

Effect of spin-orbit interactions on an off-center D^0 impurity in an asymmetric quantum dot under a magnetic field

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ABSTRACT We evaluate the ground state energy of an off-centre Coulomb impurity in an asymmetric Gaussian GaAs quantum dot under a constant magnetic field. Using a unitary transformation, we take into account the combined effect of the Rashba and Dresselhaus interactions treating the coupling terms up to quadratic order. We next treat the transformed Hamiltonian using a variational approach with a suitable ansatz. The findings reveal that the ground state energy increases with growing impurity distance from the confinement centre and with increasing potential asymmetry. Finally, we examine how the asymmetry in the Gaussian confinement influences the magnetic moment and susceptibility of the impurity.

KEYWORDS Asymmetric GaAs quantum dot, off-centre impurity energy, spin-orbit interactions, magnetic field and susceptibility

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1. Introduction

Quantum dot (QD) systems have attracted significant interest due to their applications in semiconductor technology and their ability to probe quantum phenomena at the nanoscale [1–10]. Their discrete energy spectrum and strong confinement lead to a range of novel physical properties, as demonstrated in numerous studies [11–29].

In realistic systems, the presence of impurities is unavoidable, making their influence on the QD properties particularly important. In this regard, two key aspects are of primary interest: the role of impurities on quantum dot characteristics and the modification of impurity states under quantum confinement. Bastard [30] first considered a Coulomb impurity in a quantum well and employed a variational approach to determine its energy spectrum. Following his work, various authors have investigated impurity-related effects in low-dimensional nanostructures. For example, Shaer *et al.* [31] have examined the influence of a Gaussian impurity on the electrons confined in a lateral QD.

Another important interaction in QD systems is the relativistic spin-dependent interaction or the spin-orbit (SO) interaction (SOI) which forms a fundamental driving factor in the development of spintronic research. The key idea in spintronic devices is to utilize the electron spin, rather than its charge, as the primary carrier of information [32–35]. In solid-state systems, SO coupling generally arises from two primary mechanisms. The first is the Rashba spin-orbit interaction (RSOI) [36], which originates from structural inversion asymmetry, while the second is the Dresselhaus spin-orbit interaction (DSOI) [37], resulting from bulk inversion asymmetry. The degree of coupling of the DSOI is essentially determined by the intrinsic properties of the QD, whereas the RSOI can be tuned by both structural parameters and external fields [38]. This tunability of RSOI has been widely exploited in spintronic applications [39, 40].

In recent years, many authors have analysed how SOIs shape critical material characteristics and key attributes of quantum-confined geometries. Kumar *et al.* [41] have examined the role of RSOI on the energetics and magnetic properties of a many-electron harmonic QD using the Johnson-Payne model potential [42] for the electron-electron interaction. Elsaid and collaborators [43] have examined the thermodynamic and magnetic features of a parabolic GaAs QD under electric and magnetic fields factoring in the impact of impurities and the Rashba coupling. Li *et al.* [44] have investigated the influence of RSOI on the electronic states of a D^0 impurity in a GaAs/GaAlAs quantum well. Khordad [45] has examined the combined effects of electron–electron interaction, Rashba coupling, and an external magnetic field on the magnetic susceptibility of a parabolic QD at finite temperature. Gisi *et al.* [46] have analyzed the role of Rashba interaction and magnetic field on the optical absorption properties of a parabolic quantum wire. Saini *et al.* [47, 48] have reported

the binding energy of the D^0 and D^- impurities in a GaAs QD considering the Gaussian function as the confinement potential under the influence of RSOI and DSOI.

In most of the earlier investigations, the impurity has been assumed to be at the bottom of the confinement potential, and such an impurity is normally referred to as an on-centre impurity. Later, of course, quite a few investigations have also considered off-centre impurities [50–52]. However, a comprehensive analysis of both RSOI and DSOI on an off-center Coulomb impurity within an asymmetric GQD (AGQD) placed in a magnetic field has remained hitherto unexplored. Our work endeavours to explore this aspect. We will consider an off-centre hydrogenic impurity in a GaAs AGQD and calculate its ground state and binding energies, and the magnetic properties.

