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Original article

Mixed problem for a linear differential equation of parabolic type with nonlinear impulsive conditions

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ABSTRACT In this paper, we consider a linear parabolic type partial differential equation in the space of generalized functions as the equation of neutron diffusion in the presence of neutron absorption by the atomic nucleus with nonlinear impulsive effects. Spectral equation is obtained from the Dirichlet boundary value conditions and this spectral problem is studied. The Fourier method of variables separation is used. Countable system of nonlinear functional integral equations is obtained with respect to the Fourier coefficients of unknown function. Theorem on a unique solvability of the countable system of functional integral equations is proved. The method of successive approximations is used in combination with the method of contracting mapping. Criteria of uniqueness and existence of generalized solution of the impulsive mixed problem is obtained. Solution of the mixed problem is derived in the form of the Fourier series. It is shown that the Fourier series converges uniformly.

KEYWORDS Mixed problem, impulsive parabolic equation, nonlinear impulsive conditions, involution, unique solvability

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1. Formulation of the problem statement

Differential equations of mathematical physics have direct applications in the theory of nanosystems (see, for example, [1–13], and [14]). Partial differential and integro-differential equations of parabolic type with initial and boundary conditions are investigated widely by large number of scientists and have different applications in sciences and technology (see, for example, [15–28]). Note that the differential equations of parabolic type are associated with heat and diffusion processes. Neutron diffusion plays a significant role in the operation of nuclear reactors. The diffusion equation makes it possible to calculate the neutron density inside the core of a nuclear reactor, the neutron flux from the moderator surface, and the reflection and transmission of neutrons by biological protection structures.

Differential and integro-differential equations with impulse effects have applications in sciences, ecology, biotechnology, industrial robotics, pharmacokinetics, optimal control, etc. [29–38]. A lot of publications are devoted to study differential equations with impulsive effects, describing many natural and technical processes (see, for example, [39–50]). The questions of existence and uniqueness of periodic solutions of differential and integro-differential equations were studied in [51–55]. In [56–58], the Whitham type partial differential equations of the first order with impulsive effects are studied.

To date, the impulsive systems for ordinary differential and integro-differential equations have been well studied. As for the partial differential equations of mathematical physics, the authors are not aware of any work where the differential equations of mathematical physics with impulsive influences were studied. It is necessary to solve problems associated with the use of the Fourier series, the integration of impulsive countable systems in different subdomains and the definition of the class of solutions. So, in the present paper, we study the solvability of the impulsive mixed problem for a linear differential equation with involution and nonlinear impulsive conditions in the space of generalized functions.

In the domain

$$\Omega = \left\{ t \in (0,T),\ t \neq t_m,\ 0 < t_m < T,\ m = 1,2,...,p,\ \ x \in (-1,1) \right\},$$

we consider the following differential equation of neutron diffusion in the presence of neutron absorption by the atomic nucleus

$$U_t(t,x) - U_{xx}(t,x) - \varepsilon\, U_{xx}(t,-x) = a(t) U(t,x) + f(t,x) \tag{1}$$

with the Dirichlet boundary value conditions

$$U(t,-1) = U(t,1) = 0, \qquad 0 \le t \le T; \tag{2}$$

and initial value condition

$$U(0,x) = \varphi(x), \quad -1 \le x \le 1, \tag{3}$$

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where T is given positive number, unknown function $U(t,x)$ characterizes the neutron density with the impulsive effects, ε is the diffusion coefficient, $0<\varepsilon<1$, $a(t)\in C[0;T]$ is the function characterizing the lifetime of a neutron in a medium before absorption, the function $f(t,x)\in L_2(\Omega)$ characterizes the rate of the neutron production, $\varphi(x)$ is given function on the segment $[-1,1]$. We assume that for the initial value function $\varphi(x)\in L_2[-1,1]$ the following conditions are fulfilled $\varphi(-1)=\varphi(1)=0$. We also suppose that $f(t,-1)=f(t,1)=0$.

Since the equation (1) is impulsive and the unknown function $U(t,x)$ has some discontinuity points $t_1,t_2,...,t_p$ on the interval $(0,T)$, in integration processes, we need to know the difference between the right and the left side limit values of unknown function at these discontinuity points. However, in practice, it is often impossible to determine these differences explicitly. So, we use them in the form of nonlinear functions. Therefore, we consider equation (1) with the following impulsive conditions

$$U\left(t_m^+,x\right)-U\left(t_m^-,x\right)=I_m\left(x,\int_{-1}^{1}G(y)U\big(t_m,y\big)dy\right),\quad m=1,2,...,p, \tag{4}$$

where $I_m(x,\cdot)$ are continuous functions on x, $\int_{-1}^{1}|G(x)|dx<\infty$, $0<t_1<...<t_p<T<\infty$, $U\big(t_m^+,x\big)=\lim_{\nu\to 0^+}U\left(t_m+\zeta,x\right)$, $U\left(t_m^-,x\right)=\lim_{\nu\to 0^-}U\left(t_m-\zeta,x\right)$ are the right-hand side and the left-hand side limits of function $U(t,x)$ at the points $t=t_m$, respectively.

