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Original article

Features of surface Bessel plasmon-polaritons in optical anisotropic hyperbolic metamaterials

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ABSTRACT The features of generation and properties of surface Bessel Plasmon-polaritons (SBPPs) in optical anisotropic hyperbolic metamaterials formed by a periodic lattice of metal nanowires made of gold and silver embedded into pores of aluminum oxide is studied. We investigate the influence of the thickness of the porous material matrix on the generated plasmon-polaritons. Calculation of energy flows in the structure is made.

KEYWORDS surface plasmons, Bessel light beam, surface Bessel plasmon-polaritons, hyperbolic metamaterials

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1. Introduction

In recent years, surface plasmon-polaritons (SPPs) that arise at the interface between a dielectric and a conducting medium have been actively studied. Surface plasmon-polaritons are coupled oscillations of the electromagnetic wave and the density of free oscillations of the electron gas [1]. SPPs propagate along the interface between dielectric and metal. The metal in this system is necessary for the existence of electronic plasma, and the dielectric is necessary to bind the electronic plasma to the electromagnetic field [2]. Surface plasmon-polaritons are two-dimensional objects: they propagate along the interface and decay on the both sides of it [3–6]. This leads to unique properties of SPPs: high spatial localization and the ability to significantly enhance the intensity of the field near the boundary [7–9]. Due to these properties, it is promising to use surface plasmon-polaritons for creation of photonic and electronic devices of the nanometer scale, plasmonic modulators and switches, and for transmitting information over metallic wires in plasmonic microcircuits [10–13].

Currently, interest in the study of nanostructures that excite surface plasmon-polaritons is associated with the need of creation a new class of devices characterized by small dimension and a significantly lower resolution threshold that meets the challenges of nanotechnology. The use of metamaterials (MM), new artificial materials with electromagnetic properties that do not occur in nature, is one of the approaches to this problem [14–20].

Among anisotropic metamaterials, the most simple in technological implementation are uniaxial hyperbolic metamaterials (HMMs), which have isofrequency surfaces in the space of wave vectors in the form of a hyperboloid [21]. These metamaterials are of significant interest due to their ability to be used for obtaining an image with sub-wavelength (up to 15 nm or less) resolution, amplifying spontaneous emission, and increasing the density of photon states. A number of important practical applications of HMMs are associated with the possibility of ex-citing plasmon-polaritons (PP), which represent coupled oscillations of an electro-magnetic wave and the density of free oscillations of the electron gas. Tasks related to the study of the excitation and localization of plasmon-polaritons at the boundary (in the layer) of HMMs are of interest for optical manipulation of micro- and nano-particles, quantum informatics, photonics, and astrophysics [21–23]. The use of plasmon-polariton light fields in HMMs is one of the ways to solve the problem of creating new optical devices characterized by significantly lower resolution thresholds and smaller sizes.

At present, the attention of many researchers is turned to quasi-nondiffracting light fields. Among them, the Bessel light beams (BLBs) occupy a special place, having a divergence in the axial region significantly smaller than that of traditional (e.g., Gaussian) beams, as well as the ability to self-reconstruct the wavefront [24–29]. However, the evanescent BLBs investigated in works [24–29] have a significant disadvantage: they are weak. This leads to the need for strong laser fields to generate them. One of the possibilities of using BSPs for microscopy is to form Bessel plasmon-polaritons (BPPs) – quasi-nondiffracting light fields formed at the boundary of media with different dielectric permittivities.

This paper investigates the interaction of Bessel light beams with hyperbolic metamaterials and studies the possibility of generating quasi-nondiffracting SBPPs in hyperbolic metamaterial.

2. Conditions for the formation of Bessel plasmon-polariton in hyperbolic metamaterials

Let's consider a structure created using nanoporous aluminum oxide with pores filled with metal (Fig. 1). Thus, we obtain a medium that represents a set of periodically arranged metallic nanorods in a dielectric matrix. The controlled parameters are the radius of the metallic nanocylinders, the dielectric permittivity of the metal, and the average distance between the centers of two adjacent nanocylinders .

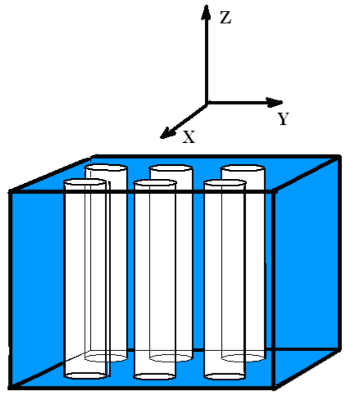


FIG. 1. Matrix of nanoporous aluminum oxide with pores filled with metal

In the effective medium approximation, the structure shown in Fig. 1 can be regarded as a uniaxial medium with effective dielectric parameters [30].