2. Theoretical model and Mathematical formalism

An off-centre D^0 impurity situated in an AGQD with RSOI and DSOI and placed under a magnetic field can be modelled by the following Hamiltonian:

$$\mathcal{H} = \left(\frac{1}{2m^*} \left(\mathbf{p} + \frac{e}{c} \mathbf{A} \right)^2 - \frac{e^2}{\varepsilon |\mathbf{r} - \mathbf{D}|} \right) I - IV_0 e^{-\left[\frac{(a(x^2+y^2)+b z^2)}{2R^2} \right]} + H_R + H_D \quad (1)$$

with

$$H_R = \frac{\alpha_R}{\hbar} \left[\boldsymbol{\sigma} \times \left(\mathbf{p} + \frac{e}{c} \mathbf{A} \right) \right]_Z, \quad (2)$$

$$H_D = \frac{\beta_D}{\hbar} \left[\sigma_x \left(p_x + \frac{e}{c} A_x \right) - \sigma_y \left(p_y + \frac{e}{c} A_y \right) \right]. \quad (3)$$

In Eq. (1), $e(m^*)$ refers to the charge (Bloch mass) of the electron, $\mathbf{r}(x, y, z)$ represents the position of the electron, $\mathbf{p}(p_x, p_y, p_z)$ its momentum, $\mathbf{D}$ is the position of the D^0 impurity, ε is the dielectric constant of the medium, $V_0(R)$ represents the depth (range) of the confinement potential, $\mathbf{A}$ denotes the vector potential defined by the equation: $\mathbf{B} = \nabla \times \mathbf{A}$, where $\mathbf{B}(0, 0, B)$ is the magnetic field applied in the z -direction, I is a unit matrix of order 2 and H_R and H_D describe respectively the Rashba and Dresselhaus interactions. In Eq. (2) and Eq. (3), $\sigma_x, \sigma_y, \sigma_z$ are the Pauli spin matrices, α_R and β_D denote the RSOI and DSOI constants. We use the symmetric gauge and choose: $\mathbf{A} = B(-y/2, x/2, 0)$.

We first deal with the SOI's. In order to incorporate the effect of SOI's, we apply on $\mathcal{H}$ a unitary transformation: $U = e^S$ with $S = i(m^*/\hbar^2) [\alpha_R(y\sigma_x - x\sigma_y) + \beta_D(x\sigma_x - y\sigma_y)]$. This transformation was first used by Aleiner and Falco [53] and later by Rodriguez *et al.* [54] and Kumar *et al.* [55]. This diagonalizes the Hamiltonian in the spin space to order α_R^2 and β_D^2 . In the transformed frame, the eigenstates are orbitals with well-defined spin and orbital angular momentum. In this method, terms higher than α_R^2 and β_D^2 are neglected and we get

$$\begin{aligned} \tilde{\mathcal{H}} = & \left(\frac{p^2}{2m^*} + \frac{1}{8} m^* \omega_c^2 \rho^2 - \frac{e^2}{\varepsilon |r - D|} \right) I - IV_0 e^{-\frac{a(x^2+y^2)+b z^2}{2R^2}} - \frac{m^*}{\hbar^2} (\alpha_R^2 + \beta_D^2) I - \\ & - \frac{m^*}{2\hbar^3} (\alpha_R^2 - \beta_D^2) [\omega_c \sigma_z \rho^2 + 2L_z] \sigma_z + \frac{\omega_c}{2} L_z, \end{aligned} \quad (4)$$

where $\omega_c = (eB/m^*c)$, $\rho^2 = (x^2 + y^2)$ and $L_z = -i\hbar(\partial/\partial\varphi)$. We are interested in a variational GS of $\tilde{\mathcal{H}}$ and so we adopt an appropriate variation ansatz: $\psi(\mathbf{r}) = [1 + \gamma e^{-\alpha(r-D)}] e^{-\beta r^2 - im\varphi}$, where γ, α and β are to be selected by the optimization process. The proposed variational ansatz has been designed to adequately describe the dominant physical characteristics of the GS of (2). Let $E(e^-)$ and $E(D^0)$ denote the lowest energy states of a single electron and a neutral donor complex, respectively, confined within the AGQD. We also calculate the magnetic moment (M) and susceptibility (S) of the D^0 complex, which are defined as: $M = -(\partial E/\partial B)$; $S = (\partial M/\partial B)$.