2. Formal solution of the mixed problem

First, consider the homogeneous partial differential equation

$$U_t(t,x)-U_{xx}(t,x)-\varepsilon\,U_{xx}(t,-x)=0 \tag{5}$$

with boundary value conditions of the Dirichlet type

$$U(t,-1)=U(t,1)=0,\qquad 0\le t\le T. \tag{6}$$

Problem (5), (6) will be solved by the method of separation of variables: $U(t,x)=u(t)\,\vartheta(x)$. After separation of variables, from (5), (6), we arrive at the following spectral problem for an ordinary differential equation

$$\vartheta''(x)+\varepsilon\vartheta''(-x)+\lambda\vartheta(x)=0 \tag{7}$$

with boundary value conditions

$$\vartheta(-1)=0,\quad \vartheta(1)=0. \tag{8}$$

It is obvious that for the case of even eigenfunctions, equation (7) takes the form

$$(1+\varepsilon)\vartheta_1''(x)+\lambda_1\vartheta_1(x)=0. \tag{9}$$

Solving differential equation (9) with conditions (8), we find the eigenvalues

$$\lambda_{1,n}=(1+\varepsilon)\,\pi^2(n+0.5)^2 \tag{10}$$

and eigenfunctions of problem (7), (8):

$$\vartheta_{1,n}(x)=\cos\pi(n+0.5)x,\quad n\in\mathbb{N}. \tag{11}$$

In the case of odd eigenfunctions, equation (7) takes another form

$$(1-\varepsilon)\vartheta_2''(x)+\lambda_2\vartheta_2(x)=0. \tag{12}$$

Solving differential equation (12) with spectral (zero) conditions (8), we find the eigenvalues and the corresponding eigenfunctions of problem (12), (8):

$$\lambda_{2,n}=(1-\varepsilon)\,\pi^2n^2,\quad 0<\varepsilon<1, \tag{13}$$

$$\vartheta_{2,n}(x)=\sin\pi n\,x,\quad n\in\mathbb{N}. \tag{14}$$

Note that the eigenfunctions $\vartheta_{i,n}(x)\,(i=1,2)$ determined by (11) and (14) form a complete system of orthonormal eigenfunctions in the space $L_2[-1,1]$. Therefore, we seek desired solutions to the nonhomogeneous partial differential equation (1) in the forms $U(t,x)=U_1(t,x)+U_2(t,x)$ of the Fourier series

$$U_i(t,x)=\sum_{n=1}^{\infty}u_{i,n}(t)\,\vartheta_{i,n}(x),\quad i=1,2, \tag{15}$$

where $U_1(t,x)$ and $U_2(t,x)$ satisfy the given differential equation (1)

$$U_{it}(t,x)-U_{ixx}(t,x)-\varepsilon\,U_{ixx}(t,-x)=a(t)U_i(t,x)+f_i(t,x),$$

$$u_{i,n}(t)=\int\limits_{-1}^{1}U_i(t,x)\,\vartheta_{i,n}(x)\,dx,\quad i=1,2. \tag{16}$$

We also suppose that the function $f(t,x)=f_1(t,x)+f_2(t,x)\in L_2(\Omega)$ expands to the Fourier series in eigenfunctions $\vartheta_{i,n}(x)$:

$$f_i(t,x)=\sum_{n=1}^{\infty}f_{i,n}(t)\,\vartheta_{i,n}(x),\;\; i=1,2, \tag{17}$$

where

$$f_{i,n}(t)=\int\limits_{-1}^{1}f_i(t,x)\,\vartheta_{i,n}(x)\,dx,\quad i=1,2. \tag{18}$$

By using the Fourier series (15), from the impulsive conditions (4), we obtain

$$\sum_{n=1}^{\infty}u_{i,n}(t_m^+)\vartheta_{i,n}(x)-\sum_{n=1}^{\infty}u_{i,n}(t_m^-)\vartheta_{i,n}(x)=\sum_{n=1}^{\infty}I_{m,i,n}\vartheta_{i,n}(x),\quad m=1,2,...,p,$$

where

$$I_{m,i,n}=\int\limits_{-1}^{1}I_{m,i}\left(x,\int\limits_{-1}^{1}G(y)\sum_{j=1}^{\infty}u_{i,j}(t_m)\,\vartheta_{i,j}(y)dy\right)\vartheta_{i,n}(x)\,dx,\quad i=1,2. \tag{19}$$

