

On the validity of 2D reduction of the Fournier-Galatola planar anchoring model for polymer-dispersed nematic liquid crystals

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ABSTRACT The validity of directly reducing the three-dimensional Fournier-Galatola model of planar anchoring to two-dimensional nematic systems is investigated analytically and numerically. An alternative two-dimensional formulation based on a direct target surface Q -tensor is introduced for comparison. Analytical calculations reveal that the reduced Fournier-Galatola functional yields a cubic restoring torque at the poles of an elliptical droplet, whereas the direct tensor formulation preserves a linear response along the entire boundary. The influence of this difference on orientational relaxation is studied using the Landau-de Gennes simulations of mesoscale elliptical 5CB droplets. The reduced Fournier-Galatola model is shown to enhance boojum mobility, alter the final bipolar orientation at moderate aspect ratios, and systematically predict lower equilibrium energies. The results indicate that direct projection-based reduction can lead to qualitatively different relaxation scenarios in two-dimensional nematic models.

KEYWORDS bipolar structure, planar anchoring, Q -tensor, boojums, mesoscale nematic droplets.

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1. Introduction

Polymer-dispersed liquid crystals (PDLC) represent a class of composite materials in which liquid crystal droplets are dispersed within a polymer matrix. Due to a combination of a pronounced electro-optic response, ease of fabrication, and the ability to retain formed orientational states for a long time, such systems are widely considered as a basis for information display devices, switchable optical elements, and memory materials [1–3]. In composites based on 5CB within a polymer matrix, mesoscale nematic droplets are formed, the internal orientational structure of which is determined by the interplay of elastic interactions, surface anchoring, and boundary geometry. Elliptical and elongated droplets are characterized by the formation of bipolar textures with surface defects of the boojum type, whose position significantly affects the configuration stability, optical characteristics, and relaxation processes after the external field is removed [4]. Consequently, a correct description of boundary orientational processes and surface defect dynamics is a prerequisite for building predictive models of such systems, especially since different surface anchoring models may predict qualitatively different relaxation scenarios. Establishing the range of applicability of these models is therefore important before using them for quantitative comparison with experiment.

The Landau-de Gennes theory, based on the tensor order parameter Q , serves as the most universal continuum approach to describing defect structures in nematics. Unlike the Frank-Oseen approach, this method allows for a local reduction of the order parameter down to its vanishing at defect cores, thereby naturally capturing the topological features of the texture [5–7]. Several surface anchoring formulations have been developed within the Q -tensor framework. One of the most well-known is the Nobili-Durand model [8], designed to specify a fixed preferred director orientation at the surface. For the case of degenerate planar anchoring, the Fournier-Galatola projection formulation [9] is widely used; it does not select a preferred direction in the tangent plane and is therefore convenient for modeling droplets and colloidal systems.

When moving to two-dimensional (2D) and quasi-2D formulations, the corresponding surface functional must be transferred to a 2D geometry, raising the question of the validity of such an operation. During the transition from 3D to 2D, the dimensionality of the tangent space decreases from two to one, causing the structure of the projection operator to change qualitatively, not just quantitatively. It remains unclear whether the 2D reduction of the Fournier-Galatola functional retains the same restoring torque properties inherent in the original 3D formulation, or if such a reduction leads to additional dynamical effects. This question is of particular interest for systems with a curved boundary and pronounced

geometric anisotropy, specifically for elliptical nematic droplets, where even small changes in the surface torque structure can significantly affect the dynamics of surface defects and the selection of the final equilibrium state. To the authors' knowledge, this issue has not received systematic analytical investigation to date.

The aim of this work is to investigate the 2D reduction of the Fournier-Galatola projection model and determine its affect on the relaxation dynamics of the orientational structure in an elliptical nematic liquid crystal droplet. To this end, an analytical investigation of the restoring torque is performed and compared with an alternative anchoring formulation based on the direct specification of a target surface Q -tensor. Next, using a numerical solution of the relaxation equations, the director field evolution after switching off the external aligning field is investigated, and the influence of the chosen anchoring model on boojum migration and the formation of the final bipolar configuration is analyzed. The theoretical formulation is presented in Section 2. Section 3 analyzes the boundary torque structure. Numerical implementation and simulation results are discussed in Sections 4 and 5, respectively.

