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Oscillation results for second-order delay differential equation with several deviating arguments

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ABSTRACT The oscillatory behaviour of all solutions to the second-order delay differential equation with several deviating arguments and non negative coefficients is studied. Some sufficient oscillation conditions are obtained. An example is also given to illustrate the significance of our main results.

KEYWORDS second-order delay differential equation; non-monotone arguments; oscillatory solution.

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1. Introduction

Delay differential equations (DDEs) play an essential role in the mathematical modeling of dynamical systems where aftereffects, memory, or delayed responses are inherent. In nanosystems, delay effects frequently appear in nanoelectromechanical systems (NEMS), where the coupling between electronic circuits and mechanical oscillators introduces time-delayed feedback that significantly influences system stability and oscillatory behavior. For instance, time-delayed feedback control has been shown to govern the dynamics of nonlinear nanomechanical resonators and can induce or suppress oscillations depending on system parameters. Moreover, nanoscale heat transport and thermodynamic processes often exhibit memory effects that are effectively described using delay-differential formulations, especially in materials with internal relaxation and thermal lag.

Delay differential models also arise in nanoscale chemical kinetics and nanoparticle synthesis processes, where reaction rates and precipitation mechanisms depend on delayed interactions and diffusion effects. Such delay-driven models have been successfully applied to describe precipitation reactions, industrial chemical synthesis, and nanoscale reaction-diffusion systems. A comprehensive overview of the relevance of delay differential equations in nanoscale systems and their applications in physics and engineering can be found in recent interdisciplinary studies linking delay dynamics with nanoscience, see for example [1–5] and the references therein.

Motivated by both theoretical significance and practical applications in nanoscale systems, this paper investigates the oscillatory behavior of solutions of the second-order delay differential equation with several deviating arguments of the form

$$y''(x) + \sum_{i=1}^{m} p_i(x) y(u_i(x)) = 0, \qquad for \quad x \geq x_0 > 0, \tag{1}$$

where, p_i, $i = 1, 2, 3, ..., m$, are functions of non negative real numbers and u_i, $1 \leq i \leq m$ are non-monotone functions of positive real numbers such that

$$u_i(x) < x,\, x \geq 0 \quad and \quad \lim_{x\to\infty} u_i(x) = \infty,\, 1 \leq i \leq m. \tag{2}$$

and

$$\begin{gathered} v_i(x) = \sup_{s \leq x} u_i(s), \quad v(x) = \max_{1\leq i\leq m} v_i(x), \quad x \geq x_0, \\ u(x) = \max_{1\leq i\leq m} u_i(x) \quad with \quad u_i(x) \leq v_i(x). \end{gathered} \tag{3}$$

A continuously differentiable function defined on $[x_0, \infty)$, is called a solution of (1), if it satisfies (1) for almost all $x > x_0$. Any solution of (1) belongs oscillate class, if it has infinitely many zeros or large number of zeros, otherwise it is non oscillatory. Our equation (1) is oscillatory if all of its solutions oscillate, otherwise it is non oscillatory.

The first systematic study for the oscillation of all solutions of the first order delay differential equation,

$$y'(x) + p(x)y(u(x)) = 0, \quad x > x_0 > 0, \tag{4}$$

was made by Myshkis [6]. Later, several authors have studied (4) the situation when $u(x)$ is non-increasing and not necessarily monotone see, for example [7–10]. Some authors obtained oscillation results for (4) with constant delay $\beta_i > 0$, see [11]. A few authors have also investigated (4) with variable delays of the form $(x - \alpha_i(x))$, see [12]. In recent years, Gyori and Ladas studies (4) under the assumptions that $\alpha_i(x)$ are non-decreasing functions, for details, see [13]. Later, many authors considered the equation

$$y'(x) + \sum_{i=1}^{m} p_i(x)y(u_i(x)) = 0, \tag{5}$$

and investigated oscillation properties for non-decreasing and non-monotone arguments $u_i(x)$, for example, see [14–22]. Some authors have discussed the special case of (1) when $m = 1$, see [23] for more details.

In the present paper, we are interested in obtaining oscillation conditions for (5) instead of the first-order equation we take the second-order equation of the form (1).