3. Quantitative findings and discussion

Although the formalism developed here is fairly broad in scope and applicable to any QD system, we specifically consider a GaAs QD dot for concreteness. All physical quantities will be expressed in SI units, with energy in meV and length in nm. For GaAs, the electrical permittivity is taken as $\varepsilon = 12.4$ and the electron band mass as $m^* = 0.06m_0$, where m_0 denotes the free electron mass. The magnetic field is given in Tesla (T), and the SOI parameters are given in meV·nm.

Figure 1 illustrates how the GS energy (E) of an off-centre donor D^0 within a GaAs AGQD evolves relative to the donor position (D), comparing three distinct pairs of asymmetry parameters a and b . As D enhances, E rises and gradually approaches a constant value at large D . This behaviour can be interpreted using a straightforward semiclassical framework where the effective potential function on minimization provides the system's lowest energy. The GS energy decreases with decreasing asymmetry along the z -direction. Furthermore, in cases when D^0 is separated farther from the confinement centre, the influence of asymmetry becomes increasingly significant.

Figure 2 demonstrates how E changes with V_0 in the case of a GaAs AGQD for different values of α_R and β_D and a certain value of D ($D = 10$ meV). The GS energy decreases with increasing V_0 , which is consistent with expectations. Additionally, both RSOI and DSOI lower the GS energy, with the RSOI contribution being more significant.

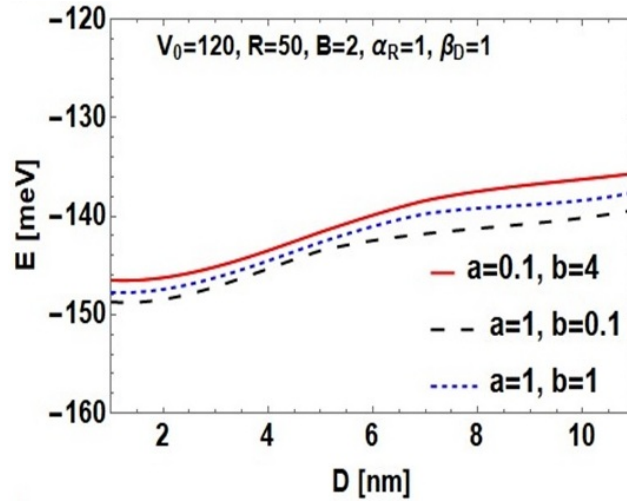


FIG. 1. Plot showing the variation of the GS energy E (in meV) of an off-centre D^0 impurity within a GaAs AGQD with respect to the distance of D^0 from the stable equilibrium potential minimum (D in nm) for three sets of asymmetry parameters

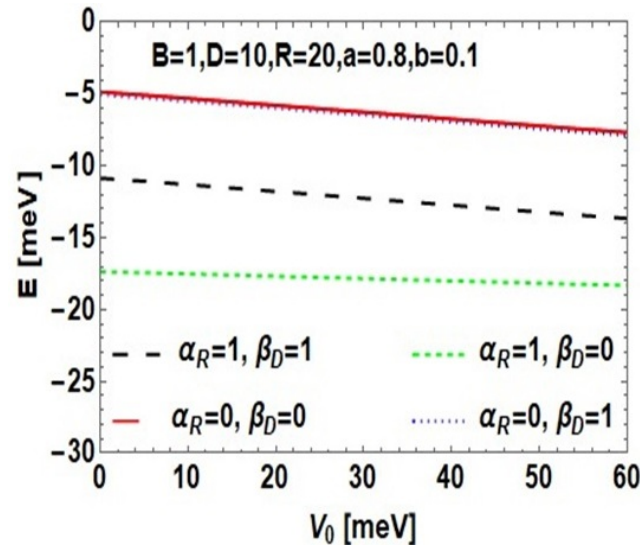


FIG. 2. Plot showing the variation of GS energy E with respect to V_0 for a D^0 impurity within GaAs AGQD with $D = 10$ nm and for four sets of SOI parameters

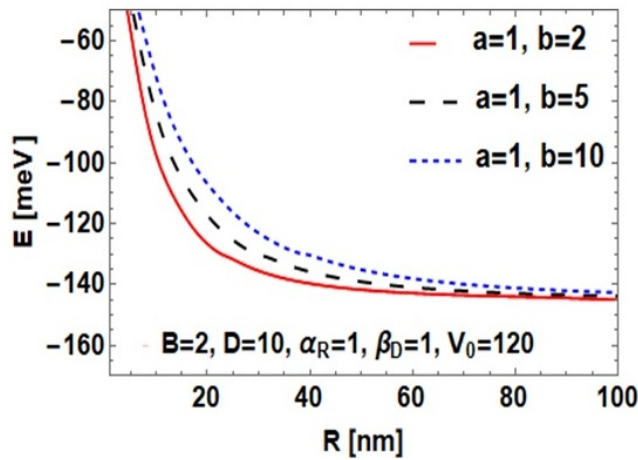


FIG. 3. Plot of E against R in the case of a D^0 impurity in a GaAs AGQD for a few sets of asymmetry parameters