2. Theoretical Formulation

2.1. Q -tensor parameterization

The tensor order parameter Q_{ij} is a symmetric traceless matrix that describes the local degree of orientational order and the direction of the preferred molecular orientation in a nematic. In the 3D case, a nematic is characterized by five independent degrees of freedom, and the Q -tensor is typically written as:

$$Q_{ij} = \frac{3}{2}S \left(n_i n_j - \frac{1}{3} \delta_{ij} \right), \quad (1)$$

where S is the scalar order parameter, and $\mathbf{n}$ is the unit director vector. This normalization ensures that in the fully ordered state ($S = 1$), the largest eigenvalue of the tensor equals one, and the other two equal $-1/2$.

Upon transitioning to 2D geometry, the system possesses only two translational degrees of freedom, and the tensor Q_{ij} becomes a 2×2 matrix. Symmetry and tracelessness requirements are preserved, but the condition on the eigenvalues changes: there are now only two, and they must equal $\pm S$. This leads to a different normalization factor. In the 2D case, the Q -tensor is parameterized as:

$$Q_{ij} = S \begin{pmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{pmatrix} = S (2n_i n_j - \delta_{ij}), \quad (2)$$

where ϕ is the angle of preferred molecular orientation relative to the x -axis, and $\mathbf{n} = (\cos \phi, \sin \phi)$ is the unit director vector. The eigenvectors coincide with $\mathbf{n}$ and the direction orthogonal to it.

2.2. Bulk free energy

The free energy in the domain Ω includes bulk and elastic contributions. The bulk contribution governs the thermodynamics of the isotropic-nematic phase transition and in the 2D reduction takes the form [11]:

$$f_{bulk} = \frac{A}{2} \left(1 - \frac{U}{3} \right) Q_{ij} Q_{ij} - \frac{AU}{3} Q_{ij} Q_{jk} Q_{ki} + \frac{AU}{4} (Q_{ij} Q_{ij})^2 = A \left(1 - \frac{U}{3} \right) S^2 + AU S^4, \quad (3)$$

here A is the energy scale (for 5CB $A = 1.17 \times 10^5$ J/m³), and U is a dimensionless parameter controlling the depth of the nematic minimum. Minimization with respect to S shows that for $U > 3$, the isotropic state $S = 0$ loses stability, and a non-zero equilibrium value emerges:

$$S_0 = \sqrt{\frac{U-3}{6U}}. \quad (4)$$

For the characteristic value $U = 3.5$, corresponding to 5CB [11], we obtain $S_0 \approx 0.154$. This value is lower than the typical one for bulk 5CB ($S_0 \approx 0.53$), which is due to the specifics of the 2D reduction of the bulk functional and does not contradict the physics of thin films, where the order parameter can be reduced.

The elastic contribution in the one-constant approximation is written in terms of the Q -tensor gradients [11]:

$$f_{elastic} = \frac{L}{2} \frac{\partial Q_{ij}}{\partial x_k} \frac{\partial Q_{ij}}{\partial x_k}, \quad (5)$$

where L is the elastic constant (for 5CB $L \approx 6$ pN). Direct calculation of the gradients of Q_{ij} leads to the form:

$$f_{elastic} = L [(\nabla S)^2 + 4S^2(\nabla \phi)^2]. \quad (6)$$

The first term is associated with inhomogeneities of the order amplitude, and the second with orientational distortions. As $S \rightarrow 0$ (defect core), the orientational elasticity contribution automatically vanishes, eliminating the elastic energy singularity in regions with disrupted order.

