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Original article

Hidden polarization in open quantum systems

Alexander D. Vatutin, George P. Miroshnichenko, Alexander I. Trifanov

ITMO University, St.-Petersburg, Russia

Corresponding author: Alexander Vatutin, advatutin@yandex.ru

ABSTRACT In this work, we explore the master equation governing open quantum systems dynamics in an alternative form, which preserves the normal-ordered representation of the averaged normal-ordered operators. We derive a linear system of differential equations for the fourth-order moments of corresponding bosonic operators. Polarization moments of the first and the second orders are investigated using plane rotation transformation. We also evaluate the dynamics of the hidden polarization in comparison with the dynamics of usual polarization within open quantum dynamics.

KEYWORDS master equation, normal-ordered correlators, hidden polarization, fourth-order moments

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1. Introduction

The transfer, storage, and conversion of quantum information constitute three primary challenges in the field of quantum information processing. These are crucial for various quantum protocols such as quantum teleportation [1], quantum computing [2], quantum key distribution [3], dense coding, and quantum memory [4]. While numerous specific solutions address a variety of problems within these protocols, a general approach that resolves all types of issues remains unattainable.

The theory of open quantum systems addresses the problems of transport and storage of quantum information. This theory is typically articulated in terms of Completely Positive Trace-Preserving (CPTP) mappings, or trace-non-increasing linear mappings known as quantum channels. The dynamics of open quantum systems are most efficiently characterized by the master equation. Specifically, the Lindblad-type master equation [5, 6] is employed in this paper. There are two primary avenues of investigating this type of equations: the physical approach [7–9] and the mathematical techniques for the single-mode Lindblad equation [6, 10–17]. These mathematical techniques are extended to the case of multi-mode bosonic systems and have been utilized to construct the Fock-like eigenstates of Lindblad superoperators using the Lie algebras [18].

These mathematical techniques facilitate the determination of the dynamics of the averaged moments of the Stokes operators and the polarization of light [19]. Many physical processes are accurately described by these parameters. However, the averaged moments of the Stokes operator fall short in cases where not the average intensity, but other statistical parameters of the wave are recorded [20]. Therefore, higher-order moment correlators are essential for a more thorough description of these quantum processes. Furthermore, the introduction of a parameter, equivalent to polarization P_2 for higher-order moments, is both relevant and challenging. Numerous authors have demonstrated the existence of this process regardless of the existence of usual polarization [23, 24]. A more generalized approach for the determination of hidden polarization [20] is thoroughly considered in this paper.

The non-commutativity of the Stokes operators leads to quantum noise manifested through fourth-order correlators. The extent of variation in these correlators serves as an indicator of hidden polarization. The study of hidden polarization characteristics in the emission of exciton-polariton lasers is an area of research that holds fundamental interest [25]. Additionally, a comparison of the dynamics of hidden polarization to the dynamics of usual polarization in open quantum systems is meaningfully reviewed in this paper.

2. A model

We start with the master equation in the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) form for polarized light in the two-mode bosonic system in optical fibers:

$$\frac{\partial\hat{\rho}}{\partial t} = -i\sum_{n,m}\frac{1}{2}\Omega_{n,m}[\hat{a}_n^\dagger\hat{a}_m,\hat{\rho}] - \sum_{n,m}\frac{1}{2}\Gamma_{n,m}((n_T+1)(\hat{a}_n^\dagger\hat{a}_m\hat{\rho}+\hat{\rho}\hat{a}_n^\dagger\hat{a}_m-2\hat{a}_m\hat{\rho}\hat{a}_n^\dagger)+ + n_T(\hat{a}_m\hat{a}_n^\dagger\hat{\rho}+\hat{\rho}\hat{a}_m\hat{a}_n^\dagger-2\hat{a}_n^\dagger\hat{\rho}\hat{a}_m)), \quad (1)$$

where $n, m \in \{1, 2\}$, dagger denotes the Hermitian conjuration, $\hat{\rho}$ is the density matrix of a quantum state, $\hat{a}_n^\dagger$ and $\hat{a}_n$ are the creation and the annihilation operators of the n-th mode. Ω and Γ are frequency and relaxation matrixes [21], and n_T is a mean number of thermal photons:

$$n_T = \frac{1}{e^{\frac{\hbar\Omega_0}{k_B T}} - 1},$$

where Ω_0 is the bare(free-space) frequency, $\hbar$ is the reduced Planck constant, k_B is the Boltzmann constant, T is the temperature of the environment. The frequency and the relaxation matrices are given by:

$$\Omega = \begin{pmatrix} \omega_1 & \omega_3^* \\ \omega_3 & \omega_2 \end{pmatrix}, \ \Gamma = \begin{pmatrix} \gamma_1 & \gamma_3^* \\ \gamma_3 & \gamma_2 \end{pmatrix}, \tag{2}$$

where an asterisk stands for the complex conjuration. The matrices Ω and Γ are Hermitian: $\Omega = \Omega^\dagger, \Gamma = \Gamma^\dagger$. The relaxation matrix Γ is positively definite: $z^\dagger \Gamma z \geq 0$, $z \in \mathbb{C}^2$.

Equation (1) can be converted to the following form for any operator $\hat{\zeta}$ with the mean value $Z = \mathrm{tr}(\hat{\zeta}\hat{\rho}) = \langle\hat{\zeta}\rangle$:

$$\frac{\partial Z}{\partial t} = \frac{1}{2}\sum_{n,m}\Big(\Big\langle[\hat{\zeta}, \hat{a}_n^\dagger]\hat{a}_m\Big\rangle(-i\Omega_{n,m} - \Gamma_{n,m}) + \Big\langle\hat{a}_n^\dagger[\hat{a}_m, \hat{\zeta}]\Big\rangle(i\Omega_{n,m} - \Gamma_{n,m})\Big) - n_t\sum_{n,m}\Gamma_{n,m}\Big\langle[[\hat{\zeta}, \hat{a}_n^\dagger], \hat{a}_m]\Big\rangle. \tag{3}$$

If the operator $\hat{\zeta}$ with the mean value Z on the left-hand side of the equation is normal-ordered, the form of master equation (3) preserves the normal form of the moments [22].

3. Dynamics of normal-ordered forth moments

If $\hat{\zeta} = a_p^\dagger a_q^\dagger \hat{a}_s \hat{a}_t$ is the normal-ordered fourth moment, we can see that the relaxation is caused by fourth moments (the first sum) and the second moments (the second sum). Let us assume:

$$\begin{gathered}
Z_1 = \Big\langle\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1\Big\rangle = A; \quad Z_2 = Z_4^* = \Big\langle\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_1\hat{a}_1\Big\rangle = D^*; \quad Z_3 = \Big\langle\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_1\hat{a}_2\Big\rangle = C; \\
Z_4 = \Big\langle\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_2\Big\rangle = D; \quad Z_5 = \Big\langle\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_2\hat{a}_2\Big\rangle = E; \quad Z_6 = Z_5^* = \Big\langle\hat{a}_2^\dagger\hat{a}_2^\dagger\hat{a}_1\hat{a}_1\Big\rangle = E^*; \\
Z_7 = \Big\langle\hat{a}_2^\dagger\hat{a}_2^\dagger\hat{a}_1\hat{a}_2\Big\rangle = F; \quad Z_8 = Z_7^* = \Big\langle\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_2\hat{a}_2\Big\rangle = F^*; \quad Z_9 = \Big\langle\hat{a}_2^\dagger\hat{a}_2^\dagger\hat{a}_2\hat{a}_2\Big\rangle = B; \\
\vec{Z} = (Z_1, Z_2, Z_3 ..., Z_9)^T; \\
B_1 = \Big\langle\hat{a}_1^\dagger\hat{a}_1\Big\rangle; \quad B_2 = \Big\langle\hat{a}_1^\dagger\hat{a}_2\Big\rangle; \quad B_3 = B_2^* = \Big\langle\hat{a}_2^\dagger\hat{a}_1\Big\rangle; \quad B_4 = \Big\langle\hat{a}_2^\dagger\hat{a}_2\Big\rangle; \\
\vec{B} = (B_1, B_2, B_3, B_4)^T,
\end{gathered} \tag{4}$$

where A^T denotes the transposed matrix for matrix A.

