

Theoretical investigation of the effects of viscosity variation, couple stress, and slip velocity on squeeze film lubrication of porous conical bearings

Kashinath Biradar^{1,a} (Rtd.), Adinatha C. Upadhy^{2,b}

¹Department of Mathematics, Government First Grade College, Sedam, Karnataka, India

²Department of Mathematics, Sangolli Rayanna First Grade Constituent College, Belagavi, Karnataka, India

^akbbiradar@yahoo.co.in, ^badinatha06@gmail.com

Corresponding author: Adinatha C. Upadhy, adinatha06@gmail.com

ABSTRACT This study examines the effects of viscosity variation and velocity slip on the performance of porous conical bearings lubricated with a couple stress fluid. Darcy's law is used to describe fluid flow through the porous medium, while Stokes' couple stress theory accounts for the microstructure of the lubricant. A modified Reynolds equation is developed by incorporating viscosity variation and slip effects. Numerical results illustrate the influence of couple stress, slip velocity, permeability, and viscosity variation on pressure distribution, load-carrying capacity, and squeeze film time. It is observed that an increase in the couple stress parameter enhances the pressure and load capacity, while higher permeability tends to reduce the supporting ability of the bearing. The presence of slip at the boundary modifies the flow behaviour and contributes to improved lubrication characteristics. The combined effects of porous structure and slip lead to better aggregate performance, and the use of couple stress fluids significantly improves lubrication efficiency and operational reliability.

KEYWORDS couple stress fluid; viscosity variation; velocity slip; porous conical bearing.

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1. Introduction

Hydrodynamic lubrication of conical bearings is a significant area of research in tribology due to their ability to support both radial and axial loads in high-speed rotating machinery. The performance of such bearings is governed by the rheological properties of the lubricant and the flow characteristics within the thin film region. Classical lubrication theory assumes Newtonian fluids with constant viscosity; however, this assumption is not suitable for modern lubricants containing microstructural additives. To overcome this limitation, Stokes [1] introduced the couple stress fluid theory, which incorporates micro-continuum effects such as size dependency and particle rotation. This theory provides a more realistic representation of lubricant behaviour and improves the prediction of pressure distribution and load-carrying capacity in bearing systems. It is critical to note that Stokes' couple stress model specifically isolates fluid microstructure effects. It does not account for shear-thinning or non-linear shear-rate dependence. The viscosity coefficient in this formulation remains independent of the shear rate tensor. In engineering practice, this specialized model describes real-world lubricants containing long-chain polymer additives, synthetic oils with micro-particle suspensions, or bio-fluids where particle-rotation effects dominate.

The rheological behaviour of lubricants significantly affects the performance of tribological systems, particularly under high load and thin-film conditions. Modern lubricants containing polymer additives and microstructural particles often exhibit non-Newtonian characteristics such as shear-dependent viscosity and viscoelastic effects. Experimental studies have shown that lubricant viscosity varies with pressure, temperature, and shear rate, thereby influencing film formation and load-carrying capacity. Bair [2] experimentally emphasized the importance of pressure-dependent viscosity, while Spikes [3] reported that lubricant additives strongly affect rheological behaviour under severe operating conditions. These findings support the need for advanced lubrication models incorporating viscosity variation and non-Newtonian effects.

Viscosity variation is another crucial factor that significantly influences lubrication performance. In the present study, the lubricant dynamic viscosity (μ) is assumed to vary with pressure (p) according to the Barus viscosity – pressure relation,

$$\mu = \mu_0 e^{\beta p} \quad (1)$$

where μ_0 is the reference viscosity at atmospheric pressure and β represents the pressure – viscosity coefficient. This piezo-viscous behaviour represents the increase in lubricant viscosity under high-pressure operating conditions. Furthermore, the lubricant rheology is described using Stokes' couple stress fluid theory, which models the lubricant as a non-Newtonian fluid with microstructural effects represented through the couple stress parameter material constant (η), which

characterizes the microstructural properties of the fluid. The non-dimensional couple stress parameter (l^*) is defined as $l^* = \frac{l}{h_0}$, where $l = \sqrt{\frac{\eta}{\mu}}$ is the characteristic couple stress length scale and h_0 is the initial lubricant film thickness. This relationship was first established by Barus [4] in his study on viscosity – pressure dependence. The importance of incorporating viscosity variation in lubrication analysis has been demonstrated by Sangeetha and Govindarajan [5] in their work on viscosity variation with magnetohydrodynamic effects, and by Naduvinamani and Kadadi [6] in their investigation of micropolar fluid lubrication in journal bearings. Dass et al. [7] further emphasized that variable viscosity plays a key role in enhancing load-carrying capacity. Additional studies by Naduvinamani et al. [8] and Salma and Hanumagowda [9] confirmed that neglecting viscosity variation leads to inaccurate predictions, particularly in thin film lubrication systems.

The boundary condition at the lubricant–surface interface is equally important. The classical no-slip condition becomes invalid in micro-scale flows, where velocity slip occurs due to fluid–surface interactions. Lauga et al. [10], in their work on microfluidics and the no-slip boundary condition, highlighted the limitations of classical assumptions. Similarly, Vinogradova [11] and Choi and Kim [12] demonstrated that slip flow significantly alters fluid motion and shear behaviour. The combined influence of viscosity variation and slip velocity has been studied by Dass et al. [13] and Kousar et al. [14], who showed that these effects improve pressure distribution and load-carrying capacity in lubrication systems.

