NANOSYSTEMS:
PHYSICS, CHEMISTRY, MATHEMATICS

Akhmadjanov T., Rakhimov E.Yu. *Nanosystems: Phys. Chem. Math.*, 2023, **14** (2), 158–163.
http://nanojournal.ifmo.ru
DOI 10.17586/2220-8054-2023-14-2-158-163

Original article

Corrugated non-stationary optical fiber

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ABSTRACT Using numerical methods, we study the fractal properties of the optical paths difference for rays propagating in a model of a homogeneous optical fiber with periodically curved (corrugated) wall and other wall periodically oscillating according to the sine law. Also the angle of entry of the rays into the optical fiber and their coordinates in the exit plane is investigated.

KEYWORDS numerical analysis, dynamics, optical fiber, corrugation, phase image, optical path difference, fractal property

FOR CITATION Akhmadjanov T., Rakhimov E.Yu. Corrugated non-stationary optical fiber. *Nanosystems: Phys. Chem. Math.*, 2023, **14** (2), 158–163.

1. Introduction

One of the main factors limiting the range of wave propagation (for example, low-frequency sound in the ocean [1] or optical rays in an optical fiber [2]) is the medium inhomogeneity in the direction of wave propagation. As a result of the influence of such inhomogeneity on the propagation time of modes in an optical fiber, the intermode dispersion and chaos appear which affect on the possibility of using the optical fiber in telecommunications and many other areas. This requires a reduction of the dispersion effect, i.e., the propagation time of the mode. At present, there are various ways of compensation the dispersion in optical fibers [3–5].

Interesting theoretical and experimental results concerning to the propagation of electromagnetic waves in inhomogeneous or corrugated optical fibers were obtained [6–10]. The problems of such type are intensively studied last decade [11–15]. In some cases, the inhomogeneity can be described as periodic perturbation along the optical fiber axis (analogously to internal waves in the ocean [1]). In this case, a group of waves, the length of which is in resonance with the inhomogeneity of the medium, can be caught in the effective optical fiber channel [16]. This effect is similar to nonlinear resonance in classical mechanics [17].

Numerical methods were used in the simplest model of homogeneously filled optical fiber, one wall of which is periodically curved (corrugated) and absolutely reflective. Fine characteristics of the optical fiber dynamics, in particular, the fractal properties of the propagation time and spatial frequency, were determined [18]. In the model of a cosine bent optical fiber, a phase map and a histogram of optical fibers were obtained and the level of chaos in them was studied [19]. The dynamics of particles in this system, the change of their energy in time were studied, and their phase images were determined using the reflection obtained during the space spreading of the stationary and non-stationary billiard plane of the stadium [20–22].

In this work, using numerical methods, we studied the fractal properties of the difference in the optical paths of rays propagating in the model of uniformly filled optical fiber one wall of which is periodically curved (corrugated) and the second wall oscillates periodically according to the sinusoidal law. The walls assumed to be absolutely reflective. The dependence of the coordinates on the optical fiber exit plane of the angle of entry of rays into the optical fiber is studied.

2. Numerical experiment

Let us consider the simplest model of an absolutely reflecting, homogeneously filled optical fiber with one wall periodically curved (corrugated) (Fig. 1). Here a is the non-corrugated channel width, L is the spatial period of the corrugated wall, and z is the longitudinal coordinate. The curve describing the difference between the wall and the non-corrugated level in one period is determined by the function $f(\xi)$, where $\xi = \{z/L\}$ is the fractional part of the normalized longitudinal coordinate z/L. Naturally, one the following inequality holds: $0 < \xi < 1$. Let the non-corrugated upper wall periodically oscillate according to the following law: $y_0 = a_0 \sin(\omega t)$. Correspondingly, the optical fiber width equals to $a + y_0$. Here a_0 is the vibration amplitude for the wall, and ω is the oscillation frequency.