Hence, due to the orthonormality of the eigenfunctions,

$$(\vartheta_{i,n}(x),\vartheta_{i,j}(x))=\begin{cases}1,\;\; n=j,\\ 0,\;\; n\neq j,\end{cases}$$

we derive that

$$u_{i,n}(t_m^+)-u_{i,n}(t_m^-)=I_{m,i,n},\quad m=1,2,...,p. \tag{20}$$

Substituting the Fourier series (15) into the given differential equation (1), we obtain a countable system of the first order ordinary differential equations

$$u'_{i,n}(t)+\lambda_{i,n}\,u_{i,n}(t)=a(t)\,u_{i,n}(t)+f_{i,n}(t), \tag{21}$$

where $\lambda_{i,n}$ are eigenvalues determined by (10) and (13). We rewrite the countable system of the first order ordinary differential equations (21) as

$$\left(e^{\lambda_{i,n}t}u_{i,n}(t)\right)'=e^{\lambda_{i,n}t}\left[a(t)\,u_{i,n}(t)+f_{i,n}(t)\right],\;\; i=1,2. \tag{22}$$

By integration of the last equation (22) on the intervals: $(0,t_1]\,,\;(t_1,t_2]\,,\;\ldots\,,\;(t_p,t_{p+1}]$, we obtain:

$$\int\limits_0^{t_1}e^{\lambda_{i,n}s}\left[a(s)\,u_{i,n}(s)+f_{i,n}(s)\right]ds=\int\limits_0^{t_1}\left(e^{\lambda_{i,n}s}u_{i,n}(s)\right)'ds=$$

$$=e^{\lambda_{i,n}t_1^-}u_{i,n}(t_1^-)-e^{\lambda_{i,n}0^+}u_{i,n}(0^+)=e^{\lambda_{i,n}t_1}u_{i,n}(t_1)-u_{i,n}(0),\quad i=1,2,\quad t\in(0,t_1]\,,$$

$$\int\limits_{t_1}^{t_2}e^{\lambda_{i,n}s}\left[a(s)\,u_{i,n}(s)+f_{i,n}(s)\right]ds=\int\limits_{t_1}^{t_2}\left(e^{\lambda_{i,n}s}u_{i,n}(s)\right)'ds=$$

$$=e^{\lambda_{i,n}t_2^-}u_{i,n}(t_2^-)-e^{\lambda_{i,n}t_1^+}u_{i,n}(t_1^+)=e^{\lambda_{i,n}t_2}u_{i,n}(t_2)-e^{\lambda_{i,n}t_1}u_{i,n}(t_1^+),\quad t\in(t_1,t_2]\,,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\int\limits_{t_p}^{t_{p+1}}e^{\lambda_{i,n}s}\left[a(s)\,u_{i,n}(s)+f_{i,n}(s)\right]ds=\int\limits_{t_p}^{t_{p+1}}\left(e^{\lambda_{i,n}s}u_{i,n}(s)\right)'ds=$$

$$=e^{\lambda_{i,n}t_{p+1}^-}u_{i,n}(t_{p+1}^-)-e^{\lambda_{i,n}t_p^+}u_{i,n}(t_p^+)=e^{\lambda_{i,n}t_{p+1}}u_{i,n}(t)-e^{\lambda_{i,n}t_p}u_{i,n}(t_p^+),\;\; t\in(t_p,t_{p+1}]\,.$$

Here we took into account that $u_{i,n}(0^+)=u_{i,n}(0),\;\; u_{i,n}(t_{p+1}^-)=u_{i,n}(t)$. So, taking the impulsive conditions (20) into account, on the interval $(0,T]$, we have

$$\int\limits_0^{t}e^{\lambda_{i,n}s}\left[a(s)\,u_{i,n}(s)+f_{i,n}(s)\right]ds=$$

$$=e^{\lambda_{i,n}t_1}u_{i,n}(t_1)-u_{i,n}(0)+e^{\lambda_{i,n}t_2}u_{i,n}(t_2)-e^{\lambda_{i,n}t_1}u_{i,n}(t_1^+)+e^{\lambda_{i,n}t_3}u_{i,n}(t_3)-e^{\lambda_{i,n}t_2}u_{i,n}(t_2^+)+$$
$$+...+e^{\lambda_{i,n}t_{p+1}}u_{i,n}(t)-e^{\lambda_{i,n}t_p}u_{i,n}(t_p^+)=$$

$$= -u_{i,n}(0) - e^{\lambda_{i,n} t_1}\left[u_{i,n}(t_1^+) - u_{i,n}(t_1)\right] - e^{\lambda_{i,n} t_2}\left[u_{i,n}(t_2^+) - u_{i,n}(t_2)\right] -$$

$$- \ldots - e^{\lambda_{i,n} t_p}\left[u_{i,n}(t_p^+) - u_{i,n}(t_p)\right] + e^{\lambda_{i,n} t} u_{i,n}(t) =$$

$$= -u_{i,n}(0) - \sum_{0<t_m<t}^{p} e^{\lambda_{i,n} t_m} I_{m,i,n} + e^{\lambda_{i,n} t} u_{i,n}(t).$$