2. Main results

The proof of our main results are essentially based on the following Lemmas.

Gronwall Inequality: If $y'(x) + p(x)y(x) \le 0$, $x \ge x_0$ where $p(x) > 0$ and $y(x) \ge 0$, then one has

$$y(x) \le y(t) \exp\left\{ \int_x^t p(s)ds \right\}, \; x \ge t \ge x_0. \tag{6}$$

Lemma 1. *Suppose that $\Delta > 0$ and (3) holds. Then*

$$\liminf_{x\to\infty} \int_{v(x)}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)ds_1ds = \liminf_{x\to\infty} \int_{u(x)}^{x} \sum_{j=1}^{m} \int_{u_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)ds_1ds = \Delta. \tag{7}$$

Proof. Use Lemma 2.2.1 in [24], to complete the proof. □

Lemma 2. *Let (3) hold and $y(x)$ be an eventually positive solution of (1).*
If $\Delta > \frac{1}{e}$ then

$$\liminf_{x\to\infty} \frac{y(v(x))}{y(x)} = \infty,$$

If $\Delta \le \frac{1}{e}$ then

$$\liminf_{x\to\infty} \frac{y(v(x))}{y(x)} \ge \Theta, \tag{8}$$

where Θ is the least root of

$$\Theta = e^{\Delta\Theta}. \tag{9}$$

Proof. Use Lemma 2.2.2 in [24], to complete the proof. □

Lemma 3. *Assume that $y(x)$ is a positive solution of (1). Denote,*

$$\varphi_{r+1}(x,t) := \exp\left\{ \int_t^x \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)\varphi_r(s_1, u_i(s_1))ds_1ds \right\}, \quad r \in \mathbb{N}. \tag{10}$$

with $\varphi_0(x,t) = 1$. Then

$$y(x)\varphi_r(x,t) \le y(t), \quad 0 \le t \le x. \tag{11}$$

Proof. The proof is similar to that of Lemma 1, in [15]. □

Theorem 1. *Assume that $p_i(x) \ge 0$, $1 \le i \le m$ and (2), (3) are valid. If*

$$\liminf_{x\to\infty} \int_{v(x)}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)ds_1ds > 1, \tag{12}$$

then all solutions of (1) oscillate.

Proof. Assume that $y(x)$ be an eventually positive solution of (1). From (1), $y''(x) \leq 0$ and then $y'(x) > 0$, if suppose, $y' < 0$, then for some $l \in \mathbb{R}^+$ we see $y'(x) = -l$ and integrating this over $[x_1, x]$, we obtain

$$y(x) = (y(v(x_1)) - lx_1) - x.$$

Hence, $y(x) \to -\infty$ as $x \to \infty$ but this makes no sense with $y(x) > 0$, as was assumed. So $y'(x) > 0$ which means $y(x)$ is non-decreasing. Integrating (1) over $[u_j(x), x]$, one gets

$$y'(x) - y'(v(x)) \leq y'(x) \leq -y(v(x)) \int_{v_j(x)}^{x} \sum_{i=1}^{m} p_i(s_1) ds_1 \leq -\int_{v_j(x)}^{x} \sum_{i=1}^{m} p_i(s_1) y(u_i(s_1)) ds_1 ds. \tag{13}$$

From the above inequality, one takes

$$\sum_{j=1}^{m} \int_{v_j(x)}^{x} \sum_{i=1}^{m} p_i(s_1) y(u_i(s_1)) ds_1 \leq y'(v(x)).$$

Again integrating over $[v(x), x]$,

$$\left\{ \int_{v(x)}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1) ds_1 ds \right\} y(v(x)) \leq y(v(x)) - y(v(v(x))) \tag{14}$$