Figure 3 illustrates the dependence of E on R in the case of a GaAs AGQD for a three sets of asymmetry parameters with the impurity position $D = 10$ meV. The values of other parameters are specified within the figure. The figure shows that for small QD sizes, the effect of the asymmetry parameter is not very significant. However, as the QD size increases, the GS energy is impacted by the asymmetry parameter in a more pronounced way. The graph, of course, also demonstrates the accepted behavior that the GS energy rises with increasing QD size.

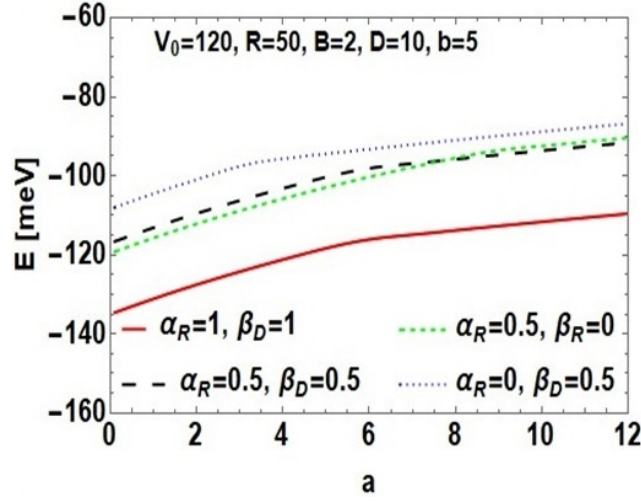


FIG. 4. Plot showing how E depends on the asymmetry parameter a in the case of an off-centre hydrogenic complex within a GaAs AGQD

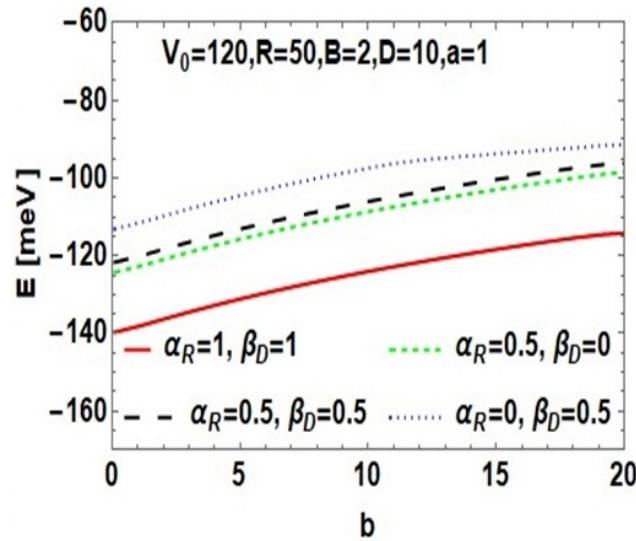


FIG. 5. Plot showing how E depends on the asymmetry parameter b in the case of an off-centre hydrogenic complex within a GaAs AGQD

Figure 4 and Fig. 5 describe how E depends on the asymmetric parameters a and b for four pairs of values of α_R and β_D with $V_0 = 120$ meV, $R = 50$ nm, $D = 10$ nm and $B = 2T$. The graphs indicate that the energy is a growing function of the asymmetry parameters a and b . Furthermore, the directional asymmetry along the Z exhibits a more pronounced impact in lowering the energy than the x-y asymmetry. The combined impact of RSOI and DSOI gives rise to a further reduction in the GS energy in both cases.

Figure 6 displays the trend of the GS energy against B for $V_0 = 300$ meV, $D = 4$ nm, $R = 10$ nm, $\alpha_R = 1$, $\beta_D = 1$ with different values of the asymmetry parameters. In all cases, a positive correlation is observed between E and B . Moreover, reduction in asymmetry along the z-direction results in lower energy.