We can rewrite eq. (3) for all fourth moments like the nonhomogeneous system of linear differential equations:

$$\frac{\partial\vec{Z}}{\partial t} = H\vec{Z} + J\vec{B}, \tag{5}$$

where H is matrix 9×9 and J is 9×4 matrix. You can see matrices H and J in the Appendix. Dynamic of vector $\vec{B}$ is determined by the following equation [22]:

$$\frac{\partial\left\langle\hat{a}_p^\dagger\hat{a}_q\right\rangle}{\partial t} = \frac{1}{2}\sum_m\left\langle\hat{a}_p^\dagger\hat{a}_m\right\rangle(-i\Omega_{q,m} - \Gamma_{q,m}) + \frac{1}{2}\sum_n\left\langle\hat{a}_n^\dagger\hat{a}_q\right\rangle(i\Omega_{n,p} - \Gamma_{n,p}) + n_t\Gamma_{q,p}. \tag{6}$$

4. Polarization conversion as a rotation

If the phase factor common to the both modes is neglected, then the effect of a loss-free converter can be represented in the Heisenberg representation in the form:

$$\begin{pmatrix} \hat{a}_1' \\ \hat{a}_2' \end{pmatrix} = \begin{pmatrix} t^* & r^* \\ -r & t \end{pmatrix} \cdot \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}, \tag{7}$$

where $t(r)$ is the amplitude transmission (conversion) coefficients of the converter. The standard parametrization can be introduced in the form:

$$\begin{gathered} t = \cos(\theta/2) \cdot \exp[i(\phi + \psi)/2], \\ r = -\sin(\theta/2) \cdot \exp[i(\phi - \psi)/2], \end{gathered} \tag{8}$$

where $0 \leq \theta < \pi$, $0 \leq \psi < 2\pi$ and $0 \leq \phi < 4\pi$. Using (8), we can find action of a converter on the operator $\hat{n}_1' = \hat{a}_1'^\dagger \hat{a}_1'$:

$$\hat{n}_1' = t^* t \cdot \hat{n}_1 + r^* r \cdot \hat{n}_2 + r^* t \cdot \hat{s}_+ + t^* r \cdot \hat{s}_+^*, \tag{9}$$

where $\hat{n}_1 = \hat{a}_1^\dagger \hat{a}_1$, $\hat{n}_2 = \hat{a}_2^\dagger \hat{a}_2$, and $\hat{s}_+ = \hat{a}_1^\dagger \hat{a}_2$. By averaging equations (9), one obtains the same equations for observables $N_1' = \langle \hat{n}_1' \rangle$, $N_1 = \langle \hat{n}_1 \rangle$, $N_2 = \langle \hat{n}_2 \rangle$ and $S_+ = \langle \hat{s}_+ \rangle$. We can rewrite N_1' in terms of (9):

$$N_1' = \begin{pmatrix} t & r \end{pmatrix}^* \cdot \begin{pmatrix} N_1 & S_+^* \\ S_+ & N_2 \end{pmatrix} \cdot \begin{pmatrix} t \\ r \end{pmatrix}. \tag{10}$$

If one varies one of the parameters, for example, a variation produced by rotating a $\lambda/4$ phase plate, the observed intensity varies periodically [20]: $N_1 \propto 1 + V_2 \cos(2\chi)$, $0 \le V_2 \le 1$. Here χ is the angle of the plane. The parameter V is called the *visibility factor* of the polarization interference. We can interpret Eq. (10) as a quadratic function with matrix K and normalized complex vector $(t, r)^T$, $|t|^2 + |r|^2 = 1$. Matrix K is the coherence matrix:

$$K = \begin{pmatrix} N_1 & S_+^* \\ S_+ & N_2 \end{pmatrix}. \tag{11}$$

Extremal values of the hermitian quadratic function are determined by minimum $\lambda_{\min}$ and maximum $\lambda_{\max}$ eigenvalues. These values can be found us follows:

$$(N_1')_{\max,\min} = \lambda_{\max,\min} = \frac{1}{2}\left(\operatorname{tr} K \pm [(\operatorname{tr} K)^2 - 4\det K]^{1/2}\right).$$

Hence, the maximum possible interference visibility assumes the form:

$$V_{2_{\max}} = \frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda_{\min}} = \sqrt{1 - \frac{4\det K}{(\operatorname{tr} K)^2}} = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} = P_2,$$

Therefore $V_{2_{\max}}$ equals to the degree of polarization P_2. We have to mention that $S_i = \operatorname{tr}(\hat{s}_i \cdot \hat{\rho})$ for $i \in \{0, 1, 2, 3\}$ and

$$\hat{s}_0 = \hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2, \quad \hat{s}_1 = \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2,$$
$$\hat{s}_2 = \hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1, \quad \hat{s}_3 = -i(\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1).$$

