013, **88**, P. 125122.
[15] Vijay S., Haah J., Fu L. A new kind of topological quantum order: A dimensional hierarchy of quasiparticles built from stationary excitations. *Phys. Rev. B*, 2015, **92**, P. 235136.
[16] Williamson D.J. Fractal symmetries: Ungauging the cubic code, *Phys. Rev. B*, 2016, **94**, P. 155128.
[17] Ma H., Schmitz A.T., Parameswaran S.A., Hermele M., Nandkishore R.M. Topological entanglement entropy of fracton stabilizer codes. *Phys. Rev. B*, 2018, **97**, P. 125101.
[18] Slagle K., Kim Y.B. Fracton topological order from nearest-neighbor two-spin interactions and dualities. *Phys. Rev. B*, 2017, **96**, P. 165106.
[19] Anderson P.W. Absence of diffusion in certain random lattices, *Physical Review*, 1958, **109**(5), P. 1492–1505.
[20] Konobeeva N.N., Belonenko M.B. Sensitivity characteristics of germanene. *Nanosystems: physics, chemistry, mathematics*, 2018, **9**(6), P. 770–774.
[21] Zosimov V.V., Lyamshev L.M. Fractals and scaling in acoustics. *Acoustical Physics*, 1994, **40**(5), P. 627–653.
[22] Zhikharev L.A. Fractal dimensionalities. *Geometry and Graphics*, 2018, **6**(3), P. 33–48.
[23] Adams H., Aminian M., Farnell E., Kirby M., Peterson C., Mirth J., Neville R., Shipman P., Shonkwiler C. A fractal dimension for measures via persistent homology. In: Baas N., Carlsson G., Quick G., Szymik M., Thaule M. (eds), Topological Data Analysis. Abel Symposia, vol 15. Springer, 2020.
[24] Levitov L.S., Shitov A.V. *Green's functions. Problems with solutions*. Fizmatlit, Moscow, 2003, 392 p.
[25] Dalrymple K., Strichartz R.S., Vinson J.P. Fractal differential equations on the Sierpinski gasket. *The Journal of Fourier Analysis and Applications*, 1999, **5**(73), P. 203–284.
[26] Fukushima M., Shima T. On a spectral analysis for the Sierpinski gasket. *Potential Analysis*, 1992, **1**, P. 1-35.
[27] Strichartz R.S. Analysis on Fractals. *Notices of the AMS*, **46**(10), P. 1199–1208.

[28] Strichartz R.S. Some properties of Laplacians on fractals. *Journal of Functional Analysis*, 1999, **164**, P. 181–208.

[29] Kim M.H., Yoon D.H., Kim I. Lower and upper bounds for the anomalous diffusion exponent on Sierpinski carpets. *J. Phys. A Math. Gen.*, 1993, **26**, P. 5655–5660.

[30] Reinhardt H., Kroll M., Karstens S.L., Schlabach S., Hampp N.A., Tallarek U. Nanoscaled fractal superstructures via laser patterning – A versatile route to metallic hierarchical porous materials. *Adv. Mater. Interfaces*, 2020, **8**, P. 2000253.

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Information about the authors:

Natalia N. Konobeeva – Volgograd State University, University avenue, 100, Volgograd, 400062, Russia;
ORCID 0000-0002-6043-9555; yana_nn@volsu.ru

Mikhail B. Belonenko – Volgograd State University, University avenue, 100, Volgograd, 400062, Russia;
ORCID 0000-0003-2003-3793; belonenko@volsu.ru

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