2.3. Relaxation equations

The system dynamics are defined within the framework of linear nonequilibrium thermodynamics. The dissipative function, quadratic in the rate of change of the Q-tensor, takes the following form in the 2D case:

$$R = \frac{\gamma}{2} \text{tr}(\dot{Q}^2) = \gamma \dot{S}^2 + 4\gamma S^2 \dot{\phi}^2. \quad (7)$$

Here and below, the overdot denotes differentiation with respect to time. In the Onsager formalism, the generalized dissipative forces are balanced by the variational derivatives of the total free energy $F = \int_{\Omega} (f_{\text{bulk}} + f_{\text{elastic}}) d\Omega$ [11]:

$$\frac{\partial R}{\partial \dot{S}} = -\frac{\delta F}{\delta S}, \quad \frac{\partial R}{\partial \dot{\phi}} = -\frac{\delta F}{\delta \phi}. \quad (8)$$

Calculating the variational derivatives yields the following system of coupled relaxation equations:

$$2\gamma \frac{\partial S}{\partial t} = 2L\nabla^2 S - 2A \left(1 - \frac{U}{3}\right) S - 4AUS^3 - 8LS(\nabla\phi)^2, \quad (9)$$

$$8\gamma S^2 \frac{\partial \phi}{\partial t} = 8L\nabla \cdot (S^2 \nabla \phi). \quad (10)$$

The coefficient of $\partial\phi/\partial t$ is proportional to S^2 , so the director dynamics slow down in regions with a small order parameter. This property is particularly important when modeling topological defect cores.

2.4. Surface anchoring

The surface free energy is given by an integral over the boundary:

$$F_{\text{surface}} = \int_{\partial\Omega} f_{\text{surface}} dS. \quad (11)$$

This work compares two formulations of degenerate planar anchoring that translate boundary conditions into bulk equations via fluxes: the projection model (hereafter the FG model) and the direct target tensor model (DT model).

The FG model is based on the standard Fournier-Galatola approach [9]. The surface energy has the form:

$$f_{\text{surface}}^{FG} = W \left(\tilde{Q}_{ij} - \tilde{Q}_{ij}^{\perp} \right)^2, \quad (12)$$

where $W > 0$ is the anchoring energy,

$$\tilde{Q}_{ij} = Q_{ij} + \frac{1}{2} S_0 \delta_{ij}, \quad (13)$$

$$\tilde{Q}_{ij}^{\perp} = P_{ik} \tilde{Q}_{kl} P_{lj}, \quad (14)$$

$$P_{ij} = \delta_{ij} - \nu_i \nu_j, \quad (15)$$

ν_i are the components of the outward unit normal to the boundary. In the 2D case, the tensor P_{ij} projections onto the tangent direction $t_i = (-\nu_y, \nu_x)$. Substituting the parameterization of Q_{ij} leads to the expression for the torque $g_{\phi} = -\partial f_{\text{surface}} / \partial \phi$:

$$g_{FG} = -4WS \sin 2\phi (S \cos 2\phi \mp S_0), \quad (16)$$

where the “−” sign applies at boundary points where the equilibrium director is perpendicular to the normal (ellipse equator), and the “+” sign applies at points where the equilibrium director is parallel to the normal (pole).

Direct 2D anchoring (DT model) is based on explicitly specifying a target surface Q-tensor. The requirement that the director be tangent to the boundary yields the form:

$$Q_{ij}^{\text{surf}} = S_0 (2t_i t_j - \delta_{ij}). \quad (17)$$

In components relative to the global basis (x, y) , taking into account $t_x = -\nu_y$, $t_y = \nu_x$:

$$Q_{xx}^{\text{surf}} = S_0 (\nu_y^2 - \nu_x^2), \quad Q_{xy}^{\text{surf}} = -2S_0 \nu_x \nu_y. \quad (18)$$

The surface energy in the DT model is defined as a quadratic form of the difference between the actual and target tensors:

$$f_{\text{surface}}^{DT} = 2WS_0^2 (\Delta_{xx}^2 + \Delta_{xy}^2), \quad (19)$$

where the dimensionless residuals are:

$$\Delta_{xx} = \frac{S}{S_0} \cos 2\phi - (\nu_y^2 - \nu_x^2), \quad \Delta_{xy} = \frac{S}{S_0} \sin 2\phi + 2\nu_x \nu_y. \quad (20)$$

The corresponding torque is:

$$g_{DT} = 4WS_0^2 \frac{S}{S_0} (-\Delta_{xx} \sin 2\phi + \Delta_{xy} \cos 2\phi). \quad (21)$$

For $S = S_0$, the expression simplifies to the compact form:

$$g_{DT} = -2WS_0^2 \sin 2(\phi - \phi_{eq}), \quad (22)$$

where ϕ_{eq} is the angle of the tangent to the boundary at the given point. This form is used in Section 3 for analytical comparison with the projection model.