Hence, we obtain that

$$e^{\lambda_{i,n} t} u_{i,n}(t) = u_{i,n}(0) + \int_0^t e^{\lambda_{i,n} s}\left[a(s)\, u_{i,n}(s) + f_{i,n}(s)\right] ds + \sum_{0<t_m<t}^{p} e^{\lambda_{i,n} t_m} I_{m,i,n}$$

or

$$u_{i,n}(t) = u_{i,n}(0) e^{-\lambda_{i,n} t} + \int_0^t e^{-\lambda_{i,n}(t-s)}\left[a(s)\, u_{i,n}(s) + f_{i,n}(s)\right] ds + \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} I_{m,i,n}. \tag{23}$$

For given function $\varphi(x)$ in (3), we set $\varphi(x) = \varphi_1(x) + \varphi_2(x)$. Now, supposing that the functions $\varphi_1(x)$ and $\varphi_2(x)$ are expanded in the Fourier series and using the Fourier coefficients (16), from condition (3), we obtain

$$u_{i,n}(0) = \int_{-1}^{1} U_i(0,x)\, \vartheta_{i,n}(x)\, dx = \int_{-1}^{1} \varphi_i(x)\, \vartheta_{i,n}(x)\, dx = \varphi_{i,n}, \quad i = 1, 2. \tag{24}$$

To find the unknown coefficients $u_{i,n}(0)$ in the presentations (23), we use the initial value conditions (24). Then we have

$$u_{i,n}(t) = \varphi_{i,n} e^{-\lambda_{i,n} t} + \int_0^t e^{-\lambda_{i,n}(t-s)}\left[a(s)\, u_{i,n}(s) + f_{i,n}(s)\right] ds + \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} I_{m,i,n}. \tag{25}$$

We note that representations (25) are the Fourier coefficients of the solution to problem (1)–(4) and have been considered as a countable system of nonlinear functional and integral equations. Taking into account (18) and (19), we rewrite it as

$$u_{i,n}(t) = A(t; u_{i,n}) \equiv \varphi_{i,n} e^{-\lambda_{i,n} t} + \int_0^t e^{-\lambda_{i,n}(t-s)} a(s) u_{i,n}(s) ds +$$

$$+ \int_0^t e^{-\lambda_{i,n}(t-s)} \int_{-1}^{1} f_i(s,x)\, \vartheta_{i,n}(x)\, dx ds +$$

$$+ \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} \int_{-1}^{1} I_{m,i}\left(x, \int_{-1}^{1} G(y) \sum_{j=1}^{\infty} u_{i,j}(t_m)\, \vartheta_{i,j}(y) dy\right) \vartheta_{i,n}(x)\, dx. \tag{26}$$

The representation (26) is countable system of functional equations. Substituting representation (26) into the Fourier series (15), we obtain a formal solution of the problem (1)–(4) on the domain Ω

$$U_i(t,x) = \sum_{n=1}^{\infty} \vartheta_{i,n}(x) \left[\varphi_{i,n} e^{-\lambda_{i,n} t} + \int_0^t e^{-\lambda_{i,n}(t-s)} a(s)\, u_{i,n}(s) ds + \right.$$

$$+ \int_0^t e^{-\lambda_{i,n}(t-s)} \int_{-1}^{1} f_i(s,x)\, \vartheta_{i,n}(x)\, dx ds +$$

$$\left. + \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} \int_{-1}^{1} I_{m,i}\left(x, \int_{-1}^{1} G(y) \sum_{j=1}^{\infty} u_{i,j}(t_m)\, \vartheta_{i,j}(y) dy\right) \vartheta_{i,n}(x)\, dx \right]. \tag{27}$$

3. Solvability of the countable system of nonlinear functional equations

Let us investigate the countable system of nonlinear functional equations (26) from the point of view of its unique solvability. Consider the following well-known Banach spaces, which will be used below:
the space $B_2[0,T]$ of function sequences $\{\, u_n(t)\,\}_{n=1}^{\infty}$ on the segment $[0,T]$ with the norm

$$\|\,\vec{u}(t)\,\|_{B_2[0,T]} = \sqrt{\sum_{n=1}^{\infty}\left(\max_{t\in[0,T]}|\,u_n(t)\,|\right)^2} < \infty;$$

the Hilbert coordinate space ℓ_2 of number sequences $\{\varphi_n\}_{n=1}^{\infty}$ with the norm

$$\|\,\varphi\,\|_{\ell_2} = \sqrt{\sum_{n=1}^{\infty}|\,\varphi_n\,|^2} < \infty;$$