Let the light source be located on the upper wall of the optical fiber at the point $z_0 = 0$ along the longitudinal coordinate. The optical fiber path consists of straight segments that return sequentially from the walls of the optical fiber. Let's denote by z_n the nth ray return from the oscillating upper wall, and denote the angle between the ray path and the z

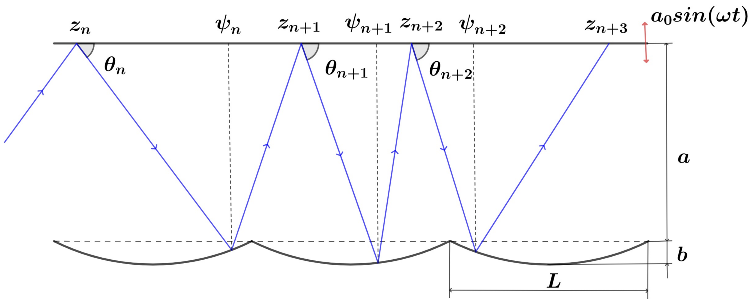


FIG. 1. The model of the optical fiber channel

axis by θ_n. The relationship between the values $(z_n,\ \theta_n)$ and $(z_{n+1},\ \theta_{n+1})$ for single return from the corrugated wall is expressed by the following system of equations obtained from simple geometry:

$$\begin{cases} \psi_n = z_n + (a + y_0 + f(\psi_n)) \cot\theta_n, \\ \theta_{n+1} = \theta_n - 2\arctan\left(f'(\psi_n)\right), \\ z_{n+1} = z_n + (a + y_0 + f(\psi_n)) \cot\theta_{n+1}. \end{cases} \tag{1}$$

In this expression, ψ_n is the longitudinal coordinate of the point of collision of the nth time collision of the ray coming from the vibrating upper wall with the lower corrugated wall. The phase frequency of the optical fiber is:

$$\Omega = \frac{2\pi}{L}. \tag{2}$$

The system of equations (1) is written with hidden function $f(\xi)$. To write it explicitly, one needs an explicit representation of the function $f(\xi)$. We use the following profile at one period: $f(\xi) = 4b\xi(1-\xi)$. The maximum deviation of the wall from the non-corrugated level is b.

Solving the system (1), one obtains the results presented in Fig. 2(a,b). This figure shows the results of a numerical calculation of the relationship between the optical path difference $\Delta S(\theta_0, z) = S_0(\theta_0, z) - z$ at distance z and the initial exit angle θ_0 from the light source at distance $z_0 = 0$. In this work, when the difference in the optical paths of the rays entering an optical fiber of length z at close angles to each other is equal to the difference $\Delta S_n(\theta_{0n}, z) - \Delta S_{n-1}(\theta_{0n-1}, z) = 10^{-12}$ in the optical paths of the rays leaving the optical fiber The difference in the optical paths of the rays $\Delta\theta_0$ entering at these two angles was considered mutually equal. The difference of the input angle from each other is the sample time. N is the number of resonant input angles (i.e., the pitch width). It is possible to exit back for some optical fibers. Their number was set to Nr, and thus their optical path difference was taken to be zero. It should be noted that the rays can return to the corrugated wall one or more times in one cycle [16].

It can be seen from Fig. 2 that the difference between the optical paths of the rays entering the optical fiber is the same in a certain range of entry angles. But it is not generally. In Fig. 2(a), the optical path differences are not close to each other in the above interval, i.e., $N = 0.01 \approx 0.6^\circ$ From Fig. 2(b), it can be seen that the optical fiber entry angles $N = 0.46 \approx 23^\circ$ are close to each other in the main resonant part.

In general, the dependence of the number of resonant rays on the parameters of the oscillating wall can be seen in the dynamic map (Fig. 3). It is not difficult to see from Fig. 3 that large values of N correspond to the interval $0.75 < \omega/\Omega < 0.85$. If the value $\omega/\Omega = 0.8$ of the ratio a_0/a increases, the value of N will also increase, and accordingly, the value of Nr will also change (Fig. 4).

It can be seen from Fig. 5 that the resonance capture of the rays does not change after the value of the optical fiber length $z > 300L$, and the stair width is maintained.