5. Fourth-order polarization parameters

Let us similarly introduce the fourth-order polarization. Let vector $\hat{\vec{s}}$ and its average $\vec{S}$ be as follows: $\hat{\vec{s}} = (\hat{s}_1, \hat{s}_2, \hat{s}_3)$, $\vec{S} = \langle \hat{\vec{s}} \rangle = (S_1, S_2, S_3)$. The definition of fluctuations ΔS_X of the observable, for example, ΔS_1 along a definite direction of $X = (\cos(\Theta), -\sin(\Theta)\cos(\Phi), \cos(\Theta)\cos(\Phi))^T$ with coordinates (Θ, Φ) in the Stokes–Poincare space is as follows:

$$\begin{aligned}(\Delta S_X)^2 &= \sum_{k,l=1}^{3} \left\langle \Delta\hat{\vec{s}}_k \Delta\hat{\vec{s}}_l \right\rangle X_k X_l = \sum_{k,l=1}^{3} (Q_{k,l} - S_k S_l + S_0 I) X_k X_l + \sum_{k,l,m=1}^{3} i\varepsilon_{k,l,m} S_m X_k X_l = \\ &= \sum_{k,l=1}^{3} (Q_{k,l} - S_k S_l + S_0 I) X_k X_l = \sum_{k,l=1}^{3} (\Delta Q)_{k,l} X_k X_l,\end{aligned} \tag{12}$$

where $\Delta\hat{\vec{s}} = \hat{\vec{s}} - \vec{S}$. The second sum is always equal to zero, because of $X \in \mathbb{R}$ and antisymmetry of symbol Levi–Civita $\varepsilon_{k,l,m}$. Matrix $Q = \left\langle : \hat{\vec{s}} \otimes \hat{\vec{s}} : \right\rangle$ is the normal-ordered second moments of the Stokes operators:

$$Q = 2\begin{pmatrix} (A+B)/2 - C & \operatorname{Re}(D-F) & \operatorname{Im}(D-F) \\ \operatorname{Re}(D-F) & C + \operatorname{Re}(E) & \operatorname{Im} E \\ \operatorname{Im}(D-F) & \operatorname{Im} E & C - \operatorname{Re} E \end{pmatrix}, \tag{13}$$

where A, B, C, D E and F are taken from Eq. (4).

The matrix $\Delta Q \equiv Q - \vec{S} \otimes \vec{S} + S_0 I$ expresses the variances of the Stokes parameters. Hence, the last expression from Eq.(12) can be reviewed as quadratic form with real vector X: $|X| = 1$. The observed S_1 varies periodically in the same way, as N_1: $\Delta S_1 \propto 1 + V_4 \cos(2\theta)$, $0 \le V_4 \le 1$, where V_4 is visibility factor of fourth moments. Hence, the degree of fourth-order polarization P_4 can be defined as the maximum possible visibility of the fourth-order interference, observed according to the fluctuations of the Stokes parameter:

$$V_{4_{\max}} = \frac{\mu_{\max} - \mu_{\min}}{\mu_{\max} + \mu_{\min}} = P_4, \tag{14}$$

where $\mu_{\max}$ and $\mu_{\min}$ are the maximum and the minimum eigenvalues of the matrix ΔQ.

It should be mentioned, that the fluctuation of any observable can be reviewed. For ΔS_2 under consideration, only the representation of direction X changes, but the most important part ΔQ and $|X| = 1$ remains the same. In addition, the fluctuations of $A = \left\langle \hat{a}_1^\dagger \hat{a}_1^\dagger \hat{a}_1 \hat{a}_1 \right\rangle$ can be taken in consideration. The formalism of quadratic forms is unusable in this case. However, the maximum possible visibility of the fourth-order interference $V_{4_{\max}}$ gives the same result, as the fluctuations of components of Stock's vector.