The incorporation of porous materials into bearing design further enhances lubrication performance by enabling fluid flow through the bearing matrix. This behaviour is described using Darcy's law, as presented by Bear [15] in his comprehensive study on fluid flow in porous media. The non-dimensional permeability parameter (ψ) is defined as $\psi = \frac{k\delta}{h_0^3}$, where k is the Darcy permeability of the porous matrix and δ denotes the porous layer thickness. Boussaha et al. [16] investigated non-Newtonian couple-stress squeeze film behaviour between porous circular discs and demonstrated improved stability. Similarly, Naduvinamani and Siddangouda [17] analyzed porous lubrication with couple stress fluids and reported enhanced load-carrying capacity. Ramesh and Devakar [18] analyzed fluid transport within a non-Darcy porous matrix under electroosmotic forcing, utilizing lubrication approximation and perturbation techniques. Their study highlights how porous permeability and moving boundary interfaces significantly restrict the flow field profile. Salma and Hanumagowda [19] also emphasized the importance of viscosity variation in porous lubrication systems.

Recent studies have focused on advanced modeling approaches and lubrication configurations. Vinutha et al. [20] examined the effect of couple stress and slip velocity on squeeze film lubrication and reported improved performance characteristics. Hanumagowda B.N. et al. [21] studied the combined influence of surface roughness and MHD on porous conical bearings lubricated with a conducting couple-stress fluid and observed enhanced lubrication performance. Kashinath et al. [22] investigated the squeeze film characteristics of porous conical bearings with combined effects of piezo-viscous dependency and non-Newtonian couple stresses. Vadher et al. [23] studied the effect of transverse surface roughness on the performance of hydromagnetic squeeze films between conducting truncated conical plates, while Hanumagowda et al. [24] analyzed the effects of MHD and couple stress on conical bearings. More recently, Kempepatil et al. [25] examined the influence of magneto-hydrodynamic and couple stress squeeze film lubrication on conical bearing: A slip velocity model and reported significant improvements in pressure distribution and load-carrying capacity. B.Kashinath and Upadhya A.C. [26] recently analyzed the combined impacts of surface roughness and couple stress fluids on the performance of porous slider bearings. Their findings provide a foundation for extending these non-Newtonian squeeze film characteristics to more complex porous conical geometries.

A comprehensive analysis that incorporates viscosity variation, slip velocity, couple stress fluid behaviour, and porous effects in conical bearings remains limited. Since the present analysis considers laminar thin-film lubrication flow, the governing formulation is based on a modified Reynolds lubrication equation, derived from the continuity and momentum equations under lubrication approximations. The equation is modified to incorporate the effects of couple stress fluid rheology, viscosity variation, velocity slip, and porous media characteristics.

The present study develops a modified Reynolds equation, obtained by extending the classical Reynolds lubrication equation to incorporate the effects of couple stress fluid behaviour, pressure-dependent viscosity, velocity slip, and porous medium flow. This unified formulation enables the combined influence of these effects on bearing performance to be investigated. The model is employed to examine the combined influence of viscosity variation, slip velocity, permeability, and couple stress parameters on key performance characteristics, including pressure distribution, load-carrying capacity, and squeeze film time. This approach offers a more realistic and detailed understanding of lubrication behaviour, which is important for the optimal design and improved efficiency of porous conical bearing systems.

Based on the literature review, no study has yet analyzed the combined effects of viscosity variation, couple stress, and slip velocity on porous conical bearings. Therefore, the objective of this paper is to investigate these coupled effects on the squeeze film lubrication characteristics of porous conical bearings to provide a more realistic prediction of bearing performance.

2. Mathematical formulation and solution of the problem

Fig. 1 illustrates the geometrical configuration of the porous conical bearing and lubricant film region considered in the present study. The effects of non-Newtonian couple stress fluid behaviour and velocity slip are incorporated through

the governing equations and boundary conditions. The local lubricant film thickness between the conical surfaces is denoted by h and the non-dimensional film thickness parameter is expressed as $h^* = \frac{h}{h_0}$. The squeezing velocity of the upper surface is given by $\sin \theta \frac{\partial h}{\partial t}$, where θ represents the semi-cone angle of the bearing. The present study examines the squeeze film lubrication characteristics of a porous conical bearing lubricated with a non-Newtonian couple stress fluid under viscosity variation and velocity slip. The lubricant is assumed to be thin, incompressible, axisymmetric, and laminar. The lower surface is porous and governed by Darcy's law, while the upper surface moves with squeezing motion.

The lubricant is modeled as an incompressible non-Newtonian couple stress fluid based on Stokes' couple stress fluid theory [1]. The constitutive relation for the stress tensor is given by

$$\tau_{ij} = -p\delta_{ij} + 2\mu d_{ij} - \eta \nabla^2 d_{ij} \quad (2)$$

where τ_{ij} is the stress tensor, p is the pressure, μ is the dynamic viscosity, d_{ij} is the rate of deformation tensor, and η is the couple stress material constant representing microstructural effects in the lubricant. Equation (2) represents a simplified form of the general constitutive relation proposed by Stokes [1]. Under the assumptions of incompressible thin-film lubrication flow, axisymmetry, negligible body couples, and lubrication approximations, the general couple stress constitutive equation reduces to the form employed in the present analysis. Therefore, Equation (2) should be regarded as a lubrication-theory representation derived from the original Stokes model rather than a direct reproduction of Equation (3.1) presented in Stokes [1].

It should be noted that the present study employs Stokes' couple stress fluid theory rather than micropolar fluid theory. Unlike micropolar fluids, which include independent microrotation and rotational viscosity effects, the couple stress fluid model accounts for microstructural behaviour through higher-order stress gradients characterized by the material constant η . Therefore, rotational viscosity is not considered in the present formulation. In the present formulation, the lubricant exhibits non-Newtonian rheology based on Stokes' couple stress fluid model, while viscosity variation is assumed to be pressure-dependent according to the Barus exponential law.