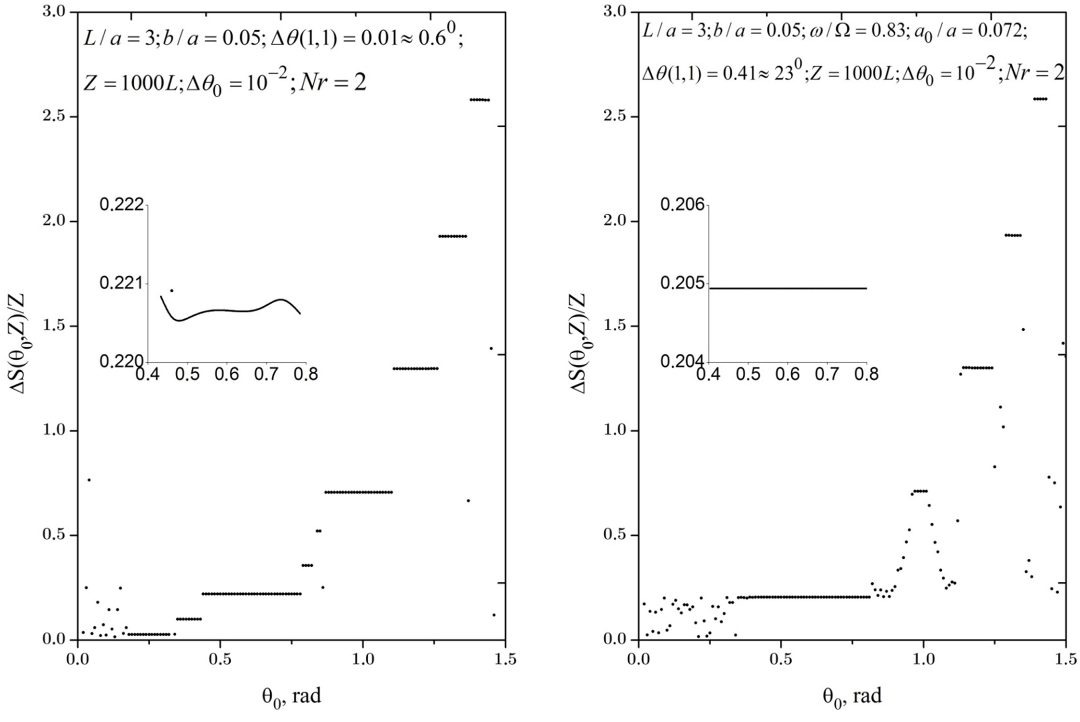
Next, we will consider the dependence of the step on the width of the stairs (Fig. 6). The purpose is to determine whether the width of the stairs will decrease or increase if the measurement step is reduced.

If the measurement step is reduced while the non-corrugated wall is not vibrating, the length of the "stair" is divided into smaller "stairs" corresponding to the small step [16, 18]. But if the wall is moving, the width of the stairs is kept unchanged. We determine the width of the stairs as follows:

$$N(\Delta\theta_0) = \left(\frac{1}{\Delta\theta_0}\right)^D. \tag{3}$$

Here $N(\Delta\theta_0)$ is the number of angles with equal optical path difference, $\Delta\theta_0$ is the measurement step, D is the fractal dimension. It can be seen from Fig. 6 that the value of D is equal to one, which means that the step width does not depend on the measurement step.

If we determine the dependence of the rays entering the optical fiber with the initial angle θ_0 on the coordinate of the exit from the optical fiber, since the length of the optical fiber is chosen the same, the x coordinate of the optical fiber



(a) (b)

FIG. 2. The dependence of the difference in the optical paths of the rays on the angle of entry: a) the upper wall does not oscillate and oscillates in b) position. In that $L/a = 3$, $b/a = 0.05$, $\omega/\Omega = 0.83$, $a_0/a = 0.072$, $z = 1000L$, $\Delta\theta_0 = 10^{-2}$ the split image was magnified 100 times and $\Delta\theta_0 = 10^{-3}$ counted step by step.