The obtained expressions for the restoring torque allow for an analytical comparison of the two anchoring models in a specific geometry. Below, this analysis is performed for an elliptical droplet.

The one-constant approximation is adopted in order to isolate the influence of the surface anchoring model on the relaxation dynamics. Although the elastic constants of 5CB are anisotropic, the same bulk elastic properties (Eq. (5)) are used for both anchoring models. Therefore, the observed differences arise solely from the form of the surface free-energy functional, as reflected in the corresponding expressions for the restoring torque (Eqs. (16) and (21)). The inclusion of elastic anisotropy would likely affect quantitative characteristics, such as relaxation times and defect trajectories, but does not affect the qualitative distinction between the anchoring models.

3. Boundary torque asymmetry: analytical derivation

Consider a 2D elliptical nematic droplet with semi-axes a (horizontal) and b (vertical), $a > b$. Degenerate planar anchoring is realized at the boundary, so the equilibrium director orientation at each point is tangent to the surface. Two characteristic boundary points define the limiting cases:

- the poles $(0, b)$ with normal $\nu = (0, 1)$, tangent $t = (1, 0)$, and equilibrium angle $\phi_{eq} = 0$;
- the equator $(a, 0)$ with normal $\nu = (1, 0)$, tangent $t = (0, 1)$, and equilibrium angle $\phi_{eq} = \pi/2$.

This section investigates the local structure of the restoring torque near the equilibrium director orientations at the boundary.

At the initial time, the director is uniform and oriented vertically. Such a configuration models the state of the droplet immediately after switching off a strong external electric field applied across the long axis of the ellipse. After the field is removed, the system must relax to an equilibrium bipolar structure, and the nature of this relaxation is determined solely by the properties of the surface anchoring.

Let us write the initial condition as:

$$\phi = \frac{\pi}{2} + \delta\psi, \quad \delta\psi \ll 1, \quad S \approx S_0, \quad (23)$$

where $\delta\psi$ is a small deviation arising from thermal fluctuations or residual field inhomogeneity.

Using expressions (16) and (22) for the restoring torques and expanding in the small parameter $\delta\psi$, we obtain the asymptotics for the two models. For the FG model, one has:

$$g_{FG}^{pole} \approx 16WS_0^2(\delta\psi)^3, \quad (24)$$

$$g_{FG}^{eq} \approx -16WS_0^2\delta\psi. \quad (25)$$

For the DT model, we obtain:

$$g_{DT}^{pole} \approx 4WS_0^2\delta\psi, \quad (26)$$

$$g_{DT}^{eq} \approx -4WS_0^2\delta\psi. \quad (27)$$

Thus, in the DT model, the torque remains linear in $\delta\psi$ in both cases and differs only in sign. Whereas in the FG model, the torque at the pole is cubic in $\delta\psi$, unlike the linear torque at the equator. For a characteristic $\delta\psi=0.1$, the ratio of the torque scales in the projection model is:

$$\frac{g_{DT}^p}{g_{FG}^p} \sim \frac{1}{(\delta\psi)^2} \sim 100. \quad (28)$$

Thus, direct reduction of the 3D Fournier-Galatola functional to the 2D case leads to a cubic dependence of the restoring torque at the poles of an elliptical droplet when starting from a vertical position.

Two qualitative predictions follow from the obtained asymptotics. First, the weakening of the restoring torque at the poles should increase the mobility of surface defects and facilitate their displacement along the boundary. Second, the influence of this effect should be most pronounced in droplets with moderate geometric anisotropy, where the competition between surface anchoring and boundary shape is most distinct. These assumptions are verified in the subsequent numerical experiments.