the space $L_2\,[-1,1]$ of square-integrable functions on the interval $[-1,1]$ with the norm

$$\|\,\vartheta\,(x)\,\|_{L_2[-1,1]} = \sqrt{\int\limits_{-1}^{1}|\,\vartheta\,(x)\,|^2\,dx} < \infty;$$

the space

$$PC_{B_2}\left([0,T];\mathbb{R}\right) = \{u_n:[0,T]\to\mathbb{R};\;\; \vec{u}(t)\in\left(B_2[t_m,t_{m+1}];\mathbb{R}\right),\;\; m=0,1,...,p\}$$

with the norm

$$\|\,\vec{u}(t)\,\|_{PC_{B_2}[0,T]} = \max\left\{\|\,\vec{u}(t)\,\|_{B_2[t_m,t_{m+1}]}\,,\;\; m=0,1,2,...,p\right\},$$

where $u_n\left(t_m^+\right)$ and $u_n\left(t_m^-\right)$ $(m=0,1,...,p)$ exist and are bounded; $u_n\left(t_m^-\right)=u_n\left(t_m\right)$.

Theorem 1. Let the estimate $\sum\limits_{j=1}^{\infty}|\,\varphi_n\,|<\infty$ be valid for the Fourier coefficients of the function $\varphi(x)\in L_2[-1,1]$. If the following conditions are fulfilled:

$$\max_{m=\overline{1,p}}\;\max_{t\in[t_m,t_{m+1}]}\sqrt{\int\limits_{-1}^{1}\left[I_{i,m}\left(x,\int\limits_{-1}^{1}G(y)\sum_{j=1}^{\infty}u_{i,j}^{0}(t)\,\vartheta_{i,j}(y)dy\right)\right]^2 dx} \le M_{0,i} = \text{const} < \infty,$$

$$\left|I_{m,i}\left(x,\int\limits_{-1}^{1}G(y)\sum_{j=1}^{\infty}u_{i,j}^{\tau}(t_m)\,\vartheta_{i,j}(y)dy\right) - I_{m,i}\left(x,\int\limits_{-1}^{1}G(y)\sum_{j=1}^{\infty}u_{i,j}^{\tau-1}(t_m)\,\vartheta_{i,j}(y)dy\right)\right| \le$$

$$\le N_{m,i}(x)\left|\int\limits_{-1}^{1}G(y)\sum_{j=1}^{\infty}\left|\,u_{i,j}^{\tau}(t_m)-u_{i,j}^{\tau-1}(t_m)\,\right|\vartheta_{i,j}(y)\,dy\right|,\;\; 0<N_{m,i}(x)\in L_2[-1,1],$$

$$\rho_i = C\cdot M_{2,i}\left[M_4\,\|\,N_i(x)\,\|_{L_2[-1,1]}\,\|\,G(x)\,\|_{L_2[-1,1]} + M_{1,i}M_5\right] < 1,\quad i=1,2, \tag{28}$$

where C, $M_{1,i}$, M_4, M_5, $M_{2,i}$ are some constants, then the countable system of nonlinear functional equations (26) is uniquely solvable in the space $PC_{B_2}[0,T]$. In this case, the desired solution can be found by the following iterative process:

$$\begin{cases} u_{i,n}^{0}(t) = \varphi_{i,n}e^{-\lambda_{i,n}t} + \int\limits_{0}^{t}e^{-\lambda_{i,n}(t-s)}\int\limits_{-1}^{1}f_i(s,x)\,\vartheta_{i,n}(x)\,dxds, \\ u_{i,n}^{\tau+1}(t) = A\left(t;u_{i,n}^{\tau}\right),\quad i=1,2,\quad \tau=0,1,2,... \end{cases} \tag{29}$$

Proof. We use the method of contracting maps in combination with the method of successive approximations in the space $PC_{B_2}[0,T]$. We take into account that

$$\int\limits_{0}^{t}e^{-\lambda_{i,n}(t-s)}ds \le \frac{1}{\lambda_{i,n}}\left[1-e^{-\lambda_{i,n}t}\right] \le \frac{1}{\lambda_{i,n}}M_{1,i}.$$