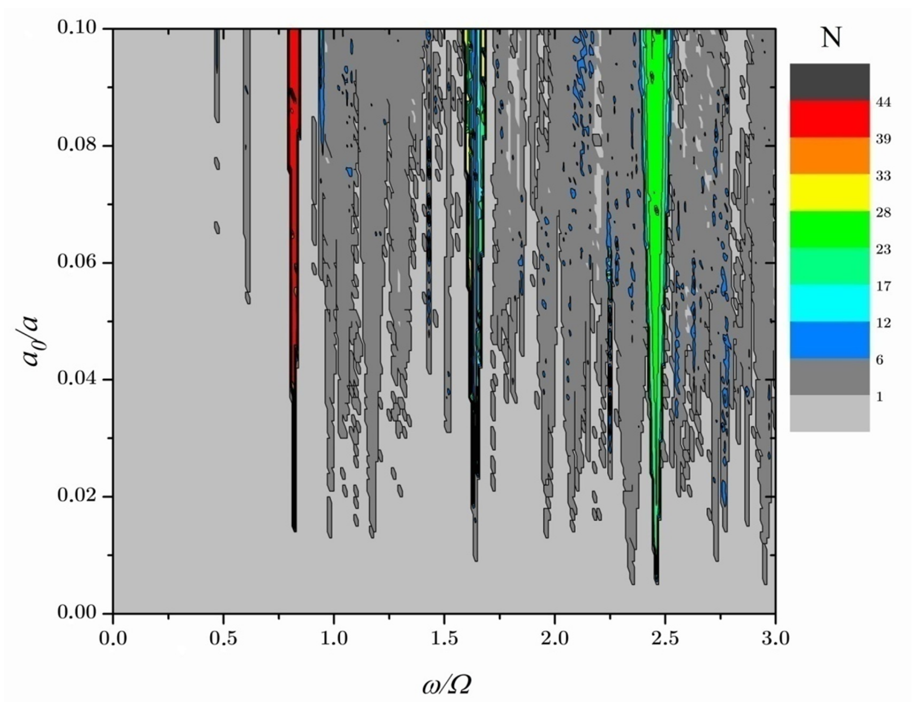


FIG. 3. Map of the dependence of the width of the main resonance $\Delta\theta(1,1)$ on the frequency and amplitude of the oscillating wall. Here, N is the number of initial resonance angles θ_0. Here $L/a = 3$, $b/a = 0.05$, $Z = 500L$, $\Delta\theta_0 = 10^{-2}$