For $y(x) > 0$, finally, we reach

$$\liminf_{x \to \infty} \int_{v(x)}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1) ds_1 ds \leq 1,$$

a contradiction to (10), and hence the proof is completed. □

Theorem 2. *Assume that* $p_i(x) \geq 0$, $1 \leq i \leq m$ *and* (2), (3) *are valid. If*

$$\liminf_{x \to \infty} \int_{v(x)}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1) ds_1 ds > \frac{1}{e} \tag{15}$$

then all solutions of (1) *oscillate.*

Proof. Assume that $y(x)$ be an eventually positive solution of (1). Divide (13) by $y(x)$ and integrate over $[v(x), x]$, then by taking simple steps, one obtains

$$\int_{v(x)}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1) ds_1 ds \leq \ln \left[\frac{y(v(x))}{y(x)} \right]. \tag{16}$$

In view of (15), there exists a constant $c > 0$ such that

$$\int_{v(x)}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1) ds_1 ds \geq c > \frac{1}{e}.$$

Using the above inequality in (16) and then taking exponential, one comes to the following inequality

$$y(v(x))/y(x) \geq e^c > ec > 1.$$

Repeating the above argument for large x, one obtains

$$y(v(x)) \geq (ec)^2 y(x).$$

Continuing these steps, we come to the issue that there exists a number $a \in \mathbb{N}$, satisfying $a > 2(\ln 2 - lnc)/(1 + \ln c)$, such that for sufficiently large x, we obtain

$$\frac{y(v(x))}{y(x)} > (ec)^a > \frac{4}{c^2}. \tag{17}$$

There exists a number $x_* \in (v(x), x)$ which satisfies the condition $y(v(x_*)) \leq v(x_*) \leq y(x)$. Consider this point as the splitting point of the integral in (15) into two equal parts as follows,

$$\int_{v(x)}^{x_*} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1) ds_1 ds \geq \frac{c}{2}; \qquad \int_{x_*}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1) ds_1 ds \geq \frac{c}{2}. \tag{18}$$

Integrating (13) over $[v(x), x_*]$ then using (18) with simple steps, one shows that

$$y(c(x_*)) \times \frac{c}{2} \leq y(v(x)). \tag{19}$$

Similarly, integrating (13) over $[x_*, x]$, by the same calculations as above we reach

$$y(v(x)) \times \frac{c}{2} \leq y(x_*). \tag{20}$$

Combining (19) and (20), we see the contradiction

$$\frac{y(v(x_*))}{y(x_*)} \leq \frac{4}{c^2}$$

to the inequality (17), and hence the Theorem is proved. □

Theorem 3. *Assume that* $p_i(x) \geq 0$, $1 \leq i \leq m$, (2), (3) *and* (7) *hold. If, for some* $r \in \mathbb{N}$

$$\liminf_{x\to\infty} \int\limits_{v(x)}^{x} \sum_{j=1}^{m} \int\limits_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))ds_1 ds > \frac{1 - \ln\Theta}{\Theta}, \tag{21}$$

where Θ *and* φ_r *are defined by* (9) *and* (10)*, respectively. Under these conditions, all solutions of* (1) *oscillate.*

Proof. Assume that $y(x)$ be an eventually positive solution of (1). By Lemma 2, the inequality (8) is fulfilled. Therefore, for an arbitrary real number $0 < \epsilon < \Theta$ and for some $x^* \in (v(x), x)$, one gets for all $x > x_1 > x_0$

$$\frac{y(v(x))}{y(x)} > (\Theta - \epsilon); \qquad \frac{y(v(x))}{y(x^*)} = (\Theta - \epsilon). \tag{22}$$

Applying (11) to (1), one obtains

$$y''(x) + \sum_{i=1}^{m} p_i(x)\varphi_r(v(x), u_i(x))y(v(x)) = 0 \tag{23}$$

Integrating (23) over $[v_j(x), x]$, one gets

$$y'(x) - y'(v(x)) + \int\limits_{v_j(x)}^{x} \sum_{i=1}^{m} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))y(v(s_1))ds_1 \leq 0 \tag{24}$$

Since $y'(x) > 0$, this implies

$$-y'(x) \leq y'(x) - y'(v(x)) + \int\limits_{v_j(x)}^{x} \sum_{i=1}^{m} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))y(v(s_1))ds_1 \leq 0,$$

and, hence,

$$\int\limits_{v_j(x)}^{x} \sum_{i=1}^{m} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))y(v(s_1))ds_1 \leq y'(x) \tag{25}$$