Figure 7 exhibits how the nature of the moment (M) is governed by B for $V_0 = 300$ meV, $D = 4$ nm, $R = 10$ nm and $\alpha_R = 1$ and $\beta_D = 1$ and for three sets of asymmetry parameters. The behaviour distinctly indicates the diamagnetic nature of M . The figure also suggests that $|M|$ is a monotonically increasing function of B . It is also visible that a lower asymmetry along the z-direction results in an increased magnitude of magnetization. However, for small B values, the dependence of M on the QD potential asymmetry is rather feeble.

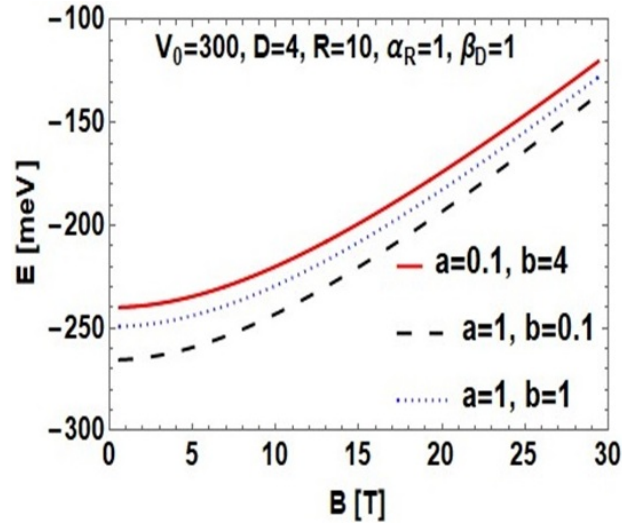


FIG. 6. Dependence of E on B in the case of an off-center hydrogenic complex in a GaAs AGQD for a few sets of asymmetry parameters

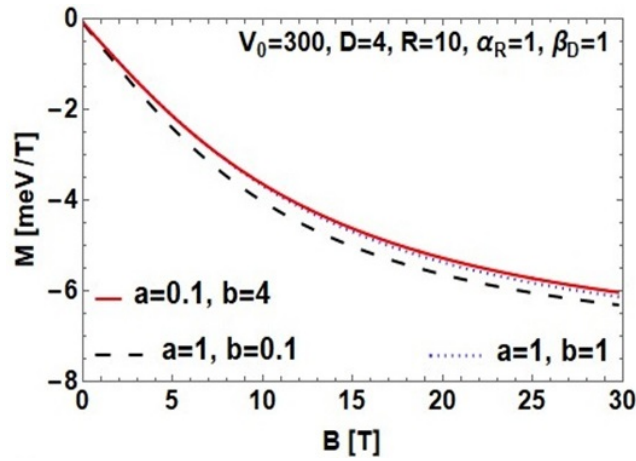


FIG. 7. Dependence of magnetic moment (M) of an off-center impurity in a GaAs AGQD on the applied field B for three sets asymmetry parameters

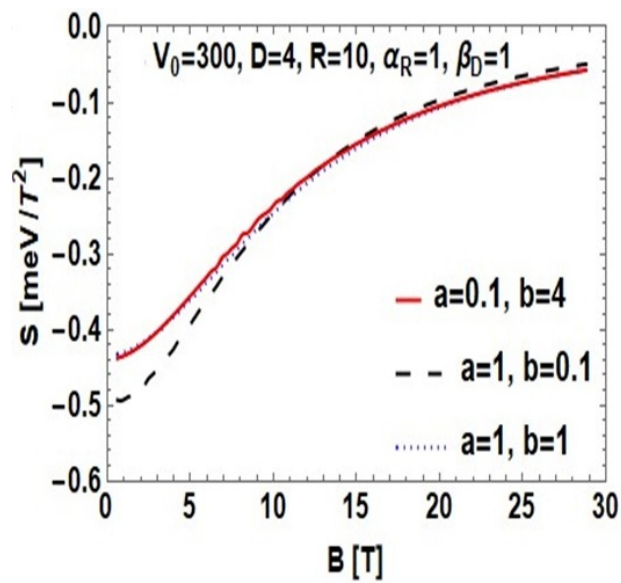


FIG. 8. Correlation between the Magnetic susceptibility (S) and B for an off-center system in a GaAs AGQD for three sets of asymmetry parameters

Figure 8 presents the correlation between the magnetic susceptibility (S) and the applied field B in the case of an off-centre D^0 system in a GaAs AGQD. The figure distinctly demonstrates the diamagnetic behaviour of the susceptibility S . In addition, the impact of the asymmetry parameters on S is significant at low small B , but becomes negligible as B increases. The susceptibility increases as a function of B for all asymmetry parameters and exhibits a gradual saturation at higher magnetic fields.