4. Numerical experiment setup

To test the analytical predictions of Section 3, numerical simulations of the director relaxation in a 2D elliptical droplet were performed after switching off an external electric field that initially oriented the director vertically. Two surface anchoring models are compared: FG (projection) and DT (direct 2D anchoring).

Calculations were carried out for two sets of semi-axes: a moderate ellipse ($a = 3.0 \mu\text{m}$, $b = 2.5 \mu\text{m}$, $a/b = 1.2$) and an elongated ellipse ($a = 6.25 \mu\text{m}$, $b = 2.5 \mu\text{m}$, $a/b = 2.5$), which corresponds to the mesoscale range typical for experimental PDLC systems. The second geometry corresponds to highly elongated droplets observed in polymer matrices [4], while the first allows for investigating the transition regime where the difference between the models is most pronounced.

The model parameters correspond to the 5CB nematic (Table 1). The anchoring energy is chosen in the strong anchoring regime ($W = 10^{-3} \text{ J/m}^2$, extrapolation length $\xi = L/W \ll a$). The initial state is specified by a vertical director with a small spatial modulation. The system of relaxation equations (Section 2.3) with the corresponding boundary conditions (Section 2.4) is integrated using the finite element method on a triangular mesh refined near the boundary. A second-order implicit time-stepping scheme is used. All quantities are non-dimensionalized using the scales $x_c = a$, $t_c = \gamma/A$, $S_c = S_0$, $f_c = AS_0^2$.

TABLE 1. Model parameters

Parameter	Symbol	Value
Thermodynamic parameter	U	3.5
Energy scale	A	$1.17 \cdot 10^5 \text{ J/m}^3$
Elastic constant	L	6 pN
Anchoring energy	W	10^{-3} J/m^2
Equilibrium order parameter	S_0	0.154
Rotational viscosity	γ	0.1 Pa·s

To prevent numerical instabilities associated with the abrupt onset of boundary conditions, the surface fluxes defined by expressions (16) and (22) were introduced smoothly. During the interval $0 \leq t \leq t_{\text{ramp}}$, they were multiplied by a monotonically increasing function $\text{ramp}(t) = t/t_{\text{ramp}}$, and for $t > t_{\text{ramp}}$, they took their full values. The ramp time is $t_{\text{ramp}} = 0.01 \text{ s}$, which in dimensionless units corresponds to approximately 10^4 .

5. Results and discussion

The numerical simulation results for the two droplet geometries and the two surface anchoring models are presented in Figs. 1-3. In all cases, after the external field is turned off, the director relaxes to a bipolar configuration; however, the nature of this relaxation and the final state qualitatively depend on the choice of the anchoring model and the aspect ratio of the ellipse semi-axes.

For moderate elongation ($a/b = 1.2$), the difference between the models is most pronounced. In the FG model, the bipolar axis is oriented horizontally (Fig. 1a): two surface defects – boojums, identified by the regions $S \rightarrow 0$ – are localized near the ends of the major axis of the ellipse at the equator, at points $(\pm a, 0)$ (Fig. 1c). In the DT model, the bipolar axis is oriented vertically (Fig. 1b), and the boojums are located at the poles $(0, \pm b)$ (Fig. 1d). Thus, with identical parameters and identical initial states, the choice of the surface anchoring model leads to different equilibrium orientations of the bipolar structure.

For strong elongation ($a/b = 2.5$), both models yield a horizontal bipolar structure (Fig. 2a,b). In both cases, boojums are localized near the ends of the equator, at points $(\pm a, 0)$ (Fig. 2c,d).