It should be emphasized that the pressure-dependent viscosity and couple stress effects represent two distinct physical mechanisms. The Stokes couple stress model accounts for the microstructural characteristics of the lubricant through the material parameter η , whereas the Barus relation $\mu = \mu_0 e^{\beta p}$ is employed to describe the experimentally observed increase in viscosity under high-pressure conditions. Although the original Stokes theory is generally formulated for fluids with constant material properties, the incorporation of the Barus viscosity law is a commonly adopted engineering approximation in lubrication analysis to account for piezo-viscous effects. Therefore, the present study considers a lubricant exhibiting both couple stress and pressure-dependent viscosity characteristics, providing a more realistic representation of practical lubricants used in bearing applications. Based on lubrication theory, the governing equations are obtained from momentum and continuity equations. Equation (3) represents the radial momentum equation for a couple stress fluid, Equation (4) assumes negligible pressure variation across the film thickness, and Equation (5) represents mass conservation within the lubricant film.

$$\mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} = \frac{\partial p}{\partial r} \quad (3)$$

$$\frac{\partial p}{\partial z} = 0 \quad (4)$$

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0 \quad (5)$$

The flow of lubricant through the porous matrix is modeled using Darcy's law. The lower conical surface is assumed to be porous, homogeneous, and isotropic, allowing lubricant seepage through the porous medium. For consistency with the governing momentum and continuity equations presented earlier, the porous-medium flow equations are first expressed in dimensional form. The seepage velocity components in the radial and transverse directions are denoted by u_p and w_p , respectively. Accordingly, the Darcy flow equations governing lubricant transport through the porous matrix are written as follows:

$$u_p = -\frac{k}{\mu(1-\beta)} \frac{\partial p}{\partial r} \quad (6)$$

$$w_p = -\frac{k}{\mu(1-\beta)} \frac{\partial p}{\partial z} \quad (7)$$

$$\frac{1}{r} \frac{\partial(ru_p)}{\partial r} + \frac{\partial w_p}{\partial z} = 0 \quad (8)$$

It should be noted that Equations (3)–(8) are expressed in dimensional form. The velocity distribution and the modified Reynolds equation are first derived using dimensional variables. Subsequently, appropriate nondimensional variables and parameters are introduced to obtain the final nondimensional governing equation.

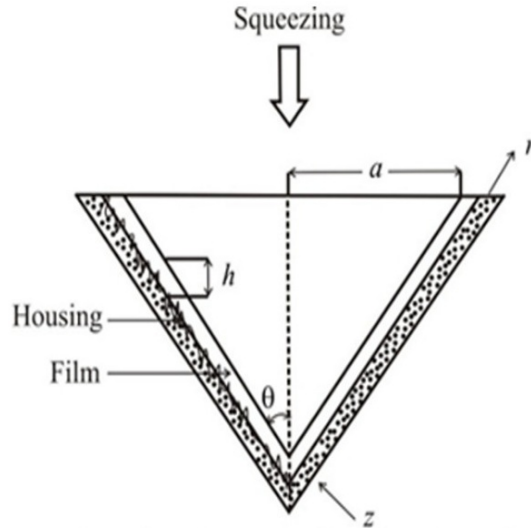


FIG. 1. Geometrical representation of the porous conical bearing

The relevant boundary conditions for the velocity components are:

(i) At the lower surface: $y = 0$

$$\frac{\alpha}{\sqrt{k}}(u - u_p) = \left(\frac{\partial u}{\partial z}\right), \quad \frac{\partial^2 u}{\partial z^2} = 0 \quad (9)$$

$$w = w_p \quad (10)$$

(ii) At the upper surface: $z = h \sin \theta$

$$u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0 \quad (11)$$

$$w = \sin \theta \frac{\partial h}{\partial t} \quad (12)$$

By solving equations (3) and (4) with given boundary conditions (9) and (11), the velocity component u can be obtained as follows.

$$u = \left[\frac{1}{2\mu_0 e^{\beta p}} \frac{\partial p}{\partial r} \right] \left\{ z(z - h) + 2l^2 \left[1 - \frac{\cosh\left(\frac{2z-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)} \right] + \xi_1 h - \xi_2 h l \tanh\left(\frac{h}{2l}\right) \right\} \quad (13)$$

where $\xi_1 = \frac{3[2s^2\alpha^2 + (1 - \beta)sh]}{h(h + s)(1 - \beta)}$ and $\xi_2 = \frac{s}{h + s}$.

To obtain the equation for pressure, we first substitute Equation (13) into the continuity equation (5) and then perform the integration. By incorporating the boundary conditions given in Equations (10) and (12), the final form of the modified Reynolds lubrication equation for laminar thin-film flow is obtained as:

The modified Reynolds type equation for pressure is:

$$\frac{1}{r} \frac{\partial}{\partial r} \left[\frac{r f(h, \xi_1, \xi_2, l)}{12\mu_0 e^{\beta p}} \frac{\partial p}{\partial r} \right] + \frac{k\delta}{1 - \beta} \nabla^2 p = 12 \frac{\partial h}{\partial t}, \quad (14)$$

where,

$$f(h, \xi_1, \xi_2, l) = (1 + \xi_1)h^3 - 6l^2h \left[h - \xi_2 l \tanh\left(\frac{h}{2l}\right) \right] - 12l^3 \tanh\left(\frac{h}{2l}\right)$$

The volume flux of the lubricant is given by the following formula:

$$Q = 2\pi r \int_0^h u \, dz \quad (15)$$

where u is the velocity component given in Eq. (13).

After obtaining the dimensional modified Reynolds equation (14), the following nondimensional variables and parameters are introduced to facilitate the analysis and numerical computation.

$$r^* = \frac{r}{R}, \quad h^* = \frac{h}{h_0}, \quad l^* = \frac{l}{h_0}, \quad p^* = \frac{ph_0^3}{\mu_0 R^2 \left(\frac{\partial h}{\partial t}\right) \sin \theta}, \quad G = \frac{\beta \mu_0 R^2 \left(\frac{\partial h}{\partial t}\right) \sin \theta}{h_0^3}, \quad \psi = \frac{k\delta}{h_0^3}, \quad \sigma^* = \frac{s}{h_0}$$

The characteristic quantities used for nondimensionalization are the bearing radius R and the initial lubricant film thickness h_0 . Thus, the radial coordinate, film thickness, couple stress parameter, and slip length are scaled by R and h_0 according to $r^* = r/R$, $h^* = h/h_0$, $l^* = l/h_0$, and $\sigma^* = s/h_0$. The dimensionless slip parameter σ^* therefore, represents the ratio of the slip length to the initial film thickness. For example, $\sigma^* = 0.6$ indicates that the slip length is 60% of the initial lubricant film thickness. Such values correspond to relatively strong slip conditions that may occur in highly smooth, porous, hydrophobic, or micro-/nano-scale lubricated interfaces.