4. Conclusion

We have examined how the GS energy (E), Magnetization and the susceptibility (S) of an off-centre D^0 complex in a GaAs AGQD are influenced by the depth, range and the asymmetry parameters of the QD potential under a magnetic field. It is observed that E increases with the increase in the impurity distance D and the asymmetry parameters of the QD potential. It is furthermore found that deepening or widening the potential well results in a lower bound-state energy, the effect of the range being much more stronger. We finally demonstrate that the magnetic field enhances the energy and also the magnetization strength and susceptibility.

References

- [1] Reed M.A. Quantum Dots. *Sci. Am.*, 1993, **268**, P. 118.
- [2] Loss D., Divencenzo D.P. Quantum computation with quantum dots. *Phys. Rev. A*, 1998, **57**, P. 120.
- [3] Pershin Y.V., Nesteroff J.A., Privman V. Effect of spin-orbit interaction and in-plane magnetic field on the conductance of a quasi-one-dimensional system. *Phys. Rev. B*, 2004, **69**, P. 121306(R).
- [4] Vaseghi B., Khordad R., Golshan M.M. Dynamical properties of spin and subbands populations in 1D quantum wire. *Phys. Stat. Sol. (B)*, 2006, **243**, P. 2772.
- [5] Zhang S., Liang R., Zhang E., Zhang L., Liu Y. Magnetosubbands of semiconductor quantum wires with Rashba and Dresselhaus spin-orbit coupling. *Phys. Rev. B*, 2006, **73**, P. 155316.
- [6] Kasapoglu E., Ungan F., Sari H., Sokmen I. Binding energies of donor impurities in modulation-doped GaAs/Al_xGa_{1-x}As double quantum wells under an electric field. *Superlatt. Microstr.*, 2009, **45**, P. 618.
- [7] Khordad R., Khaneghah S.K. Intersubband optical absorption coefficients and refractive index changes in a V-groove quantum wire. *Phys. Stat. Sol. (B)*, 2011, **248**, P. 243.
- [8] Bandyopadhyay S. *Physics of Nanostructured Solid State Devices.*, Springer, Berlin, 2012.
- [9] Boda A., Chatterjee A. Transition energies and magnetic properties of a neutral donor complex in a Gaussian GaAs quantum dot. *Superlatt. Microstr.*, 2016, **97**, P. 268.
- [10] Saini P., Kalla M., Mukhopadhyay S., Chatterjee A., Popov I.Yu. Thermoelectric properties of a correlated polar single molecular transistor in the presence of a magnetic field and dissipation. *Physica E*, 2025, **168**, P. 116175.
- [11] Merkt U., Huser J., Wagner M. Energy spectra of two electrons in a harmonic quantum dot. *Phys. Rev. B*, 1991, **43**, P. 7320.
- [12] Halonen V., Chakraborty T., Pietilainen P., Kasapoglu E., Ungan F., Sari H., Sokmen I. Binding energies of donor impurities in modulation-doped GaAs/Al_xGa_{1-x}As double quantum wells under an electric field. *Phys. Rev. E*, 1992, **45**, P. 5980.
- [13] Marzin J.Y., Bastard G. Calculation of the energy levels in InAsGaAs quantum dots. *Solid State Commun.*, 1994, **92**, P. 437.
- [14] Oh J.H., Chang K.J., Ihm G., S.J. Lee S.J. Electronic structure of three-dimensional quantum dots in tilted magnetic fields. *Phys. Rev. B*, 1994, **50**, P. 15397.