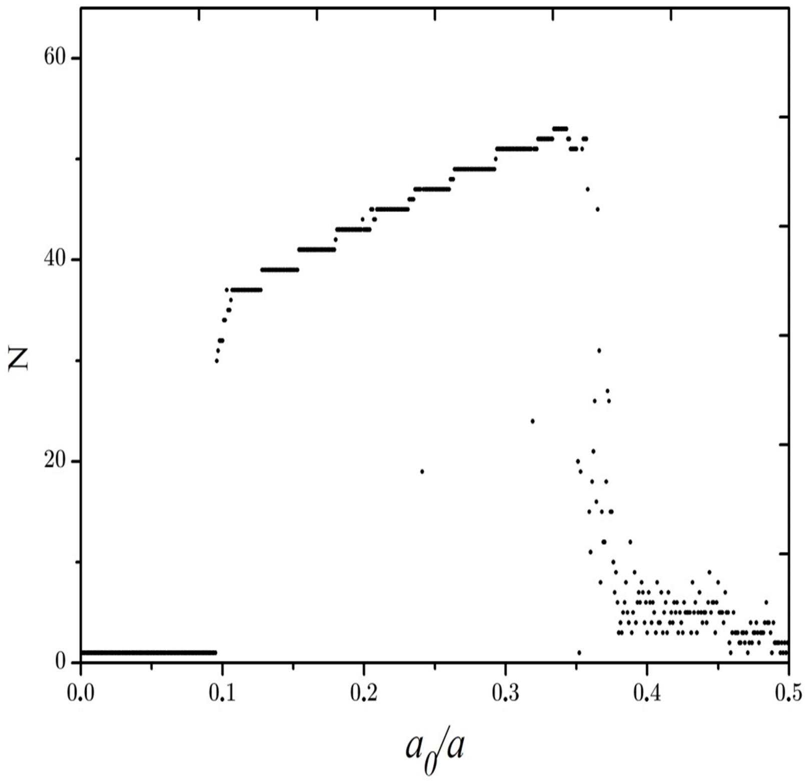


FIG. 4. Dependence of the value of N on the value of the ratio a_0/a. $L/a = 3$; $b/a = 0.03$, $\omega/\Omega = 0.8$, $z = 1000L$, $\Delta\theta_0 = 10^{-2}$