$$\varepsilon_x = \varepsilon_y = \varepsilon_o = \frac{\beta\varepsilon_m N + \varepsilon_d(1-N)}{\beta N + (1-N)}, \qquad \varepsilon_e = \varepsilon_m N + \varepsilon_d(1-N), \tag{1}$$

where $N = \pi r^2/D^2$ is the filling ratio, $\beta = 2\varepsilon_d/(\varepsilon_m + \varepsilon_d)$, ε_d is the dielectric permittivity of the dielectric. It is assumed that $N \ll 1$ and $D, r \ll \lambda_0$, λ_0 is the wavelength of radiation incident on the structure. As known, the dielectric permittivity of gold and silver within the Drude model can be described by the following formulas:

$$\varepsilon_m^{Au}(\omega) \approx \varepsilon_\infty^{Au} - \frac{\left(\omega_p^{Au}\right)^2}{\omega^2\left(1 + i\frac{\Gamma^{Au}}{\omega}\right)}, \qquad \varepsilon_m^{Ag}(\omega) \approx \varepsilon_\infty^{Ag} - \frac{\left(\omega_p^{Ag}\right)^2}{\omega^2} + i\frac{\left(\omega_p^{Ag}\right)^2\Gamma^{Ag}}{\omega^3}, \tag{2}$$

with parameters $\varepsilon_\infty^{Au} = 9$; $\omega_p^{Au} = 13.8 \cdot 10^{15}$ Hz; $\Gamma^{Au} = 0.11 \cdot 10^{15} s^{-1}$, $\varepsilon_\infty^{Ag} = 5$; $\omega_p^{Ag} = 14 \cdot 10^{15} s^{-1}$; $\Gamma^{Ag} = 0.032 \cdot 10^{15}\ \mathrm{s}^{-1}$ [30].

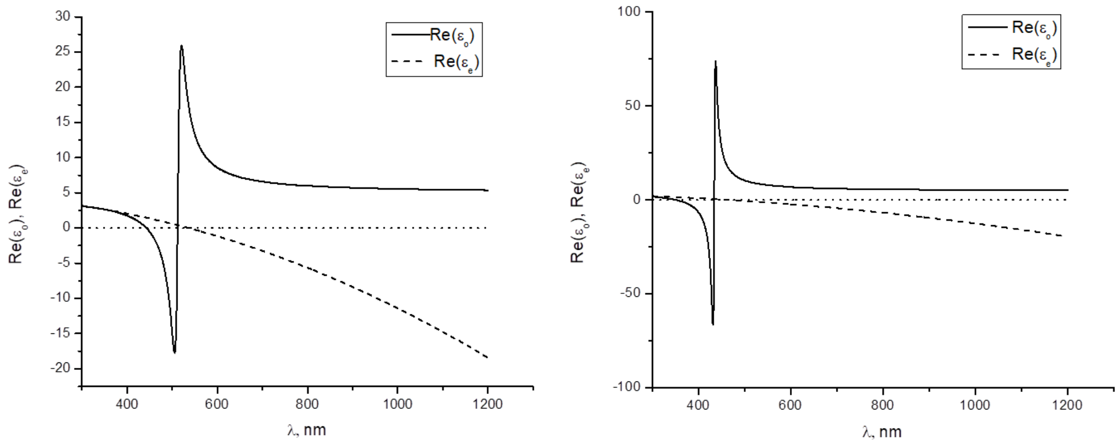
From Eqs. (1) and (2), we can obtain the spectral dependences of the permittivities of metamaterials samples. Fig. 2 shows our calculated results for the dependence of the real parts of transverse and longitudinal permittivities of Al_2O_3/Au and Al_2O_3/Ag on the wavelength. From Fig. 2, we can see that, at a wavelength of $\lambda = 633$ nm, for the fillling ratio $f = 0.3$ the real part of the transverse permittivity has a positive value and longitudinal permittivity has a negative value, then metamaterial structure Al_2O_3/Au displays the properties of the hyperbolic metamaterial. Similarly, for the metamaterial sample Al_2O_3/Ag properties of hyperbolic metamaterial are achieved when the wavelength is 589.3 nm.

Let's assume that the hyperbolic metamaterial with thickness h, depicted in Fig. 1, separates a dielectric substrate with dielectric permittivity ε_0 and an external dielectric medium with permittivity ε_1. We will use a cylindrical coordinate system, choosing it such that its origin is located at the boundary between the substrate and hyperbolic metamaterial.

Let's first consider the case when a TM-polarized m-order Bessel light beam falls on the interface between two semi-infinite media: an isotropic medium and an anisotropic uniaxial medium with its optical axis perpendicular to the interface. The electric $\vec{E}(R)$ and magnetic $\vec{H}(R)$ vectors of the Bessel light beam propagating along the z-axis in a medium with refractive index n_i can be represented as:

$$\vec{E}(R) = A\vec{E}^{\mathrm{TM}}(\rho)\exp i(k_{zi}z + m\varphi), \qquad \vec{H}(R) = A\vec{H}^{\mathrm{TM}}(\rho)\exp i(k_{zi}z + m\varphi), \tag{3}$$

where k_{zi} is the z-component of the wave vector, A is a complex constant, the common factor $\exp[i(qx - \omega t)]$ is omitted, and $q = \sqrt{k_0^2 n_i^2 - k_{zi}^2}$ is the transverse component of the wave vector.



(a) (b)

FIG. 2. Spectral dependences of the real parts of transverse ε_o and longitudinal ε_e permittivities of metamaterials made of gold (a) and silver (b) cylinders periodically with radius $r = 30$ nm embedded into aluminum oxide matrix.