- [15] Gu S.W., Guo K.X. Nonlinear optical rectification in the electric-field-biased parabolic quantum dots. *Solid State Commun.*, 1994, **89**, P. 1023.
- [16] Mukhopadhyay S., Chatterjee A. Path-Integral approach for electron-phonon interaction effects in harmonic quantum dots. *Int. J. Mod. Phys. B*, 1996, **10**, P. 2781.
- [17] Tan W.C., Inkson J.C. Magnetization, persistent currents, and their relation in quantum rings and dots. *Phys. Rev. B*, 1999, **60**, P. 5626.
- [18] Mukhopadhyay S., Chatterjee A. Polaronic effects in quantum dots. *Acta Phys. Pol. B*, 2001, **32**, P. 473.
- [19] Voskoboinikov O., Bauga O., Lee C.P., Tretyak O. Magnetic properties of parabolic quantum dots in the presence of the spin-orbit interaction. *J. Appl. Phys.*, 2003, **94**, P. 5891.
- [20] Ralko A., Truong T.T. Exact energy levels and magnetization of two identical planar charged particles in quantum dots. *Phys. Lett A*, 2004, **323**, P. 395.
- [21] Krishna P.M., Mukhopadhyay S., Chatterjee A., Polaronic effects in asymmetric quantum wire: an all-coupling variational approach. *Solid State Commun.*, 2006, **138**, P. 285.
- [22] Boyacioglu B., Saglam M., Chatterjee A. Two-electron singlet states in semiconductor quantum dots with Gaussian confinement: a single-parameter variational calculation. *J. Phys.: Condens. Matter*, 2007, **19**, P. 456217.
- [23] Boyacioglu B., Chatterjee A. Persistent current through a semiconductor quantum dot with Gaussian confinement. *Physica B*, 2012, **407**, P. 3535.
- [24] Boda A., Boyacioglu B., Chatterjee A. Ground state properties of a two-electron system in a three-dimensional GaAs quantum dot with Gaussian confinement in a magnetic field. *J. Appl. Phys.*, 2013, **114**, P. 044311.
- [25] Boda A., Madhusudhan G., Chatterjee A. Effect of external magnetic field on the ground state properties of D^- centres in a Gaussian quantum dot. *Superlattices and Microstructures*, 2014, **71**, P. 261.
- [26] Boda A., Chatterjee A. The binding energy and magnetic susceptibility of an off-centre D^0 donor in the presence of a magnetic field in a GaAs quantum dot with Gaussian confinement: An improved treatment. *J. Nanosci. Nanotech.*, 2015, **15**, P. 6472.
- [27] Boda A., Chatterjee A. Transition energies and magnetic properties of a neutral donor complex in a Gaussian GaAs quantum dot. *Superlattices and Microstructures*, 2016, **97**, P. 268.
- [28] Saini P., Boda A., Chatterjee A. Effect of Rashba and Dresselhaus spin-orbit interactions on a D^0 impurity in a two-dimensional Gaussian GaAs quantum dot in the presence of an external magnetic field. *J. Magn. Magn. Mater.*, 2019, **485**, P. 407.
- [29] Saini P., Boda A., Chatterjee A. Spin-orbit interaction effects on a D- complex in a GaAs quantum dot in a magnetic field. *Micro and Nanostructures*, 2023, **174**, P. 207487.
- [30] Bastard G. Hydrogenic impurity states in a quantum well: A simple model. *Phys. Rev. B*, 1981, **24**, P. 4714.
- [31] Shaer A., Elsaid M.K. Heat Capacity of Two Interacting Electrons in Coupled Double Quantum Dots. *Nanosystems: Phys Chem. Math.*, 2022, **13**(3), P. 265.