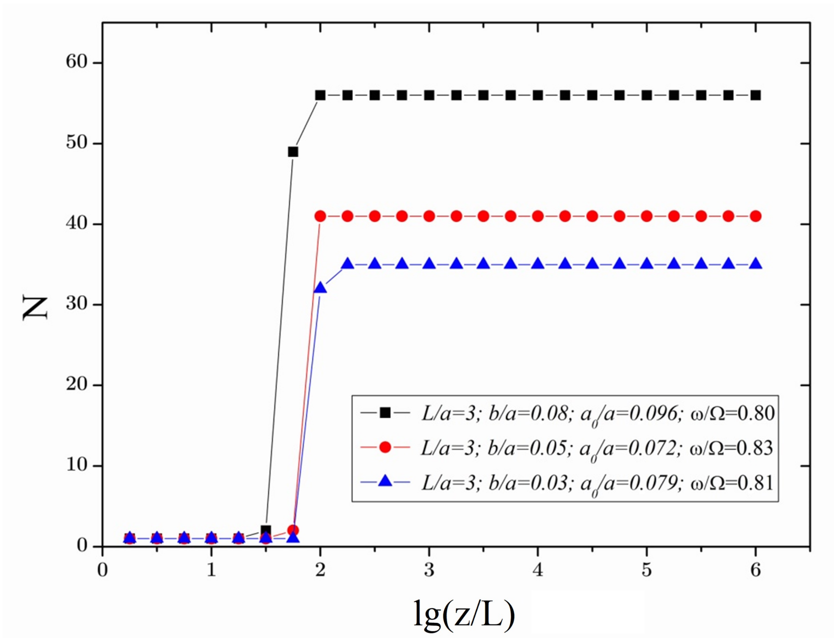


FIG. 5. Dependence of the width of the stairs on the length of the optical fiber

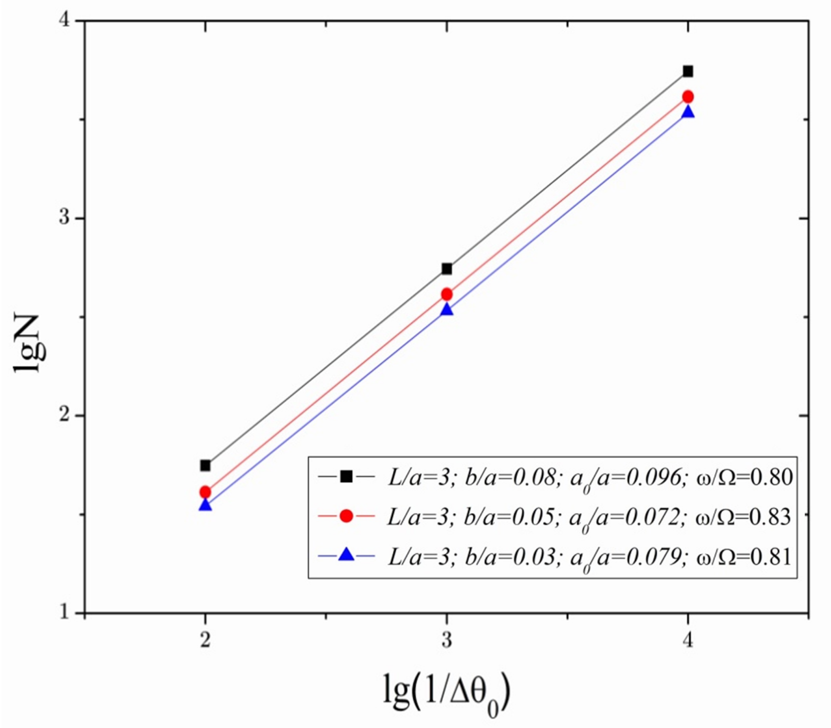


FIG. 6. Dependence of the width of the stairs on the measurement step. One has $D = 1$ in all three cases

at the exit is unchanged, and it is equal to the length of the optical fiber z. Therefore, we only observe the relationship between the Y coordinate and the angle of entry (Fig. 7).

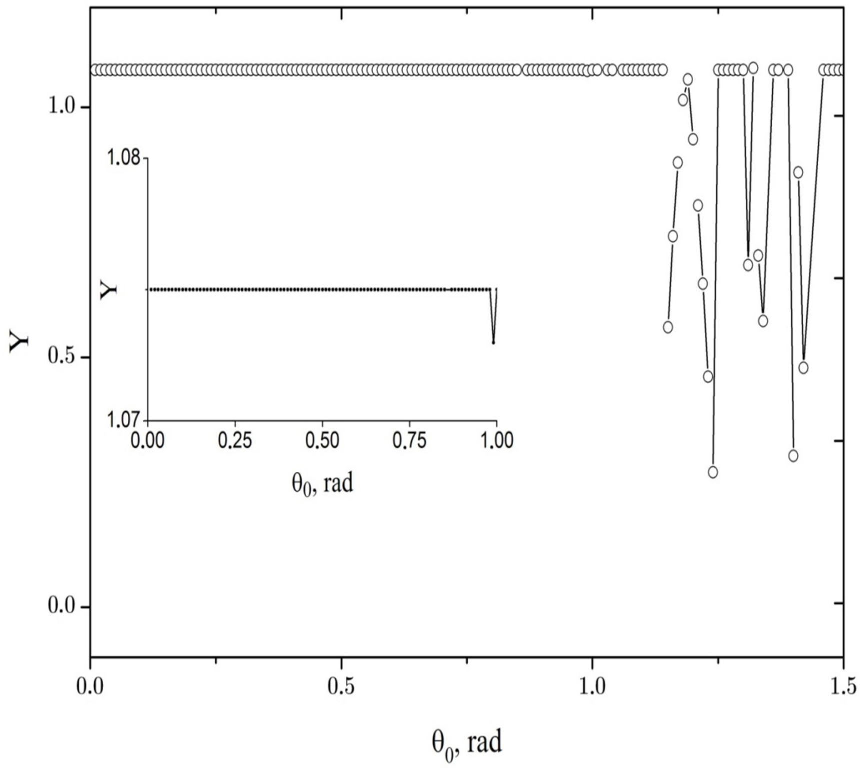


FIG. 7. Dependence of Y on θ_0. $b/a = 0.05$, $L/a = 3$, $\omega/\Omega = 0.8$, $a_0/a = 0.1$, $z = 1000L$, $\Delta\theta_0 = 10^{-2}$

In this case, it can be seen from Fig. 7 that the optical paths of the rays that resonate in the optical fiber are equal and exit from the optical fiber at the same coordinate.

3. Conclusion

When rays are scattered in an optical fiber with one wall corrugated and one wall oscillating under the influence of external periodic noise, the optical paths of the rays entering the optical fiber are equalized in a certain interval, that is, there is no time dispersion in the propagation of signals. It was found that these resulting optical channels strongly depend on external noise parameters. The width of the optical channels remains unchanged at the values of the optical fiber length $z > 300L$. Reducing the entrance angle does not affect the channel size. Also, it was found that the rays falling on the resonance come out of the optical fiber at the same coordinate.

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Submitted 24 October 2022; revised 16 March 2023; accepted 17 March 2023

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Conflict of interest: the authors declare no conflict of interest.