Substituting the non-dimensional variables into Eq. (14), the non-dimensional modified Reynolds lubrication equation, valid for laminar thin-film flow in porous conical bearings, is obtained in the form

$$\frac{\partial}{\partial r^*} \left[e^{-Gp^*} (f^*(h^*, \chi_1, \chi_2, l^*, G, \psi)) r^* \frac{\partial p^*}{\partial r^*} \right] = 12r^* \quad (16)$$

where

$$f^*(h^*, \chi_1, \chi_2, l^*, G, \psi) = \left[(h^* \sin \theta)^3 (1 + \chi_2) - 6(h^* \sin \theta)^2 l^* \chi_1 \tanh \left(\frac{h^* \sin \theta}{2l^*} \right) - 12l^{*2} \left[h^* \sin \theta - 2 \tanh \left(\frac{h^* \sin \theta}{2l^*} \right) \right] + \frac{12\psi}{1 - \beta} \right].$$

and

$$\chi_1 = \frac{\sigma^*}{h^* \sin \theta + \sigma^*}, \quad \chi_2 = \frac{3(1 - \beta)^2 (\sigma^*)^2 \alpha^2}{h^{*2} \sin^2 \theta + \sigma^* h^* \sin \theta + (1 - \beta) \sigma^* (h^* \sin \theta + \sigma^*)}.$$

The pressure boundary conditions are:

$$\frac{\partial p^*}{\partial r^*} = 0 \quad \text{at} \quad r^* = 0 \quad (17)$$

$$p^* = 0 \quad \text{at} \quad r^* = 1 \quad (18)$$

By performing an integration of the nondimensional modified lubrication equation in relation to r^* and incorporating the pressure boundary conditions, one obtains

$$p^* = -\frac{1}{G} \ln \left[\frac{3G(r^{*2} - 1)}{f^*(h^*, \chi_1, \chi_2, l^*, G, \psi)} + 1 \right] \quad (19)$$

The non-dimensional load-carrying capacity w^* , is obtained by integrating the film pressure over the bearing area:

$$W^* = \frac{2\pi}{G} \int_0^1 \ln \left[\frac{3G(r^{*2} - 1)}{f^*(h^*, \chi_1, \chi_2, l^*, G, \psi)} + 1 \right] r^* dr^* \quad (20)$$

The squeeze film time is obtained by integrating the velocity equation:

$$T^* = \frac{2\pi}{G} \int_{h_f^*}^1 \int_0^1 \ln \left[\frac{3G(r^{*2} - 1)}{f^*(h^*, \chi_1, \chi_2, l^*, G, \psi)} + 1 \right] r^* dr^* dh^* \quad (21)$$

Following the numerical resolution of the film pressure distribution, the integrated bearing metrics can be determined. The total load-carrying capacity of the bearing is evaluated in its dimensionless form as w^* , while the total operational duration of the squeezing film is represented by the non-dimensional squeeze film time (T^*).

3. Results and discussion

The effects of viscosity variation, slip velocity, couple stress fluid behaviour, and permeability on the squeeze film characteristics of porous conical bearings are analyzed using the modified Reynolds equation through numerical evaluation of pressure distribution, load-carrying capacity, and squeeze film time. Physically, an increase in the viscosity variation parameter enhances the effective lubricant viscosity under pressure, thereby increasing resistance to flow and promoting greater pressure development within the lubricant film. Consequently, both the load-carrying capacity and squeeze film time increase. Similarly, increasing the couple stress parameter strengthens the microstructural interactions within the lubricant, resulting in higher fluid stiffness and greater resistance to film thinning. This leads to improved pressure generation, enhanced load support, and longer squeeze film duration. In contrast, slip velocity reduces the shear resistance at the lubricant-surface interface, facilitating fluid motion and reducing pressure build-up. Likewise, increasing the permeability of the porous matrix allows greater lubricant seepage through the bearing surface, thereby reducing fluid retention within the film and lowering the load-carrying ability of the bearing. These observations indicate that lubricants possessing microstructural characteristics and pressure-dependent viscosity can significantly improve bearing performance compared with conventional Newtonian lubricants, while slip and permeability effects tend to moderate these improvements.

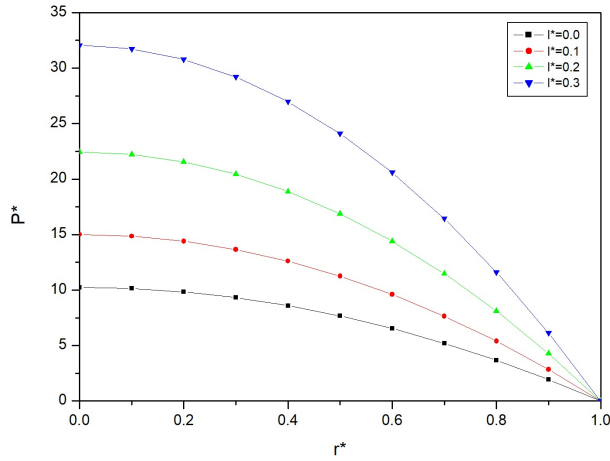


FIG. 2. Variation of non-dimensional pressure P^* with r^* for different values of l^* with $G = 0.03$, $h^* = -0.7$, $\sigma = 0.2$, $\psi = 0.01$, $\theta = \frac{\pi}{3}$.