Let us divide (25) by $y(x)$ and integrate over $[v(x), x^*]$ for $v(x) < x^* < x$. Then using (22), we get

$$\int\limits_{v(x)}^{x^*} \sum_{j=1}^{m} \int\limits_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))ds_1 ds \leq -\frac{\ln(\Theta - \epsilon)}{(\Theta - \epsilon)} \tag{26}$$

Let us integrate (25) over $[x^*, x]$ once again. Using (22), we can obtain

$$\int\limits_{x^*}^{x} \sum_{j=1}^{m} \int\limits_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))ds_1 ds \leq \frac{1}{(\Theta - \epsilon)} \tag{27}$$

Adding (26) to (27), we get

$$\int\limits_{v(x)}^{x} \sum_{j=1}^{m} \int\limits_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))ds_1 ds \leq \frac{1 - \ln(\Theta - \epsilon)}{(\Theta - \epsilon)}.$$

Then, for large x, letting $\epsilon \to 0$ and taking $liminf$, we get a contradiction with (21) and this completes the proof of the Theorem.

□

Corollary 1. *Assume that* $p_i \geq 0, 1 \leq i \leq m$, *and equations* (2), (3) *hold. If*

$$\limsup_{x\to\infty} \int_{v(x)}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))ds_1 ds > \frac{1}{\Theta}, \tag{28}$$

then all solutions of (1) *oscillate.*

Proof. Assume that $y(x)$ be an eventually positive solution of (1), so that by Lemma 2, the inequality (8) is fulfilled. Integrating (25) over $[v(x), x]$, we reach, for $y(x) > 0$,

$$\left[\int_{v(x)}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))ds_1 ds\right] y(v(x)) \leq y(x).$$

Then taking $limsup$ and using (8), we get

$$\limsup_{x\to\infty} \int_{v(x)}^{x} \sum_{j=1}^{m} \int_{v_j(s)}^{s} \sum_{i=1}^{m} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))ds_1 ds \leq \frac{1}{\Theta},$$

which contradicts with (28). Hence, the Corollary is proved. □

3. Example

Example 4. *Consider the delay differential equation*

$$y''(x) + \frac{13}{100}y(u_1(x)) + \frac{21}{100}y(u_2(x)) = 0, \tag{29}$$

where

$$u_1(x) = \begin{cases} -x + 4n - 3.5, & \textit{if} \quad x \in [2n+1, 2n+2], \\ 3x - 4n - 11.5, & \textit{if} \quad x \in [2n+2, 2n+3] \end{cases} \quad \textit{and} \quad u_2(x) = u_1(x) - 1, \quad n \in \mathbb{N}.$$

Proof. From the equation and by (3)

$$v_1(x) = \begin{cases} 2n - 4.5, & \text{if} \quad x \in [2n+1, 2n+2.33], \\ 3x - 4n - 11.5, & \text{if} \quad x \in [2n+2.33, 2n+3] \end{cases} \quad \textit{and} \quad v_2(x) = v_1(x) - 1 \quad n \in \mathbb{N}.$$

Here, $v(x)$ attains its maximum at $x = 2n+3$, so $v(2n+3) = 2n - 2.5$ and by simple calculations, we get $\Delta = 19.635$. Hence $\Delta > 1$ so that the conditions (12) and (15) are satisfied, (29) is oscillatory and oscillation of solution of (29) is plotted in Fig.1(B). □

Example 5. *Consider the equation*

$$y''(x) + \frac{1}{2.2e}y(u_1(x)) + \frac{1}{2.4e}y(u_2(x)) = 0, \tag{30}$$

where

$$u_1(x) = \begin{cases} -2x + 6n + 2, & \textit{if} \quad x \in [2n+1, 2n+2], \\ 4x - 6n - 10, & \textit{if} \quad x \in [2n+2, 2n+3] \end{cases} \quad \textit{and} \quad u_2(x) = u_1(x) - 1, \quad n \in \mathbb{N}.$$