Then, by virtue of conditions of the theorem and applying the Cauchy–Schwartz inequality and the Bessel inequality, we obtain from the approximations (29) that the following estimate is valid:

$$\|\,\vec{u}_i^{\,0}(t)\,\|_{PC_{B_2}[0,T]} \le \max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\max_{t\in[t_m,t_{m+1}]}\left|\,u_{i,n}^{0}\,(t)\,\right| \le \max_{m=\overline{1,p}}\sum_{n=1}^{\infty}|\,\varphi_{i,n}\,|\max_{t\in[t_m,t_{m+1}]}\left\{e^{-\lambda_{i,n}t}\right\}+$$

$$+C\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\sup_{t\in(t_m,t_{m+1}]}\left|\int_0^t e^{-\lambda_{i,n}(t-s)}\int_{-1}^{1} f_i(s,x)\,\vartheta_{i,n}(x)\,dxds\right|\le$$

$$\le\sum_{n=1}^{\infty}|\varphi_{i,n}|+C\cdot M_{1,i}\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\frac{1}{\lambda_{i,n}}\max_{t\in[t_m,t_{m+1}]}\left|\int_{-1}^{1} f_i(t,x)\,\vartheta_{i,n}(x)\,dx\right|\le$$

$$\le\sum_{n=1}^{\infty}|\varphi_{i,n}|+C\cdot M_{1,i}\sqrt{\sum_{n=1}^{\infty}\frac{1}{\lambda_{i,n}^2}}\max_{m=\overline{1,p}}\max_{t\in[t_m,t_{m+1}]}\sqrt{\sum_{n=1}^{\infty}\left[\int_{-1}^{1} f_i(t,x)\,\vartheta_{i,n}(x)\,dx\right]^2}\le$$

$$\le\sum_{n=1}^{\infty}|\varphi_{i,n}|+C\cdot M_{1,i}M_{2,i}M_{3,i}=\delta_{1,i}<\infty,\quad i=1,2,\tag{30}$$

where

$$M_{2,i}=\sqrt{\sum_{n=1}^{\infty}\frac{1}{\lambda_{i,n}^2}},\quad M_{3,i}=\max_{m=\overline{1,p}}\max_{t\in[t_m,t_{m+1}]}\|f_i(t,x)\|_{L_2[-1,1]},\ \ 0<C=\text{const},$$

$$\lambda_{1,n}=(1+\varepsilon)\,\pi^2(n+0.5)^2,\ \ \lambda_{2,n}=(1-\varepsilon)\,\pi^2 n^2,\ \ |\varepsilon|<1.$$

Taking into account estimate (30), applying the Cauchy–Schwartz inequality and the Bessel inequality, for the first difference of approximations (29), we obtain:

$$\left\|\vec u_i^{\,1}(t)-\vec u_i^{\,0}(t)\right\|_{PC_{B_2}[0,T]}\le\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\max_{t\in[t_m,t_{m+1}]}\left|u_{i,n}^{1}(t)-u_{i,n}^{0}(t)\right|\le$$

$$\le C\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\sup_{t\in(t_m,t_{m+1}]}\left|\sum_{0<t_m<t}^{p}e^{-\lambda_{i,n}(t-t_m)}\int_{-1}^{1}I_{m,i}\left(x,\int_{-1}^{1}G(y)\sum_{j=1}^{\infty}u_{i,j}^0(t_m)\vartheta_{i,j}(y)dy\right)\vartheta_{i,n}(x)dx\right|+$$

$$+C\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\sup_{t\in(t_m,t_{m+1}]}\left|\int_0^t e^{-\lambda_{i,n}(t-s)}a(s)u_{i,n}^0(s)ds\right|\le$$

$$\le C\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\frac{1}{\lambda_{i,n}}\sup_{t\in(t_m,t_{m+1}]}\sum_{m=1}^{p}\frac{1}{t-t_m}\left|\int_{-1}^{1}I_{m,i}\left(x,\int_{-1}^{1}G(y)\sum_{j=1}^{\infty}u_{i,j}^0(t)\,\vartheta_{i,j}(y)dy\right)\vartheta_{i,n}(x)\,dx\right|+$$

$$+C\cdot M_{1,i}\max_{m=\overline{1,p}}\max_{t\in[t_m,t_{m+1}]}|a(t)|\sum_{n=1}^{\infty}\frac{1}{\lambda_{i,n}}\max_{t\in[t_m,t_{m+1}]}\left|u_{i,n}^0(t)\right|\le$$

$$\le C\cdot M_4\sqrt{\sum_{n=1}^{\infty}\frac{1}{\lambda_{i,n}^2}}\max_{m=\overline{1,p}}\max_{t\in[t_m,t_{m+1}]}\sqrt{\sum_{n=1}^{\infty}\left[\int_{-1}^{1}I_{m,i}\left(x,\int_{-1}^{1}G(y)\sum_{j=1}^{\infty}u_{i,j}^0(t)\vartheta_{i,j}(y)dy\right)\vartheta_{i,n}(x)\,dx\right]^2}+$$

$$+C\cdot M_{1,i}M_5\sqrt{\sum_{n=1}^{\infty}\frac{1}{\lambda_{i,n}^2}}\,\left\|\vec u_i^{\,0}(t)\right\|_{PC_{B_2}[0,T]}\le$$

$$\le C\cdot M_4M_{2,i}\max_{m=\overline{1,p}}\left\|I_{m,i}\left(x,\int_{-1}^{1}G(y)\sum_{j=1}^{\infty}\delta_{1,i}\,\vartheta_{i,j}(y)dy\right)\right\|_{L_2[-1,1]}+C\cdot M_{1,i}M_{2,i}M_5\delta_{1,i}\le$$

$$\le C\cdot M_{2,i}\left[M_4M_{0,i}+M_{1,i}M_5\delta_{1,i}\right]<\infty,\tag{31}$$

$$M_4=\sup_{t\in(t_m,t_{m+1}]}\sum_{m=1}^{p}\frac{1}{t-t_m},\quad M_5=\max_{m=\overline{1,p}}\max_{t\in[t_m,t_{m+1}]}|a(t)|.$$