- [32] Datta S., Das B. Electronic analog of the electro-optic modulator. *Appl. Phys. Lett.*, 1990, **56**, P. 665.
- [33] Zutic I., Fabian J., Das Sarma S. Spintronics: Fundamentals and applications. *Rev. Mod. Phys.*, 2004, **76**, P. 223.
- [34] Liu J.F., Zhong Z.C., Chen L., Li D., Zhang C., Ma Z. Enhancement of polarization in a spin-orbit coupling quantum wire with a constriction. *Phys. Rev. B*, 2007, **76**, P. 195304.
- [35] Malet F., Pi M., Barranco M., Serra L., Lipparini E. Exchange-correlation effects on quantum wires with spin-orbit interactions under the influence of in-plane magnetic fields. *Phys. Rev. B*, 2007, **76**, P. 115306.
- [36] Rashba E.I. Spin-orbit coupling in condensed matter physics. *Sov. Phys. Solid State*, 1960, **2**, P. 1109.
- [37] Dresselhaus G. Spin-Orbit Coupling Effects in Zinc Blende Structures. *Phys. Rev.*, 1955, **100**, P. 580.
- [38] Winkler R. *Spin Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems*. Springer, Berlin, 2003.
- [39] Kurdak C., Biyikli N., Ozgur U., Morkoc H., Litvinov V.I. Weak antilocalization and zero-field electron spin splitting in Al_xGa_{1-x}N/AlN/ GaN heterostructures with a polarization-induced two dimensional electron gas. *Phys. Rev. B*, 2006, **74**, P. 113308.
- [40] Fabian J., Matos-Abiad A., Ertler C., Stano P., Zutic I. Semiconductor Spintronics. *Acta Phys. Slovaca*, 2007, **57**, P. 565.
- [41] Kumar D. S., Mukhopadhyay S., Chatterjee A. Magnetization and susceptibility of a parabolic InAs quantum dot with electron–electron and spin–orbit interactions in the presence of a magnetic field at finite temperature. *J. Mag. Mag. Mat.*, 2016, **418**, P. 169.
- [42] Johnson N.F., Payne M.C. Exactly solvable model of interacting particles in a quantum dot. *Phys. Rev. Lett.*, 1991, **67**, P. 1157.
- [43] Elsaid M.K., Alia A.A., Shaer A. Rashba spin-orbit interaction effects on thermal and magnetic properties of parabolic GaAs quantum dot in the presence of donor impurity under external electric and magnetic fields. *Chin. J. Phys.*, 2020, **66**, P. 335.
- [44] Li S.S., Xia J.B. Spin-orbit splitting of a hydrogenic donor impurity in GaAs/GaAlAs quantum wells. *Appl. Phys. Lett.*, 2008, **92**, P. 022102.
- [45] Khordad R. Simultaneous effects of electron-electron interactions, Rashba spin-orbit interaction and magnetic field on susceptibility of quantum dots. *J. Mag. Mag. Mat.*, 2018, **449**, P. 510.
- [46] Gisi B., Sakiroglu S., Kasapoglu E., Sari H., Sokmen I. Spin–orbit interaction effects on the optical properties of quantum wires under the influence of in-plane magnetic fields. *Superlatt. Microstr.*, 2015, **86**, P. 166.
- [47] Saini P., Chatterjee A. Confinement shape effect on D0 impurity in a GaAs quantum dot with spin-orbit coupling in a magnetic field. *Superlattices and Microstructures.*, 2020, **146**, P. 106641.
- [48] Saini P., Mukhopadhyay S., Chatterjee S. Spin-orbit interaction effects on binding energy and susceptibility of an off-centre D^0 impurity in a Gaussian dot in magnetic field. *Physica B*, 2021, **617**, P. 413099.
- [49] Mukhopadhyay S., Vidyullatha Ch., Saini P., Malik Z. Effect of electron-phonon interaction on the first excited energy level of a Gaussian GaAs quantum dot. *Nanosystems: Physics, Chemistry, Mathematics*, 2025, **16**(3), P. 306–310.
- [50] Yakar Y., Cakir B., Ozmen A. Off-center hydrogenic impurity in spherical quantum dot with parabolic potential. *Superlattices. Microstr.*, 2013, **60**, P. 389.
- [51] Mikhaila I.F.I., Ismail I.M.M., El Shafee M.M. Rashba effect in a quantum dot subject to a parabolic confining potential and external fields. *Eur. Phys. J. plus.*, 2022, **137**, P. 610.
- [52] Saini P., Mukhopadhyay S., Chatterjee A. Spin-orbit interaction effect on a hydrogenic D0 centre in a three-dimensional asymmetric Gaussian GaAs quantum dot in a magnetic field. *Physica B Condensed Matter*, 2023, **665**, P. 415033.
- [53] Aleiner I.L., Fal'ko V.I. Spin-orbit coupling effects on quantum transport in lateral semiconductor dots. *Phys. Rev. Lett.*, 2001, **87**, P. 256801.
- [54] Rodriguez M.V., Puente A., Serra L. Spin switching in semiconductor quantum dots through spin-orbit coupling. *Phys. Rev. B*, 2002, **66**, P. 165302.
- [55] Kumar D.S., Mukhopadhyay S., Chatterjee A. Effect of Rashba interaction and Coulomb correlation on the ground state energy of a GaAs quantum dot with parabolic confinement. *Physica E*, 2013, **47**, P. 270.

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Conflict of interest: the authors declare no conflict of interest.