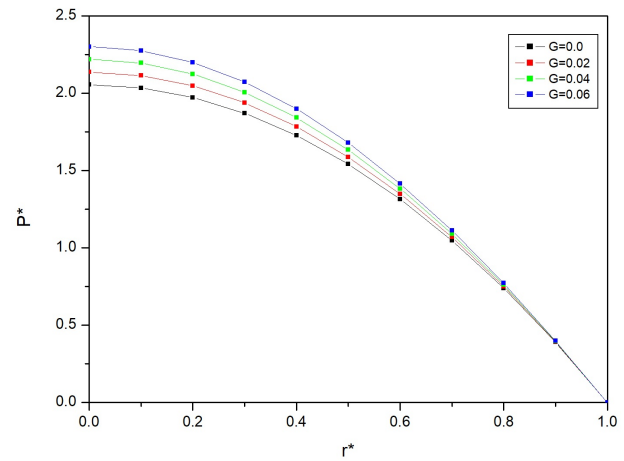


FIG. 3. Variation of non-dimensional pressure P^* with r^* for different values of G with $h^* = -0.7$, $l^* = 0.2$, $\sigma = 0.2$, $\psi = 0.01$, $\theta = \frac{\pi}{3}$.

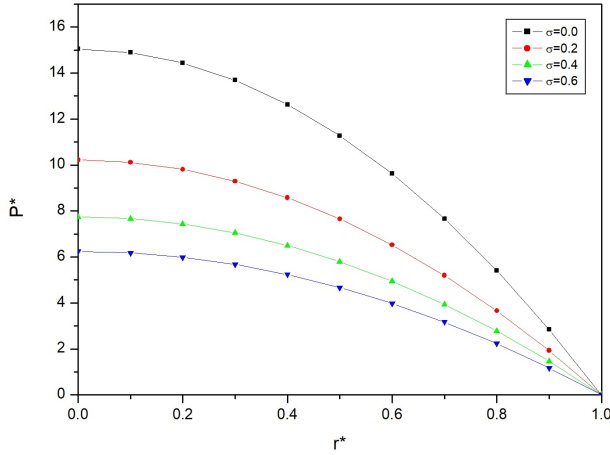


FIG. 4. Variation of non-dimensional pressure P^* with r^* for different values of σ with $l^* = 0.2$, $h^* = 0.7$, $G = 0.03$, $\psi = 0.01$, $\theta = \frac{\pi}{3}$.

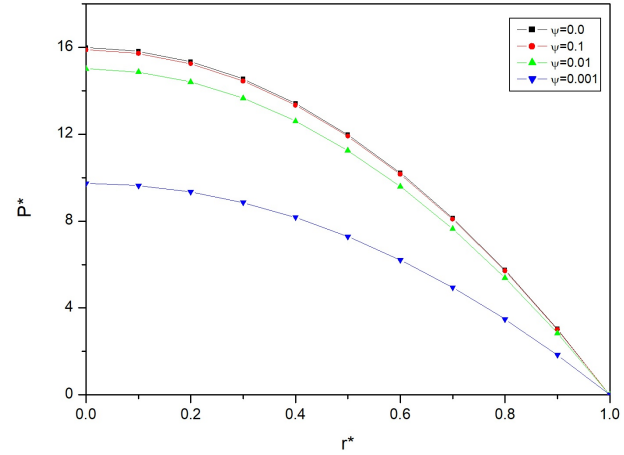


FIG. 5. Variation of non-dimensional pressure P^* with r^* for different values of ψ with $h^* = 0.7$, $l^* = 0.2$, $\sigma = 0.2$, $G = 0.03$, $\theta = \frac{\pi}{3}$.

3.1. Squeeze film pressure

Figure 2 illustrates the distribution of dimensionless pressure P^* across the dimensionless radial coordinate r^* . The data reveals that P^* rises from 0 to 0.3. This enhancement is attributed to the increased internal resistance characteristic of micro-structure fluids, which necessitates higher pressure gradients to maintain flow. Physically, increasing the couple stress parameter enhances the microstructural resistance of the lubricant due to suspended additives or long-chain molecules, thereby strengthening momentum transfer and promoting higher pressure generation within the lubricant film. Similarly, Fig. 3 indicates that an increase in the viscosity variation parameter significantly boosts the dimensionless pressure, as higher local viscosity offers greater resistance to lubricant deformation, a typical non-linear behaviour in non-Newtonian lubrication regimes. Physically, increasing viscosity under pressure increases internal flow resistance, thereby restricting lubricant motion and enhancing pressure development in the squeeze film region.

Figure 4 illustrates the effect of the dimensionless slip length parameter σ^* , varied from 0 to 0.6, where $\sigma^* = 0$ corresponds to the classical no-slip condition. The upper limit $\sigma^* = 0.6$ represents relatively strong interfacial slip conditions, which may occur in lubricated systems with highly smooth or hydrophobic surfaces, porous interfaces, polymer-enhanced lubricants, or micro-/nano-scale thin-film applications where slip effects become significant. It is observed that increasing σ^* results in a reduction of P^* , as interfacial slip decreases flow resistance at the boundary. Physically, larger slip lengths reduce shear resistance at the lubricant–surface interface, allowing easier fluid motion and reducing pressure build-up

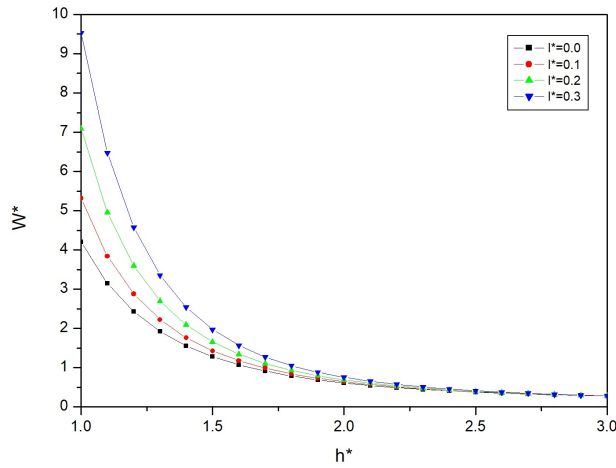


FIG. 6. Variation of non-dimensional Load carrying capacity W^* with h^* for different values of l^* with $G = 0.03$, $\sigma = 0.2$, $\psi = 0.01$, $\theta = \frac{\pi}{3}$.