Proof. From the equation and by (3), it follows that

$$v_1(x) = \begin{cases} 2n, & \text{if} \quad x \in [2n+1, 2n+2.5], \\ 4x - 6n - 10, & \text{if} \quad x \in [2n+2.5, 2n+3] \end{cases} \quad \textit{and} \quad v_2(x) = v_1(x) - 1, n \in \mathbb{N}.$$

From (3), $\Delta = 0.3205$. Clearly, $\Delta \leq \frac{1}{e} = (0.3678)$. By (9), $\Theta \approx 1.755$. Define $f_r(x) : [1, \infty) \to (0, \infty)$ as

$$f_r(x) = \int_{v(x)}^{x} \sum_{j=1}^{2} \int_{v_j(s)}^{s} \sum_{i=1}^{2} p_i(s_1)\varphi_r(v(s_1), u_i(s_1))ds_1 ds.$$

Thus, for $r = 1$ and $v(2n+3) = 2n+2$, we have $f_1(x) = 26.0111$. Hence, conditions (21) and (28) are satisfied, hence, (30) is oscillatory. □

Example 6. *Consider the delay differential equation*

$$y''(x) + \frac{5}{x^3}y(u_1(x)) + \frac{7}{x^3}y(u_2(x)) = 0, \tag{31}$$

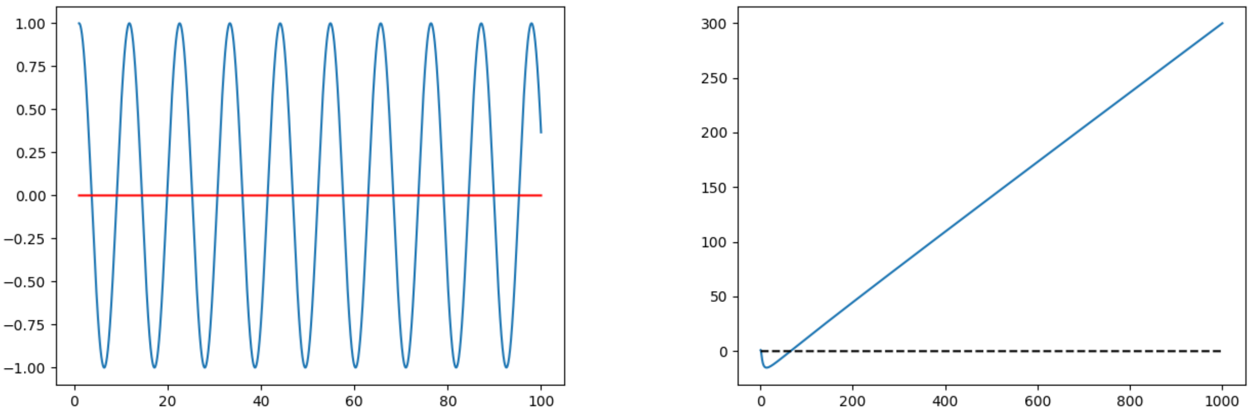
where

$$u_1(x) = \begin{cases} -x + 4n + 3, & \textit{if} \quad x \in [2n+1, 2n+2], \\ 3x - 4n - 5, & \textit{if} \quad x \in [2n+2, 2n+3] \end{cases} \quad \textit{and} \quad u_2(x) = u_1(x) - 1, n \in \mathbb{N}.$$

Proof. From the equation and by (3),

$$v_1(x) = \begin{cases} 2n + 2, & \text{if} \quad x \in [2n+1, 2n+2.35], \\ 3x - 4n - 5, & \text{if} \quad x \in [2n+2.35, 2n+3] \end{cases} \quad \textit{and} \quad v_2(x) = v_1(x) - 1, n \in \mathbb{N}.$$

Here $v(x)$ attains its maximum at $x = 2n+3$, so $v(2n+3) = 2n+4$. By simple calculation, we show that $\Delta < 0$ and so that the conditions (12) and (15) are not satisfied. Also we cannot apply conditions (21) and (28) for (31), which implies that it is non oscillatory, also non oscillation of solution of (31) is plotted in Fig.1(B). $\square$



Oscillation of solution of (29) Nonoscillation of solution of (31)

FIG. 1. Graphical representations

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