From the solution of the boundary value problem, it follows that the electric $\vec{E}_t(R)$ and magnetic $\vec{H}_t(R)$ vectors of the refracted Bessel beam can be represented as:

$$\begin{aligned} \vec{E}_t(R) &= \vec{E}_t^{tr}(R) + \vec{E}_t^{l}(R), \\ \vec{E}_t^{l}(R) &= A_{inc} t_{ij}^{\rm TM} \exp i\left[m\varphi + k_{zj}z\right] \frac{q}{k_0 n_j} J_m\left(q\rho\right)\vec{e}_z, \end{aligned} \tag{4}$$

$$\begin{aligned} \vec{E}_t^{tr}(R) &= \frac{iA_{inc}}{\sqrt{2}} \exp i[(m-1)\varphi + k_{zj}z] t_{ij}^{\rm TM}\left[J_{m-1}(q\rho)\vec{e}_+ - J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-\right], \\ \vec{H}_t(R) &= \frac{n_j A_{inc}}{\sqrt{2}} \exp i[(m-1)\varphi + k_{zj}z] t_{ij}^{\rm TM}\left[J_{m-1}(q\rho)\vec{e}_+ + J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-\right]. \end{aligned} \tag{5}$$

Here $\cos\gamma_j = k_{zj}/k_0 n_j$, $k_0 = \omega/c$, ω is the frequency of the BLB; the symbols "tr" and "l" denote the transverse and longitudinal components of the electric (magnetic) vector, respectively; $A_t^{\rm TM}$ is the amplitude coefficient; $\vec{e}_\pm = (\vec{e}_1 \pm i\vec{e}_2)/\sqrt{2}$.

Similarly for the reflected field, we obtain:

$$\begin{aligned} \vec{E}_r(R) &= \vec{E}_r^{tr}(R) + \vec{E}_r^{l}(R), \\ \vec{E}_r^{l}(R) &= A_{inc} r_{ij}^{\rm TM} \exp i\left[m\varphi - k_{zi}z\right] \sin\gamma_i J_m\left(q\rho\right)\vec{e}_z, \\ \vec{E}_r^{tr}(R) &= -\frac{iA_{inc}}{\sqrt{2}} \exp i[(m-1)\varphi - k_{zi}z] r_{ij}^{\rm TM} \cos\gamma_i \left[J_{m-1}(q\rho)\vec{e}_+ - J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-\right], \end{aligned} \tag{6}$$

$$\vec{H}_r(R) = \frac{n_i A_{inc}}{\sqrt{2}} \exp i[(m-1)\varphi - k_{zi}z] r_{ij}^{\rm TM}\left[J_{m-1}(q\rho)\vec{e}_+ + J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-\right]. \tag{7}$$

Here $r_{ij}^{\rm TM} = A_r^{\rm TM}/A_{inc}$ is the reflection coefficient; $A_r^{\rm TM}$ is the amplitude coefficient.

In this case, for the refractive and reflective coefficients of Bessel light beams at the boundary between an isotropic medium and a hyperbolic metamaterial, we have the following expressions:

$$\begin{aligned} t_{ic}^{e} &= \frac{2\sqrt{\varepsilon_0\varepsilon_e}\sqrt{1-\frac{q^2}{k_0^2\varepsilon_0}}}{\sqrt{\varepsilon_0\varepsilon_e}\sqrt{1-\frac{q^2}{k_0^2\varepsilon_e}} + \varepsilon_o\sqrt{1-\frac{q^2}{k_0^2\varepsilon_0}}}, \\ r_{ic}^{TE} &= \frac{\varepsilon_o\sqrt{1-\frac{q^2}{k_0^2\varepsilon_0}} - \sqrt{\varepsilon_0}\sqrt{\varepsilon_e - \varepsilon_o q^2/k_0^2\varepsilon_e}}{\varepsilon_o\sqrt{1-\frac{q^2}{k_0^2\varepsilon_0}} + \sqrt{\varepsilon_0}\sqrt{\varepsilon_e - \varepsilon_o q^2/k_0^2\varepsilon_e}}. \end{aligned} \tag{8}$$

For the refractive and reflective coefficients of the Bessel light beams at the boundary between the hyperbolic metamaterial and the isotropic medium, we obtain:

$$
\begin{aligned}
t_{ci}^{TE} &= \frac{2\varepsilon_o\sqrt{1-\frac{q^2}{k_0^2\varepsilon_e(q)}}}{\sqrt{\varepsilon_1}\sqrt{\varepsilon_e-\varepsilon_o q^2/k_0^2\varepsilon_e}+\varepsilon_o\sqrt{1-\frac{q^2}{k_0^2\varepsilon_1}}},\\
r_{ci}^{e} &= -\frac{\varepsilon_o\sqrt{1-\frac{q^2}{k_0^2\varepsilon_1}}-\sqrt{\varepsilon_1}\sqrt{\varepsilon_e-\varepsilon_o q^2/k_0^2\varepsilon_e}}{\varepsilon_o\sqrt{1-\frac{q^2}{k_0^2\varepsilon_1}}+\sqrt{\varepsilon_1}\sqrt{\varepsilon_e-\varepsilon_o q^2/k_0^2\varepsilon_e}}.
\end{aligned}
\tag{9}
$$

Taking into account the limited thickness of the metamaterial structure, we obtain for the electric field vector inside the hyperbolic metamaterial (HMM) and outside it:

$$
\begin{aligned}
&\vec{E}_0(R)=\vec{E}_0^{tr}(R)+\vec{E}_0^{l}(R),\\
&\vec{E}_0^{l}(R)=A_{inc}\frac{q}{k_0\sqrt{\varepsilon_0}}\exp i\left[m\varphi-k_{z0}z\right]J_m\left(q\rho\right)\vec{e}_z,\\
&\vec{E}_0^{tr}(R)=-\frac{iA_{inc}}{\sqrt{2}}\exp i[(m-1)\varphi-k_{z0}z]\sqrt{1-\frac{q^2}{k_0^2\varepsilon_0}}\left[J_{m-1}(q\rho)\vec{e}_+-J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-\right],\\
&\vec{E}_1(R)=\vec{E}_1^{tr}(R)+\vec{E}_1^{l}(R),\\
&\vec{E}_1^{l}(R)=A_{inc}\frac{q}{k_0\sqrt{\varepsilon_1}}t\exp i\left[m\varphi+k_{z1}(z-h)\right]J_m\left(q\rho\right)\vec{e}_z,\\
&\vec{E}_1^{tr}(R)=\frac{iA_{inc}}{\sqrt{2}}\exp i[(m-1)\varphi+k_{z1}(z-h)]t\sqrt{1-\frac{q^2}{k_0^2\varepsilon_1}}\left[J_{m-1}(q\rho)\vec{e}_+-J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-\right],
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
\vec{E}_{\mathrm{HMM}}(R)&=\vec{E}_{\mathrm{HMM}}^{tr}(R)+\vec{E}_{\mathrm{HMM}}^{l}(R),\\
\vec{E}_{\mathrm{HMM}}^{l}(R)&=(\vec{E}_{\mathrm{HMM}}^{l})^f+(\vec{E}_{\mathrm{HMM}}^{l})^b,\\
\vec{E}_{\mathrm{HMM}}^{tr}(R)&=(\vec{E}_{\mathrm{HMM}}^{tr})^f+(\vec{E}_{\mathrm{HMM}}^{tr})^b,\\
(\vec{E}_{\mathrm{HMM}}^{l})^{f,b}&=A_{inc}\frac{q}{k_0\sqrt{\varepsilon_e(q)}}s^{f,b}\exp i[m\varphi\pm k_{z\mathrm{HMM}}z]J_m\left(q\rho\right)\vec{e}_z,\\
(\vec{E}_{\mathrm{HMM}}^{tr}(R))^{f,b}&=\pm\frac{iA_{inc}}{\sqrt{2}}s^{f,b}\exp i[(m-1)\varphi\pm k_{z\mathrm{HMM}}z]\times\\
&\times\sqrt{1-\frac{q^2}{k_0^2\varepsilon_e(q)}}\left[J_{m-1}(q\rho)\vec{e}_+-J_{m+1}(q\rho)\exp(2i\varphi)\vec{e}_-\right].
\end{aligned}
\tag{11}
$$

Here $s^f=A_f/A_{inc}$, $s^b=A_b/A_{inc}$ are the amplitude coefficients for the BLB propagating in the forward (along the z axis) and counter-direction to it, and t is the transmittance of the metamaterial layer. Similarly, we can obtain expressions for the magnetic field vector.

In this case, it follows from the boundary conditions that:

$$
t=\frac{t_{ic}^{e}t_{ci}^{\mathrm{TM}}\exp(ik_{z\mathrm{HMM}}h)}{1+r_{ic}^{\mathrm{TM}}r_{ci}^{e}\exp(2ik_{z\mathrm{HMM}}h)},
\tag{12}
$$

$$
s^f=\frac{t_{ic}^{e}}{1+r_{ic}^{\mathrm{TM}}r_{ci}^{e}\exp(2ik_{z\mathrm{HMM}}h)},\quad s^b=\frac{t_{ic}^{e}r_{ci}^{\mathrm{TM}}\exp(2ik_{z\mathrm{HMM}}h)}{1+r_{ic}^{\mathrm{TM}}r_{ci}^{e}\exp(2ik_{z\mathrm{HMM}}h)}.
\tag{13}
$$

In addition, the boundary conditions allow us to determine the form of the dispersion equation, which defines the condition for generating Bessel plasmon-polaritons:

$$
F=1+r_{ic}^{\mathrm{TM}}r_{ci}^{e}\exp 2i(k_{z\mathrm{HMM}}h)=0.
\tag{14}
$$

3. Features of the generation and energetic characteristics of surface Bessel plasmon-polaritons in metamaterial structure.

The determination of roots for equation (14) (with the parameter of conicity q) is rather complex task, as this equation is complex. The method of reflection coefficient poles allows one to solve this problem. The essence of the method can be summarized as follows: The function F can be represented as $F=|F|\exp(if)$, where f is the phase of this function. It is evident that $f=f(q)$ corresponds to the maximum change in the phase of the function. Therefore, to determine the roots, it is necessary to construct dependencies $f(q)$ (more precisely, $f(\mathrm{Re}\,q)$). The peaks of this dependency correspond to the real part of the roots, and the full width at half maximum of each peak corresponds to the imaginary part of the corresponding root.

Let's consider a practically important scenario where the environment of the hyperbolic metamaterial is the same medium. Suppose the hyperbolic metamaterial separates semi-infinite identical dielectric media with a dielectric permittivity $\varepsilon_0 = \varepsilon_2 = (1.723)^2$ (SF10 glass). The thickness of the porous structure filled with gold is 500 nm.

The derivative of the phase of function $f(\mathrm{Re}\, q)$ versus the real part of the Bessel plasmon mode effective index is shown in Fig. 3.