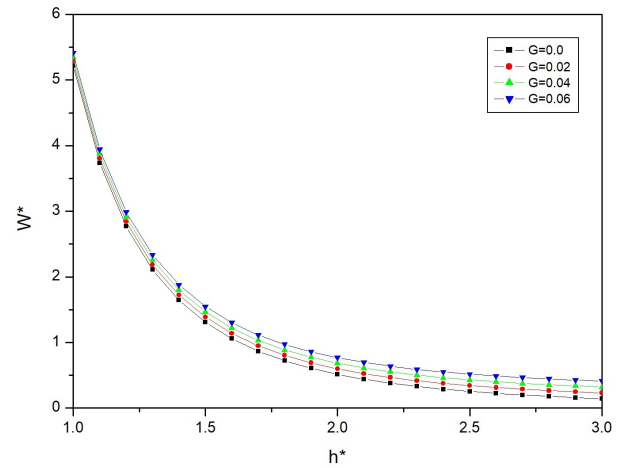


FIG. 7. Variation of non-dimensional Load carrying capacity W^* with h^* for different values of G with $l^* = 0.2$, $\sigma = 0.2$, $\psi = 0.01$, $\theta = \frac{\pi}{3}$.

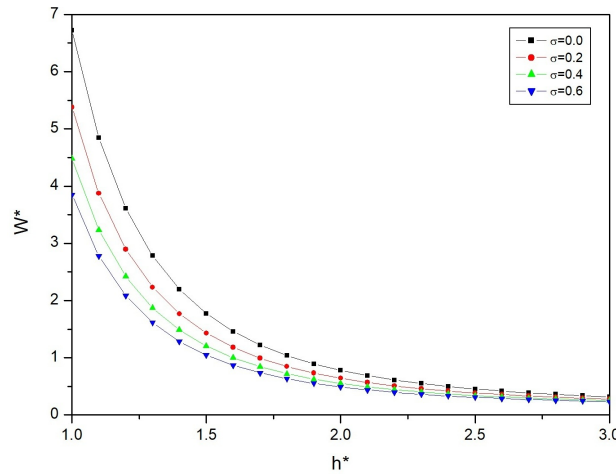


FIG. 8. Variation of non-dimensional Load carrying capacity W^* with h^* for different values of σ with $l^* = 0.2$, $G = 0.03$, $\psi = 0.01$, $\theta = \frac{\pi}{3}$.

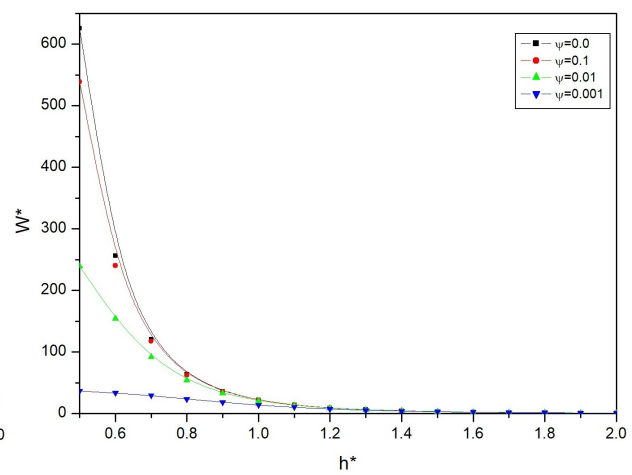


FIG. 9. Variation of non-dimensional Load carrying capacity W^* with h^* for different values of ψ with $l^* = 0.2$, $G = 0.03$, $\sigma = 0.2$, $\theta = \frac{\pi}{3}$.

within the lubricant film. Consequently, the load-carrying capacity of the bearing decreases under stronger slip conditions. Finally, in the present analysis, porous effects are represented through the non-dimensional permeability parameter ψ , which characterizes the ease of lubricant flow through the homogeneous porous matrix. Different values of ψ are considered to examine the influence of porous structure on bearing performance. Fig. 5 demonstrates that an increase in the permeability parameter ψ leads to a decrease in dimensionless pressure, highlighting that higher permeability in the porous medium facilitates fluid flow and lowers the pressure within the film. From a physical perspective, increasing permeability allows greater lubricant seepage through the porous matrix, reducing lubricant retention within the film and consequently lowering pressure generation.

3.2. Load carrying capacity

Figures 6–9 illustrate the relationship between the minimum film height and the dimensionless load-carrying capacity (W^*) for various values of the couple stress parameter (l^*), viscosity variation (G), slip velocity (σ), and permeability (ψ). As shown in Figs. 6,7, W^* increases as l^* increases and G increases. This behavior is attributed to the fact that higher couple stresses introduce internal micro-rotational effects that enhance fluid stiffness, while increased viscosity strengthens resistance to flow; both factors improve the lubricant's ability to support applied loads. Fig. 8 reveals that W^* decreases as the dimensionless slip length parameter (σ^*) increases, since greater interfacial slip reduces pressure build-up within the lubricant film. The relatively higher value $\sigma^* = 0.6$ corresponds to conditions of enhanced slip, which may

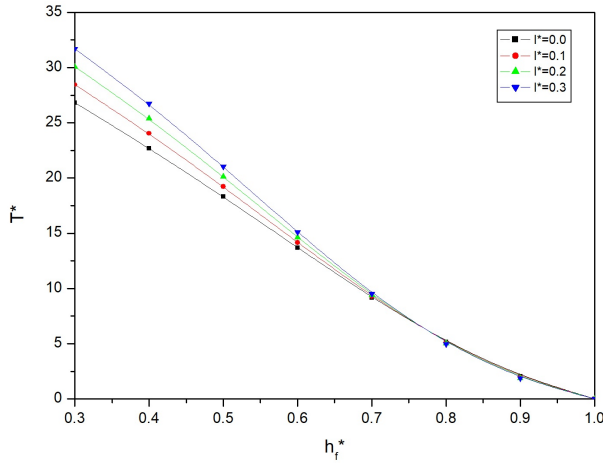


FIG. 10. Variation of squeeze film time T^* with h_f^* for different values of l^* with $G = 0.03$, $\sigma = 0.2$, $\psi = 0.01$, $\theta = \frac{\pi}{3}$.