6. Examples

The dissipative dynamics, as described by system (5), are governed by the frequency and the relaxation matrices introduced in Eq. (2). In this section, we compare the dynamics of hidden polarization and ordinary polarization under varying the initial conditions for these matrices and the quantum states. Diagonal frequency and the relaxation matrices are used:

$$\Omega = \begin{pmatrix} \omega - \dfrac{\delta}{2} & 0 \\ 0 & \omega + \dfrac{\delta}{2} \end{pmatrix}, \quad \Gamma = \begin{pmatrix} \gamma - \gamma_0 & 0 \\ 0 & \gamma + \gamma_0 \end{pmatrix}.$$

Parameter $\delta = 6.5 \cdot 10^6$ sec^{-1} represents the frequency difference between the two modes (beat lenght is $L =$ 200 meters). Only this parameter affects the hidden polarization, similar to its effect on ordinary polarization. Hence, parameter ω can be chosen to be 0. Parameter $\gamma = 1.2 \cdot 10^6$ sec^{-1} (relaxation is 25 dB/km) and $\gamma_0 = 0.2 \cdot \gamma$ sec^{-1}. Below, we show a dynamics of ordinary polarization and hidden polarization for different initial states. Any initial state relaxes to thermal state with the number of thermal photons $n_T = 1$. Examples illustrating the initial data for common states are provided in Figs. 1–5.

A particular case under consideration is the Fock state with equal numbers of photons in the vertical and horizontal modes. The initial data yield an opening polarization of $P_2(0) = 0$ and a hidden polarization of $P_4(0) = 1$. If relaxation in the both modes is equable ($\gamma_0 = 0$), polarization $P_2(t) = 0$ for any t. Hence, the result is shown in Fig. 5.

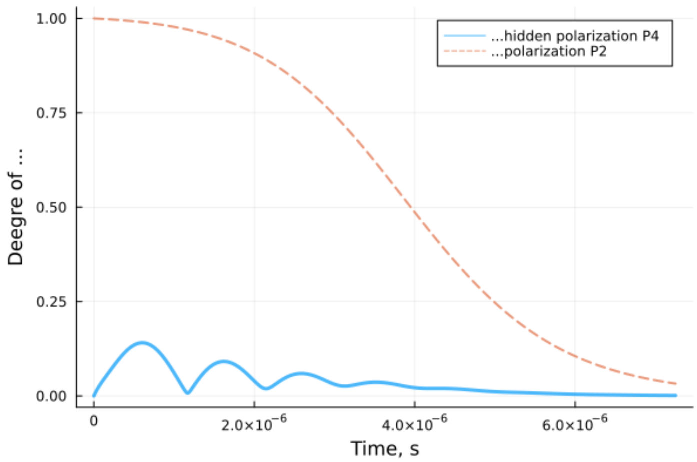

FIG. 1. Initial state is coherent state $|12, 8\rangle$

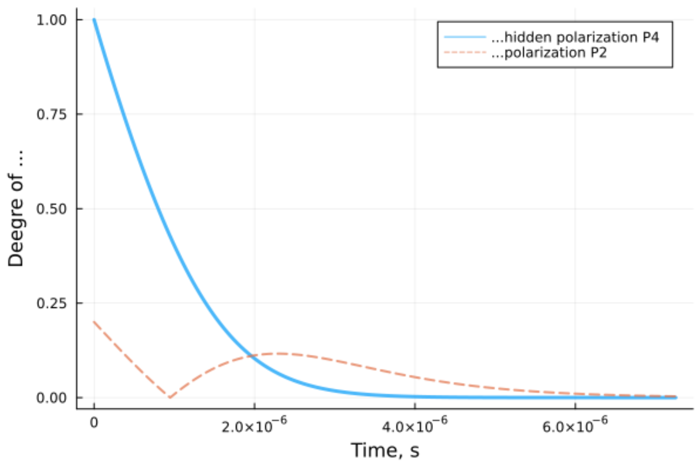

FIG. 2. Initial state is Fock state $|12, 8\rangle$

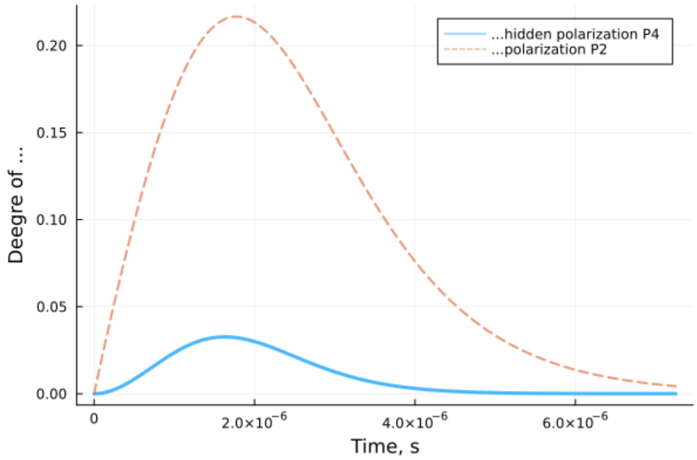

FIG. 3. Initial state is thermal state with the number of thermal photons $n_T = 10$.