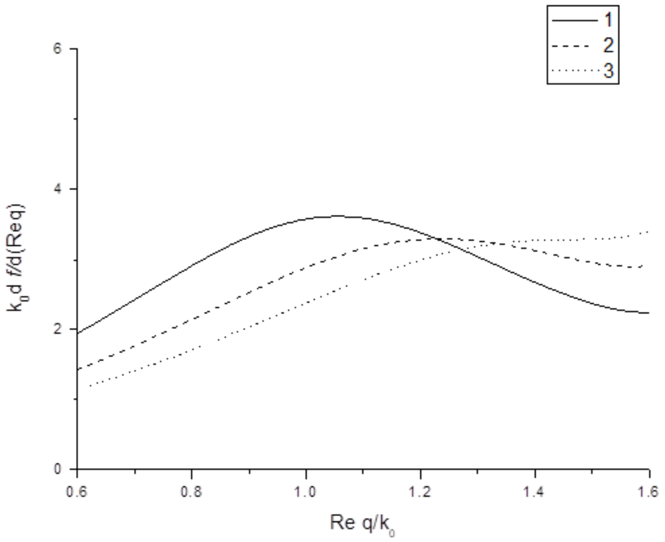


FIG. 3. Dependency of the function $k_0 df(\mathrm{Re}\, q)/d(\mathrm{Re}\, q)$ on the dimensionless parameter $\mathrm{Re}\, q/k_0$: 1 – $N = 0.3$; 2 – $N = 0.35$; 3 – $N = 0.4$. The wavelength of the incident light is 633 nm. The case of surrounding a nanoporous structure with gold nanorods by SF10 glass.

From Fig. 2, we find that: $q = (1.030 + 0.538i) \cdot 10^7$ m^{-1} for $N = 0.3$; $q = (1.187 + 0.614i) \cdot 10^7$ m^{-1} for $N = 0.35$; $q = (1.325 + 0.658i) \cdot 10^7$ m^{-1} for $N = 0.4$. It follows that, the Bessel plasmon-polaritons are generated when a Bessel light beam with cone angles of 37°, 44°, and 51° falls on a hyperbolic metamaterial based on aluminum oxide with gold-filled pores. As can be seen, the cone angle of the BLB for excitation of the surface Bessel plasmon exhibits high sensitivity to the filling ratio.

Let us now consider the dependence of the excitation conditions of plasmon polaritons on the thickness of the hyperbolic metamaterial (see Fig. 4). As can be seen from Fig. 4, $q/k_0 = 1.037+0.542i$ for $h = 500$ nm; $q/k_0 = 1.04+0.563i$ for $h = 250$ nm; $q/k_0 = 1.128+0.804i$ for $h = 100$ nm and $q/k_0 = 1.357+0.837i$ for $h = 70$ nm. Thus, as the thickness of the nanoporous structure decreases, the cone angle of the BLB for excitation of the surface Bessel plasmon-polariton increases.

Let's consider the case when a hyperbolic metamaterial is created using porous aluminum oxide with silver metallic nanowires. At a wavelength of $\lambda = 589.3$ nm and filling ratio N=0.3 one has $\varepsilon_m = -11.8736 + i1.376$. The calculation shows that $q/k_0 = 0.908 + 0.1i$ for $h = 200$ nm; $q/k_0 = 1.084 + 0.605i$ for $h = 100$ nm; $q/k_0 = 1.1253 + 0.566i$ for $h = 70$ nm. Thus, as the thickness of the nanoporous structure decreases, the parameter of the cone-shaped surface plasmon (Bessel surface plasmon) for exciting the surface Bessel plasmon-polaritons increases. Consequently, the overall nature of the established dependencies remains unchanged. However, it should be noted that the low absorption of silver, described by the imaginary part of the dielectric permittivity, leads to a reduction in the damping of Bessel plasmon-polaritons in the transverse direction.

Let us now consider the distribution of energy flows (the behavior of the Poynting vector $\vec{S} = \dfrac{c}{8\pi}\,\mathrm{Re}[\vec{E}\vec{H}^*]$). In this case, we will neglect the finiteness of the thickness of the metamaterial film. As estimated, this assumption is valid as $h \geq 500$ nm, which fully corresponds to the conditions for obtaining a hyperbolic metamaterial based on nanoporous aluminum oxide with pores filled with metal. Consider a transverse magnetic TM-polarized Bessel light beam incident on the hyperbolic metamaterial. Then, using expressions (4) and (5), where we set $t_{ij}^{\mathrm{TM}} = t_{ic}^{e}$, we obtain the following result for the longitudinal and radial components of the Poynting vector within the metamaterial:

$$S_z = S_\rho = 0. \tag{15}$$

The azimuthal component of this vector is determined by the expression:

$$S_\varphi = \frac{cq}{8\pi k_0}\,|A_{inc}|^2\,|t_{ic}^{e}|^2\,F_2(\rho)\exp\left(-2\chi_{\mathrm{HMM}}z\right), \tag{16}$$

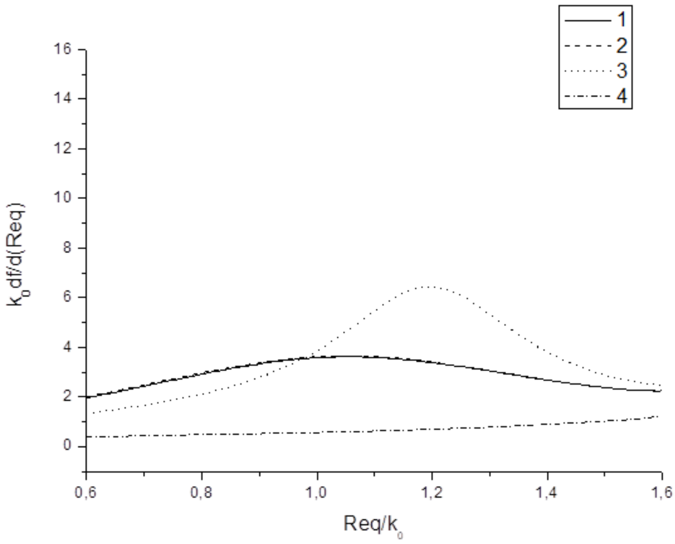


FIG. 4. Dependence $k_0 df(\mathrm{Re}\,q)/d(\mathrm{Re}\,q)$ on the dimensionless parameter $\mathrm{Re}\,q/k_0$: 1 — $h = 500$ nm; 2 – $h = 250$ nm; 3 – $h = 100$ nm; 4 – $h = 70$ nm. The wavelength of the incident light is 633 nm. Filling ratio 0.3. The case of symmetric environment of the hyperbolic metamaterial.