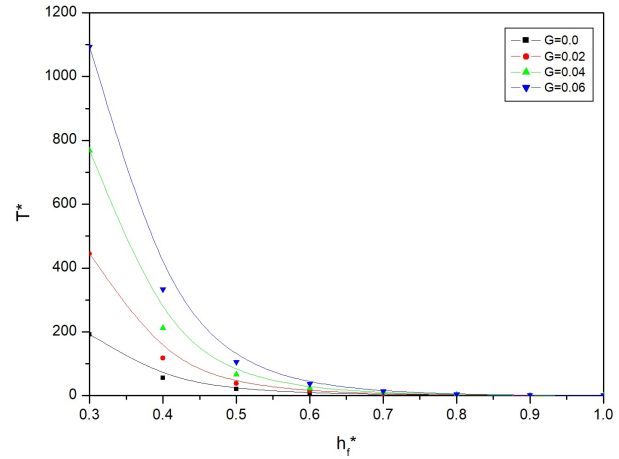


FIG. 11. Variation of squeeze film time T^* with h_f^* for different values of G with $l^* = 0.2$, $\sigma = 0.2$, $\psi = 0.01$, $\theta = \frac{\pi}{3}$.

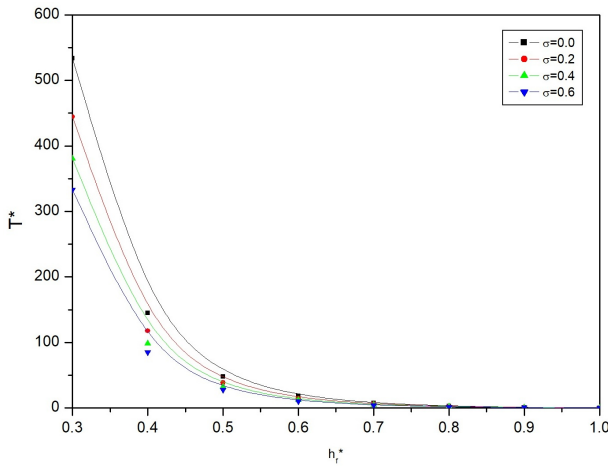


FIG. 12. Variation of squeeze film time T^* with h_f^* for different values of σ with $l^* = 0.2$, $G = 0.03$, $\psi = 0.01$, $\theta = \frac{\pi}{3}$.

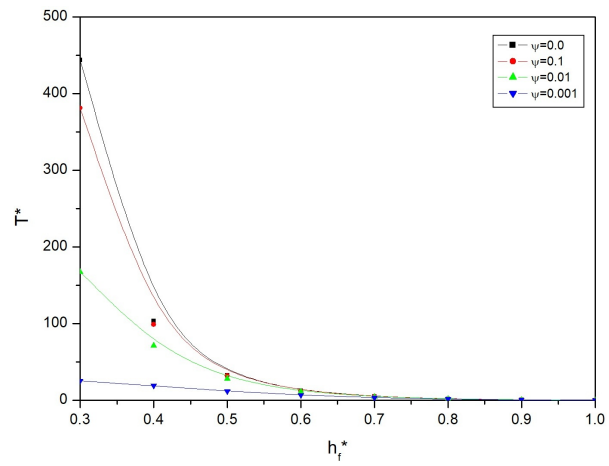


FIG. 13. Variation of squeeze film time T^* with h_f^* for different values of ψ with $l^* = 0.2$, $G = 0.03$, $\sigma = 0.2$, $\theta = \frac{\pi}{3}$.

arise in highly smooth, porous, or hydrophobic bearing interfaces and in thin-film lubrication systems containing polymer additives. Finally, Fig. 9 demonstrates that an increase in the permeability parameter (ψ) leads to a decrease in W^* , as higher permeability allows more lubricant to pass into the porous medium, thereby reducing hydrodynamic pressure and load-carrying capacity.

3.3. Squeeze film time

In Fig. 10, the dimensionless squeeze film duration (T^*) is shown to increase as the couple stress parameter (l^*) rises from 0 to 0.3. A similar trend is observed in Fig. 11, where T^* increases with the viscosity variation parameter (G). This behavior occurs because both couple stress effects and higher viscosity increase the internal resistance to fluid motion, thereby stiffening the lubricant film and extending the time required for film thinning. Fig. 12 shows that the squeeze film duration decreases as the dimensionless slip velocity (σ) increases, since boundary slip promotes faster fluid escape from the bearing gap. Finally, Fig. 13 illustrates that an increase in the permeability parameter (ψ) further reduces the squeeze film time, as the porous medium provides an additional path for fluid flow, leading to a more rapid reduction of the lubricant film thickness.

4. Conclusions

This research provides a comprehensive analysis of the squeeze film characteristics in porous conical bearings by extending the classical Reynolds lubrication equation to include the effects of pressure-dependent viscosity, couple stress fluid behaviour, velocity slip, and porous medium permeability. The results demonstrate that viscosity variation and couple stress effects significantly enhance pressure distribution, load-carrying capacity, and squeeze film time by increasing flow resistance and fluid stiffness. Conversely, slip velocity reduces these parameters by lowering interfacial shear resistance, while higher permeability decreases pressure and load support by facilitating lubricant escape through the porous matrix. The use of non-Newtonian lubricants together with viscosity variation enhances bearing performance, while slip and permeability diminish these improvements. The findings contribute to a better understanding of lubrication behaviour and may aid in the design and optimization of porous conical bearings.