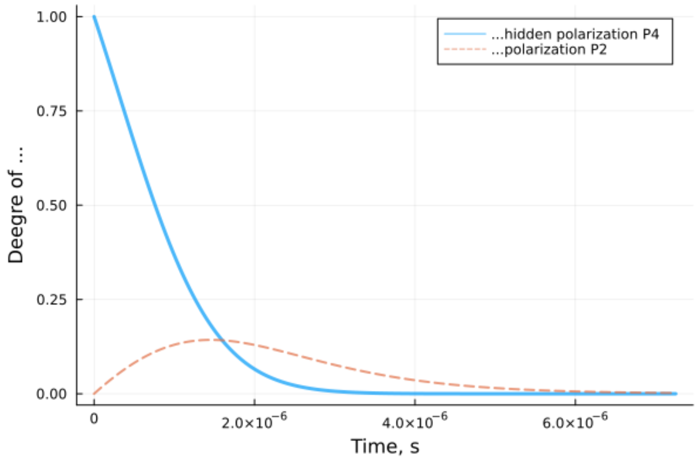

FIG. 4. Initial state is squeezed vacuum $|\xi_1, \xi_2\rangle$, where $\xi_{1(2)}$ is squeezed vacuum for 1-st(2-nd) mode. $\xi_1 = \xi_2 = 1.5 * e^{i\phi}$ is complex squeeze parameter

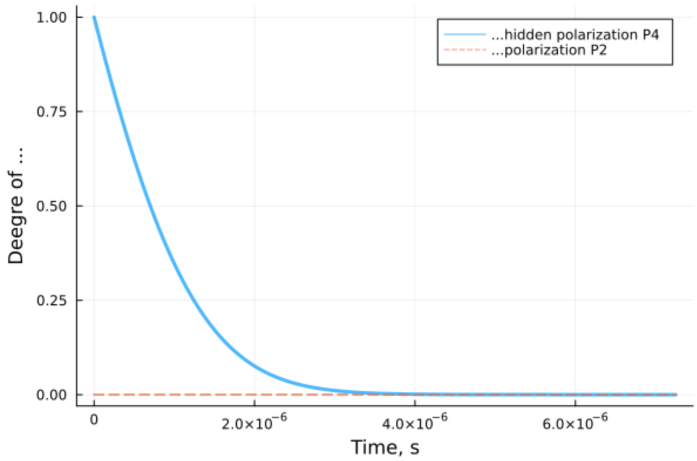

FIG. 5. Initial state is Fock state $|8, 8\rangle$. Relaxation is equal in both modes, which gives the absence of usual polarization.

7. Conclusion

In contrast to showing the existence of hidden polarization in systems without time dependence [20, 23, 24], we compare the dynamics of hidden polarization to those of usual polarization in open quantum systems. We illustrate a case where the usual polarization is absent while hidden polarization is present, as depicted in Figs. 1–4. We commence with the master equation in the GKSL form for the open quantum systems, maintain normal ordering as per equation (3). We elucidate the dynamics of the fourth-ordered observables for a more accurate description of the open quantum systems than using the dynamics of second-ordered observables [19,22]. The values of the fourth-ordered and the second-ordered observables vary depending on the plate position in the experiment. We demonstrate that both polarization P_2 and polarization P_4 can be similarly ascertained through rotation. A logical extension of this work entails the construction of quantum key distribution schemes. For instance, with the Fock state as the initial state, certain considerations regarding a qutrit arise.