where $\chi_{\mathrm{HMM}} = \mathrm{Re}\left[k_0\sqrt{\varepsilon_e^{\mathrm{eff}}(q)}[q^2/(k_0^2\varepsilon_e^{\mathrm{eff}}(q)) - 1]^{1/2}\right]$, $F_2(\rho) = \dfrac{m}{|q|\rho}(J_m(|q|\rho))^2$.

Thus, in the metamaterial, only the azimuthal energy flow is presented.

Figures 5 and 6 illustrate the 2D and 3D distribution for energy flow of the azimuthal component of the Bessel plasmon inside the hyperbolic metamaterial based on nanoporous aluminum oxide with pores filled with metal (gold and silver) at the distance $z = \lambda/3$ from the boundary with metamaterial layer. Figs. 4 and 5 show that the central peak of the distribution for energy flow of the azimuthal component of the Bessel plasmon generated in the structure decays exponentially when the cone angle of the BLB for excitation of the Bessel plasmon-polariton increases. The low absorption of silver leads to the reduction in the damping for azimuthal energy flow of the Bessel plasmon-polaritons. Besides it the field for azimuthal energy flow is characterized by more narrow near axial maxima and essential suppression of lateral maxima while moving off the field axis to its periphery.

4. Conclusion

In this paper, we have found the dispersion equation that determines the condition for exciting the Bessel plasmon-polaritons in a confined hyperbolic metamaterial based on dielectric with periodically embedded metallic nanowires. A method for solving this dispersion equation has been developed. Values for the cone-shaped parameters of the Bessel plasmons generated within the matrix of nanoporous aluminum oxide, with pores filled with gold and silver, have been obtained. Remarkably, there is a super-sensitivity of the plasmon generation conditions in this structure to the filling factor.

We have also examined the energy flows formed in the hyperbolic metamaterial when the surface Bessel plasmons are excited within it. It is demonstrated that the sole non-zero energy flow is the azimuthal flow, whose magnitude exponentially decreases as one moves away from the boundary between the isotropic medium and the hyperbolic metamaterial.