References

- [1] Stokes V.K. Couple stresses in fluids. *Physics of Fluids*, 1966, **9**(9), P. 1709–1715.
- [2] Bair S. *High Pressure Rheology for Quantitative Elastohydrodynamics*. Elsevier, Amsterdam, 2007.
- [3] Spikes H. Friction modifier additives. *Tribology Letters*, 2015, **60**(1), P. 5.
- [4] Barus C. Isothermals, isopiestic and isometrics relative to viscosity. *American Journal of Science*, 1893, **45**(266), P. 87–96.
- [5] Sangeetha S., Govindarajan A. Analysis of viscosity variation with MHD on circular stepped plates in couple stress fluid. *Journal of Interdisciplinary Mathematics*, 2019, **22**(6), P. 889–901.
- [6] Naduvinamani N.B., Kadadi A.K. Effect of viscosity variation on the micropolar fluid lubrication of a short journal bearing. *Advances in Tribology*, 2013, **2013**, P. 507821.
- [7] Dass T.D., Gunakala S.R., Comissiong D.M. Combined effect of variable viscosity on the lubrication characteristics of bearing systems. *Materials Today: Proceedings*, 2022, **56**, P. 1717–1725.
- [8] Naduvinamani N.B., Apparao S., Kadadi A.K., Biradar S.N. Combined effects of viscosity variation on lubrication characteristics. *Tribology Online*, 2014, **9**(4), P. 175–183.
- [9] Salma A., Hanumagowda B.N. Reynolds equation for porous plates accounting viscosity variation. *Partial Differential Equations in Applied Mathematics*, 2022, **6**, P. 100424.
- [10] Lauga E., Brenner M.P., Stone H.A. Microfluidics: The no-slip boundary condition. *Springer Handbook of Experimental Fluid Mechanics*, Springer, Berlin, 2007, P. 1219–1240.
- [11] Vinogradova O.I. Slippage of water over hydrophobic surfaces. *International Journal of Mineral Processing*, 1999, **56**(1–4), P. 31–60.
- [12] Choi C.H., Kim C.J. Large slip of aqueous liquid flow over a nanoengineered superhydrophobic surface. *Physical Review Letters*, 2006, **96**(6), P. 066001.
- [13] Dass T.D., Gunakala S.R., Comissiong D.M. Combined effect of variable viscosity and velocity slip on the lubrication of journal bearings. *Ain Shams Engineering Journal*, 2020, **11**(2), P. 501–518.
- [14] Kousar A., Salma A., Maruthamanikandan S., Hanumagowda B.N. Significance of slip velocity and viscosity variation on squeezed film couple-stress properties. *Partial Differential Equations in Applied Mathematics*, 2025, **14**, P. 101160.
- [15] Bear J. *Dynamics of Fluids in Porous Media*. Dover Publications, New York, 1972.
- [16] Boussaha B., Lahmar M., Bou-Said B., Boucherit H. Non-Newtonian couple-stress squeeze film behaviour between oscillating anisotropic porous circular discs with sealed boundary. *Mechanics & Industry*, 2020, **21**(3), P. 311.
- [17] Naduvinamani N.B., Siddangouda A. Effect of surface roughness on the hydrodynamic lubrication of porous step slider bearings with couple stress fluids. *Tribology International*, 2007, **40**(5), P. 780–793.
- [18] Ramesh K., Devakar M. Peristaltic pumping through non-Darcy porous medium in an electroosmotic flow. *Nanosystems: Physics, Chemistry, Mathematics*, 2020, **11**(4), P. 379–390.
- [19] Salma A., Hanumagowda B.N. Reynolds equation in stochastic form for porous plates accounting viscosity variation. *Partial Differential Equations in Applied Mathematics*, 2022, **6**, P. 100424.
- [20] Vinutha R., Hanumagowda B.N., Vasanth K.R. Theoretical analysis of effect of couple stress and slip velocity on squeeze film lubrication characteristics. *Tribology International*, 2024, **191**, P. 109164.
- [21] Hanumagowda B.N., Nair S.S. Combined effect of rough surface with MHD on porous conical bearing with conducting couple-stress fluid. *Palestine Journal of Mathematics*, 2021, **10**(2), P. 540–552.
- [22] Biradar K., Naduvinamani N.B., Upadhy A.C. Squeeze film characteristics of porous conical bearings with combined effects of piezo-viscous dependency and non-Newtonian couple stresses. *EasyChair Preprints*, 2022, P. 8305.
- [23] Vadher P.A., Deheri G., Patel R.M. Effect of transverse surface roughness on the performance of hydromagnetic squeeze film between conducting truncated conical plates. *Journal of Marine Science and Technology*, 2011, **19**(6), P. 613–621.
- [24] Hanumagowda B.N., Nair S.S., Kumar M.V. Effect of MHD and couple stress on conical bearing. *International Journal of Pure and Applied Mathematics*, 2017, **113**(6), P. 316–324.
- [25] Kempepatil R., Hiremath A.G., Hanumagowda B.N., Patil J., Tawade J.V., Khan M.I. Influence of magneto-hydrodynamic and couple stress squeeze film lubrication on conical bearing: A slip velocity model. *Alexandria Engineering Journal*, 2024, **106**, P. 735–742.
- [26] Biradar K., Upadhy A.C. Surface roughness effect on porous pivoted slider bearings with squeeze film formed by couple stress fluid. *Journal of Scientific Research*, 2025, **17**(2), P. 427–439.

Information about the authors:

Kashinath Biradar (Rtd.) – Department of Mathematics, Government First Grade College, Sedam, Karnataka, India; kbbiradar@yahoo.co.in

Adinatha C. Upadhyay – Department of Mathematics, Sangolli Rayanna First Grade Constituent College, Belagavi, Karnataka, India; ORCID 0000-0002-8433-2891; adinatha06@gmail.com

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