Appendix

With equation (3) in use, let the operator $\hat{\zeta}$ be equal to $\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1$. Then one can obtain:

$$\frac{\partial\left\langle\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1\right\rangle}{\partial t}=\frac{1}{2}\sum_{n,m}\Big(\left\langle[\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1,\hat{a}_n^\dagger]\hat{a}_m\right\rangle(-i\Omega_{n,m}-\Gamma_{n,m})+\left\langle\hat{a}_n^\dagger[\hat{a}_m,\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1]\right\rangle(i\Omega_{n,m}-\Gamma_{n,m})\Big)-$$
$$-n_t\sum_{n,m}\Gamma_{n,m}\left\langle[[\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1,\hat{a}_n^\dagger],\hat{a}_m]\right\rangle. \tag{15}$$

In the first sum of the equation (15) n equals to 1, in the second m equals to 1, in the third n and m are equal to 1. Otherwise, commutators in these sums give one 0.

$$\frac{\partial\left\langle\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1\right\rangle}{\partial t}=\sum_m\left(\left\langle\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_m\right\rangle(-i\Omega_{1,m}-\Gamma_{1,m})+\sum_n\left\langle\hat{a}_n^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1\right\rangle(i\Omega_{n,1}-\Gamma_{n,1})\right)-4n_t\Gamma_{1,1}\left\langle\hat{a}_1^\dagger\hat{a}_1\right\rangle=$$
$$=-2\gamma_1\left\langle\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1\right\rangle+(-i\omega_3^*-\gamma_3^*)\left\langle\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1\right\rangle+(i\omega_3-\gamma_3)\left\langle\hat{a}_2^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_2\right\rangle-4n_t\gamma_1\left\langle\hat{a}_1^\dagger\hat{a}_1\right\rangle. \tag{16}$$

This process can be continued for any operator $\hat{\zeta}=\hat{a}_p^\dagger\hat{a}_q^\dagger\hat{a}_s\hat{a}_t$. After some calculations, matrices H has the following form:

$$H=\begin{pmatrix}
H_1 & H_+ & 0 & H_+^* & 0 & 0 & 0 & 0 & 0\\
H_-^*/2 & H_2 & H_+^* & 0 & 0 & H_+/2 & 0 & 0 & 0\\
0 & H_+/2 & H_3 & H_-^*/2 & 0 & 0 & H_+/2 & H_+^*/2 & 0\\
H_-/2 & 0 & H_+ & H_4 & H_+^*/2 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & H_- & H_5 & 0 & 0 & H_+ & 0\\
0 & H_-^* & 0 & 0 & 0 & H_6 & H_+^* & 0 & 0\\
0 & 0 & H_-^* & 0 & 0 & H_-/2 & H_7 & 0 & H_+^*/2\\
0 & 0 & H_- & 0 & H_-^*/2 & 0 & 0 & H_8 & H_+/2\\
0 & 0 & 0 & 0 & 0 & 0 & H_- & H_-^* & H_9
\end{pmatrix}, \tag{17}$$

where

$$H_+=-\gamma_3+i\omega_3,\quad H_-=-\gamma_3-i\omega_3,\quad H_1=-2\gamma_1,$$
$$H_2=H_4^*=(-3\gamma_1-\gamma_2+i(-\omega_1+\omega_2))/2,\quad H_3=-\gamma_1-\gamma_2,$$
$$H_5=H_6^*=-\gamma_1-\gamma_2+i(+\omega_1-\omega_2),\quad H_7=H_8^*=(-\gamma_1-3\gamma_2+i(-\omega_1+\omega_2))/2,\quad H_9=-2\gamma_2.$$

For matrix J, one has:

$$J=n_T\begin{pmatrix}
4\gamma_1 & 2\gamma_3^* & \gamma_2 & \gamma_3 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \gamma_3 & \gamma_1 & 4\gamma_3 & 0 & 0 & 2\gamma_2 & 0\\
0 & 2\gamma_1 & \gamma_3^* & 0 & 0 & 4\gamma_3^* & 2\gamma_2 & 0 & 0\\
0 & 0 & \gamma_1 & 0 & 0 & 0 & 2\gamma_3^* & 2\gamma_3 & 4\gamma_2
\end{pmatrix}^T. \tag{18}$$

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Information about the authors:

Alexander D. Vatutin – ITMO University, Kronverkskiy 49, St.-Petersburg, Russia; ORCID 0000-0003-1853-2171; advatutin@yandex.ru

George P. Miroshnichenko – ITMO University, Kronverkskiy 49, St.-Petersburg, Russia; ORCID 0000-0002-4265-8818; gpmirosh@gmail.com

Alexander I. Trifanov – ITMO University, Kronverkskiy 49, St.-Petersburg, Russia; ORCID 0000-0002-5109-2086; alextrifanov@itmo.ru

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