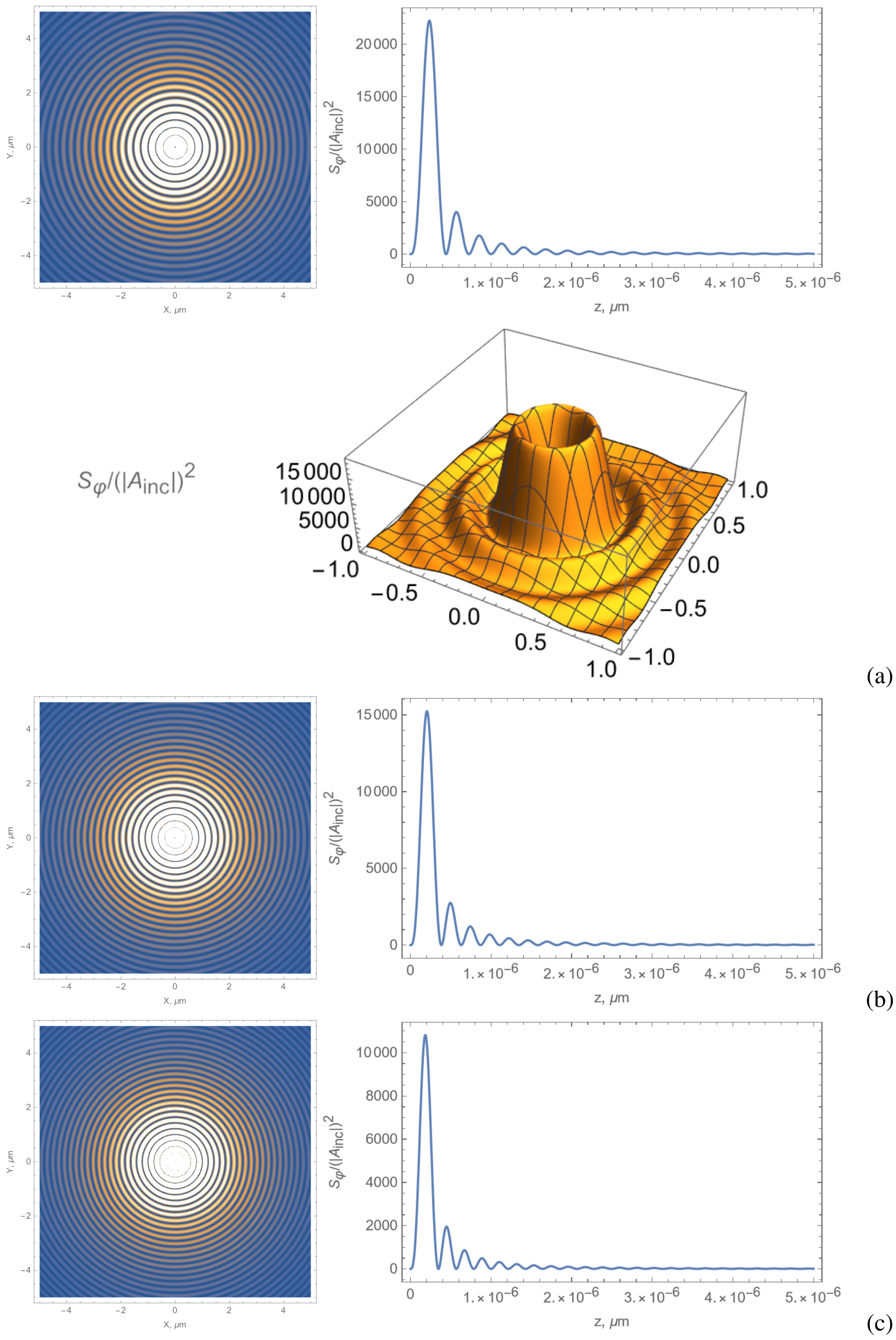
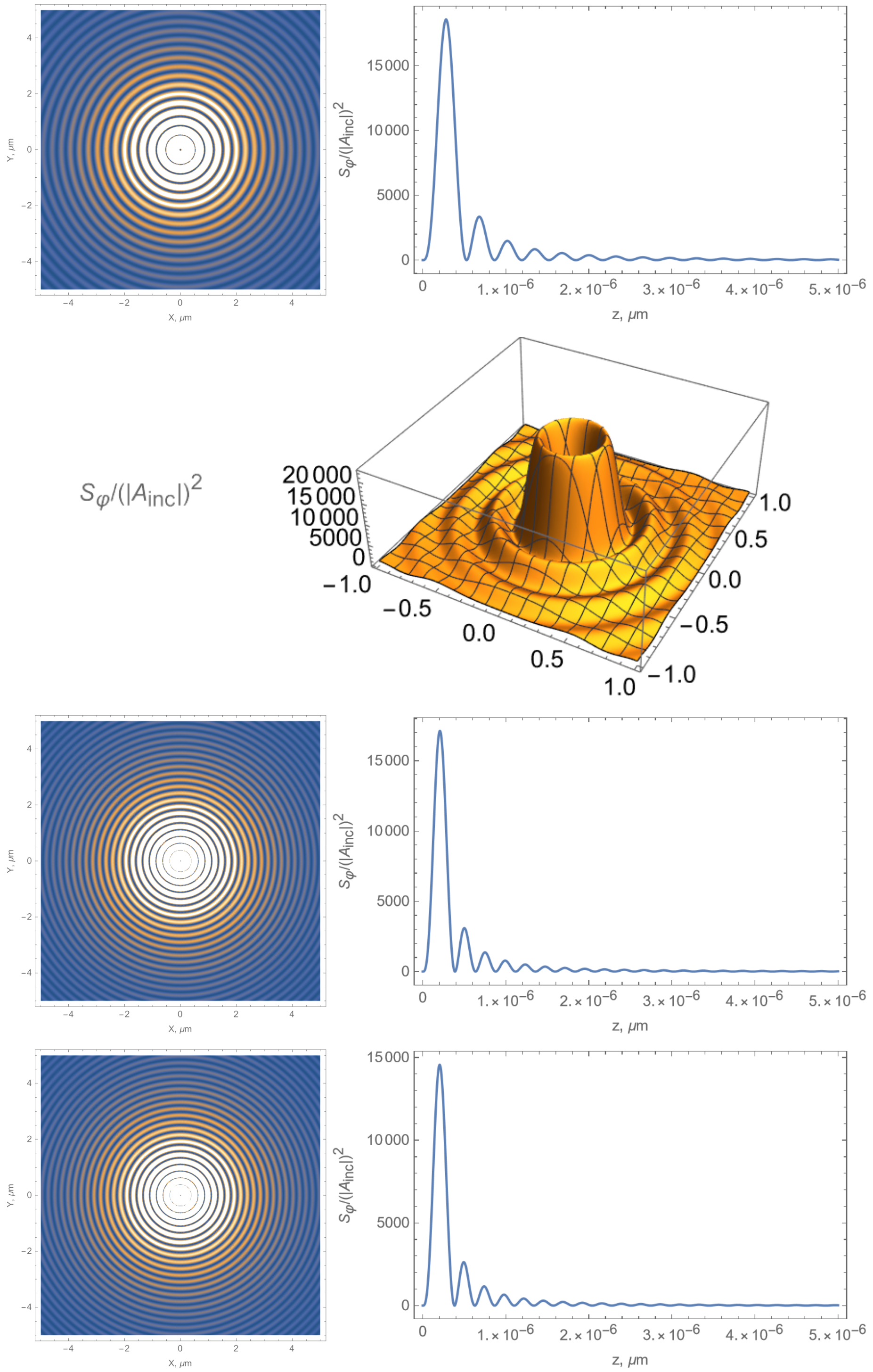


FIG. 5. 2D- and 3D- distribution for energy flow of azimuthal component of the Bessel plasmon field formed inside the hyperbolic metamaterial based on nanoporous aluminum oxide with pores filled with gold when (a) $q = (1.030 + 0.538i) \cdot 10^7$ m^{-1}, (b) $q = (1.187 + 0.614i) \cdot 10^7$ m^{-1}, (c) $q = (1.325 + 0.658i) \cdot 10^7$ m^{-1} and the wavelength $\lambda = 633$ μm. The distance from the boundary with metamaterial layer is $z = \lambda/3$.