Continuing this process, similarly to the estimate (31), we obtain

$$\left\|\vec u_i^{\,\tau+1}(t)-\vec u_i^{\,\tau}(t)\right\|_{PC_{B_2}[0,T]}\le\max_{m=\overline{1,p}}\sum_{n=1}^{\infty}\max_{t\in[t_m,t_{m+1}]}\left|u_{i,n}^{\tau}(t)-u_{i,n}^{\tau-1}(t)\right|\le$$

$$\le C\max_{m=\overline{1,p}}\left|\sum_{n=1}^{\infty}\sum_{0<t_m<t}^{p}\sup_{t\in(t_m,t_{m+1}]}e^{-\lambda_{i,n}(t-t_m)}\int_{-1}^{1}N_{m,i}(x)\times\right.$$

$$\times \left| \int_{-1}^{1} G(y) \sum_{j=1}^{\infty} \left[u_{i,j}^{\tau}(t_m) - u_{i,j}^{\tau-1}(t_m) \right] \vartheta_{i,j}(y) dy \right| \vartheta_{i,n}(x)\, dx \Bigg| +$$

$$+ C \max_{m=\overline{1,p}} \sum_{n=1}^{\infty} \sup_{t\in(t_m,t_{m+1}]} \left| \int_0^t e^{-\lambda_{i,n}(t-s)} a(s) \left| u_{i,j}^{\tau}(t) - u_{i,j}^{\tau-1}(t) \right| ds \right| \leq$$

$$\leq C \cdot M_4 \max_{m=\overline{1,p}} \sum_{n=1}^{\infty} \frac{1}{\lambda_{i,n}} \int_{-1}^{1} N_{m,i}(x) \vartheta_{i,n}(x) dx \sum_{j=1}^{\infty} \max_{t\in[t_m,t_{m+1}]} \left| u_{i,j}^{\tau}(t) - u_{i,j}^{\tau-1}(t) \right| \left| \int_{-1}^{1} G(y) \vartheta_{i,j}(y) dy \right| +$$

$$+ C \cdot M_{1,i} \max_{m=\overline{1,p}} \max_{t\in[t_m,t_{m+1}]} |\, a(t)\, | \sum_{n=1}^{\infty} \frac{1}{\lambda_{i,n}} \max_{t\in[t_m,t_{m+1}]} \left| u_{i,j}^{\tau}(t) - u_{i,j}^{\tau-1}(t) \right| \leq$$

$$\leq C \cdot M_4 M_{2,i} \| N_i(x) \|_{L_2[-1,1]} \left\| \vec{u}_i^{\,\tau}(t) - \vec{u}_i^{\,\tau-1}(t) \right\|_{PC_{B_2}[0,T]} \| G(x) \|_{L_2[-1,1]} +$$

$$+ C \cdot M_{1,i} M_5 M_{2,i} \left\| \vec{u}_i^{\,\tau}(t) - \vec{u}_i^{\,\tau-1}(t) \right\|_{PC_{B_2}[0,T]} \leq \rho_i \left\| u_i^{\tau}(t) - u_i^{\tau-1}(t) \right\|_{PC_{B_2}[0,T]}, \tag{32}$$

where

$$\rho_i = C \cdot M_{2,i} \big[M_4 \| N_i(x) \|_{L_2[-1,1]} \| G(x) \|_{L_2[-1,1]} + M_{1,i} M_5 \big],$$

$$\| N_i(x) \|_{L_2[-1,1]} = \max_{m=1,\dots,p} \| N_{m,i}(x) \|_{L_2[-1,1]}, \quad i = 1, 2.$$

According to (28), $\rho_i < 1$. Consequently, it follows from estimate (32) that the operator on the right-hand side of countable system of nonlinear functional equations (26) is contracting. It follows from estimates (30)–(32) that there is the unique fixed point, which is a solution to the countable system of functional equations (26) in space $PC_{B_2}[0, T]$. Theorem 1 is proved. □

4. Uniform convergence of the Fourier series

Theorem 2. Let the conditions of the Theorem 1 be fulfilled. Then the unknown function $U(t, x) = U_1(t, x) + U_2(t, x)$ of the mixed impulsive problem (1)–(4) is determined by the Fourier series (27). The series (27) are convergent on the domain Ω.