(a)

(b)

(c)

FIG. 6. 2D- and 3D- distribution for energy flow of azimuthal component of the Bessel plasmon field formed inside the hyperbolic metamaterial based on nanoporous aluminum oxide with pores filled with silver when (a) $q/k_0 = 0.908 + 0.1i$, (b) $q/k_0 = 1.084 + 0.605i$, (c) $q/k_0 = 1.1253 + 0.566i$ and the wavelength $\lambda = 589.3$ nm. The distancefrom the boundary with metamaterial layer $z = \lambda/3$.

References

[1] Gaponenko S.V. *Introduction to Nanophotonics*. Cambridge, Cambridge University Press, 2010, 458 p.
[2] Maier S.A. *Plasmonics Theory and Applications*. Moscow, Research Center "Regular and Chaotic Dynamics", 2011, 292 p. (in Russian)
[3] Libeson M.N. Surface electromagnetic waves in the optical range. *Soros Educational J.*, 1996, **10**, P. 92–98. (in Russian)
[4] Agranovich V.M., Mills D.L. *Surface Polaritons: Electromagnetic Waves at Surfaces and Interfaces*. Amsterdam: North-Holland, 1982, 734 p.
[5] Reather H. *Surface plasmon on Smooth and Rough Surfaces and on Gratings*. Berlin, Springer-Verlag, 1988, 140 p.
[6] Boardman A.D. *Electromagnetic Surface Modes*. New York, John Wiley & Sons, 1982, 786 p.
[7] Klimov V.V. *Nanoplasmonics*. Fizmatlit, Moscow, 2009, 480 p. (in Russian)
[8] Klyuchnik A.V., Kurganov S.Yu., Lozovik Yu.E. Plasma optics of nanostructures. *Physics of the Solid State*, 2003, **45** (7), P. 1327–1331.
[9] Ebbesen T.W., et al. Extraordinary optical transmission through sub-wavelength hole arrays. *Nature*, 1998, **391**, P. 667–669.
[10] Ozbay E. Plasmonics: Merging Photonics and Electronics at Nanoscale Dimensions. *Science*, 2006, **311**, P. 189–193.
[11] Krasavin V., Zheludev N.I. Active plasmonics: Controlling signals in Au/Ga waveguide using nanoscale structural transformations. *Applied Physics Letters*, 2004, **84**, P. 1416–1419.
[12] Bozhevolnyi S.I. *Plasmonics Nanoguides and Circuits*. Singapore, Pan Stanford Publishing, 2008, 449 p.
[13] Maier S.A., et al. Experimental demonstration of fiberaccessible metal nanoparticle plasmon waveguides for planar energy guiding and sensing. *Applied Physics Letters*, 2005, **86** (7), 071103.
[14] Rayleigh L. *The Theory of Sound*. New York, Dover, 1954, 507 p.
[15] Ritchie R. Plasma losses by fast electrons in thin films. *Physical Review*, 1957, **106**, P. 874–881.
[16] Powell C., Swan J. Origin of the characteristic electron energy losses in aluminum. *Physical Review*, 1959, **115**, P. 869–875.
[17] Liu Y., Zhang X. Metamaterials: a new frontier of science and technology. *Chemical Society Reviews*, 2011, **40** (5), P. 2494–2507.
[18] Kshetrimayum R.S. A brief Intro to Metamaterials. *IEEE Potentials*, 2004, **23** (5), P. 44–46.
[19] Soukoulis C.M. *Photonic Band Gap Materials*. Dordrecht: Springer Netherlands, 1996, 744 p.
[20] Poddubny A.N., et al. Hyperbolic metamaterials. *Nature Photonics*, 2013, **7**, P. 958–967.
[21] Jacob Z., Alekseyev L.V., Narimanov E. Optical hyperlens: far-field imaging beyond the diffraction limit. *Optics Express*, 2006, **14**, P. 8247–8256.
[22] Lu D., Liu Z. Hyperlenses and metalenses for far-field super-resolution imaging. *Nature Communications*, 2012, **3**, 1205.
[23] Poddubny A.N., et al. Microscopic model of Purcell enhancement in hyperbolic metamaterials. *Physical Review B*, 2012, **86**, 035148.
[24] Kurilkina S.N., Belyi V.N., Kazak N.S. Features of evanescent Bessel light beams formed in structures containing a dielectric layer. *Optics Communications*, 2010, **283**, P. 3860–3868.
[25] Goncharenko A.M., Khilo N.A., Petrova E.S. Evanescent Bessel light. *Proceedings of SPIE – The Int. Society for Optical Engineering*, 2001, **4517**, P. 95–99.
[26] Zhan Q. Evanescent Bessel beam generation via surface plasmon resonance excitation by a radially polarized beam. *Optics Letters*, 2006, **31**, P. 1726–1728.
[27] Jiefeng X., Quing L., Jia W. Numerical simulation of evanescent Bessel beams and apodization of evanescent field in near-field optical virtual probe. *Proceedings of the SPIE*, 2005, **5635**, P. 42–47.
[28] Novitsky A.V., Barkovsky L.M. Total internal reflection of vector Bessel beams: Imbert–Fedorov shift and intensity transformation. *J. of Optics A Pure and Applied Optics*, 2008, **10**, 075006.
[29] Al-Muhanna M.K., et al. Energy flow patterns in an optical field formed by a superposition of evanescent Bessel light beams. *J. of Optics*, 2011, **13** (10), 105703.
[30] Cai W., Shalaev V.M. *Optical Metamaterials – Fundamentals and Applications*. Springer, Berlin, 2010.

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