Proof. Let $\vec{u}_i(t) \in PC_{B_2}[0, T]$ be the unique solution of the countable system (26). As in the case of estimates (30) and (31), we obtain

$$|\, U_i(t, x)\, | \leq \sum_{n=1}^{\infty} |\, \vartheta_{i,n}(x)\, | \left[\sum_{n=1}^{\infty} |\, \varphi_{i,n}\, | \, e^{-\lambda_{i,n} t} + \right.$$

$$+ \sum_{n=1}^{\infty} \left| \int_0^t e^{-\lambda_{i,n}(t-s)} \int_{-1}^{1} f_i(s, x)\, \vartheta_{i,n}(x)\, dx ds \right| +$$

$$+ \sum_{n=1}^{\infty} \max_{t\in[t_m,t_{m+1}]} \left| \sum_{0<t_m<t}^{p} e^{-\lambda_{i,n}(t-t_m)} \int_{-1}^{1} I_{m,i} \left(x, \int_{-1}^{1} G(y) \sum_{j=1}^{\infty} u_{i,j}(t_m)\, \vartheta_{i,j}(y) dy \right) \vartheta_{i,n}(x)\, dx \right| +$$

$$\left. + C \sum_{n=1}^{\infty} \sup_{t\in(t_m,t_{m+1}]} \left| \int_0^t e^{-\lambda_{i,n}(t-s)} a(s)\, u_{i,n}(s)\, ds \right| \right] \leq$$

$$\leq \sum_{n=1}^{\infty} |\, \varphi_{i,n}\, | + M_{1,i} \sqrt{\sum_{n=1}^{\infty} \frac{1}{\lambda_{i,n}^2} \max_{m=\overline{1,p}} \max_{t\in[t_m,t_{m+1}]} \| f_i(t, x) \|_{L_2[-1,1]}} +$$

$$+ M_4 \sqrt{\sum_{n=1}^{\infty} \frac{1}{\lambda_{i,n}^2} \max_{m=\overline{1,p}} \max_{t\in[t_m,t_{m+1}]}} \left\| I_{m,i} \left(x, \int_{-1}^{1} G(y) \sum_{j=1}^{\infty} u_{i,j}(t)\, \vartheta_{i,j}(y) dy \right) \right\|_{L_2[-1,1]} +$$

$$+ C \cdot M_{1,i} M_5 \sqrt{\sum_{n=1}^{\infty} \frac{1}{\lambda_{i,n}^2}} \| \vec{u}_i(t) \|_{PC_{B_2}[0,T]} < \infty. \tag{33}$$

Due to the estimate (33) one obtains the absolute and uniform convergence of the series (27). Theorem 2 is proved. □

5. Conclusion

Neutron diffusion plays a significant role in the operation of nuclear reactors. The diffusion equation makes it possible to calculate the neutron density inside the core of a nuclear reactor, the neutron flux from the moderator surface, and the reflection and transmission of neutrons by biological protection structures.

In the domain $\Omega = \{t \in (0,T),\ t \neq t_m,\ 0 < t_m < T,\ m = 1, 2, ..., p,\ \ x \in (-1, 1)\}$, we consider a parabolic type linear differential equation (1) of neutron diffusion in the presence of neutron absorption by the atomic nucleus with nonlinear impulsive effects and involution. The Dirichlet boundary value conditions, initial value condition and nonlinear impulsive conditions are used in solving the mixed problem.

The countable system of nonlinear functional equations (26) is obtained. Theorem 1 on unique solvability of countable system of nonlinear functional integral equations (26) is proved. The Picar iteration process is constructed. The generalized solution of the mixed problem (1)–(4) is obtained in the form of the Fourier series (27). The uniform convergence of the Fourier series (27) is proved (Theorem 2). Note that the Fourier series (27) characterizes the neutron density function with the first kind discontinuities. The results of this work will make it possible to determine the neutron density inside the core of a nuclear reactor and its change in the presence of a moderator and reflectors.

Moreover, the results obtained in this work will allow us to investigate direct and inverse problems for other kinds of partial differential equations of mathematical physics with impulsive actions. Differential equations with impulsive effects often allow one to reveal common features of phenomena in different branches of science. In [29], impulsive differential equations are used for solving boundary value problems on time scales. In [30], impulsive differential equations are used in studying pulse mass measles vaccination across age cohorts. In [38], impulsive differential equations are used in studying biological problems. Parabolic type differential equations as the heat equations or as the diffusion equations have different applications. In [14], the problem of fast forward evolution of the processes described in terms of the heat equation is considered. The matter is considered on an adiabatically expanding time-dependent box. Attention is paid to acceleration of heat transfer processes. As the physical implementation, the heat transport in harmonic crystals is considered.

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