The dynamics of the free energy F are shown in Fig. 3 on a logarithmic scale of dimensionless time t/t_c . In all cases, three characteristic regions can be distinguished on the plots. The first region is a rapid energy drop by several orders of magnitude (up to $t/t_c \sim 10^{-2} - 10^{-1}$), corresponding to the initial stage of relaxation. The second region is an intermediate plateau in the interval $t/t_c \sim 10^{-1} - 10^4$, where the energy changes weakly. These two regions span the entire period of the smooth introduction of the boundary conditions (ramping), which is completed by $t/t_c \sim 10^4$ ($t = 0.01 \text{ s}$). After full anchoring is turned on, the energy increases to values corresponding to the true free energy minimum under fully developed boundary conditions, and levels off at a final plateau for $t/t_c > 10^4$.

For $a/b = 1.2$, the DT model curve, upon leaving the intermediate plateau, reaches the final stationary level $F \approx 2.1 \times 10^{-2} \text{ J}$. In the FG model, the transition from the intermediate plateau reaches a value of $F \approx 1.1 \times 10^{-2} \text{ J}$, which is 48% lower than in the DT model.

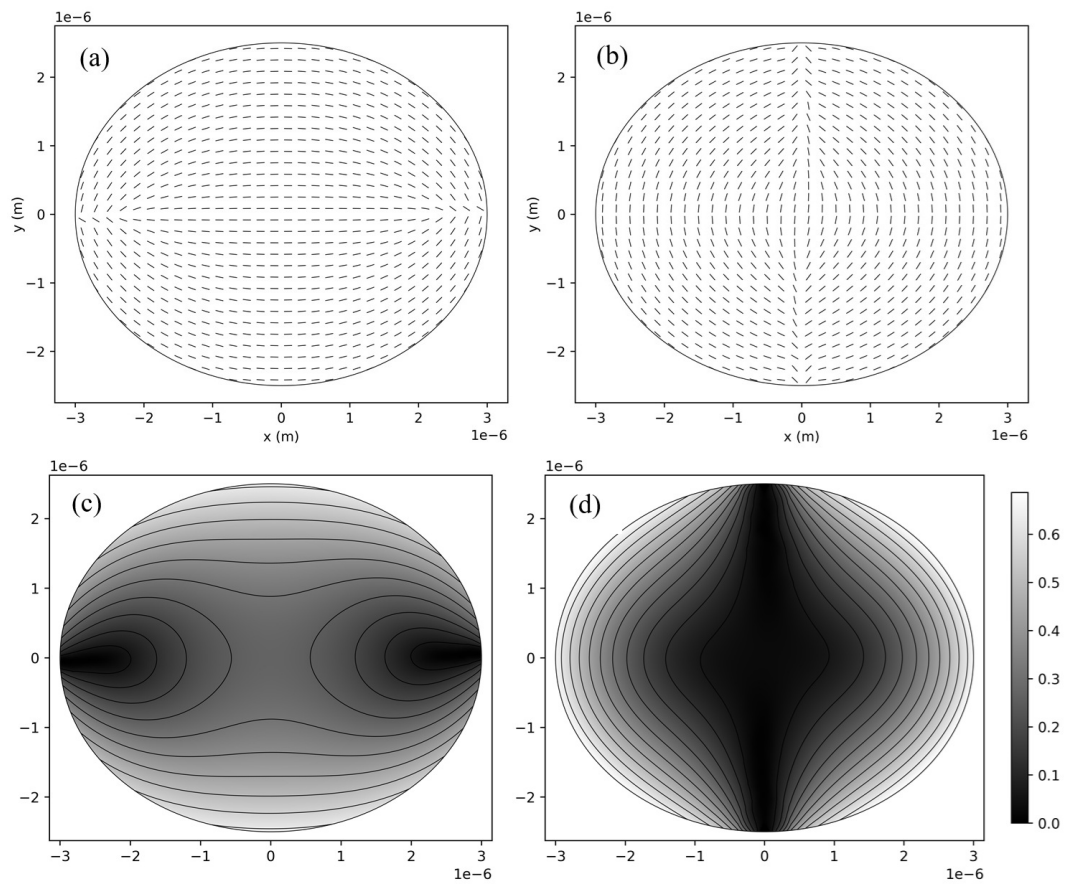


FIG. 1. Director field and order parameter for a moderately elongated droplet: FG model (a, c); DT model (b, d)

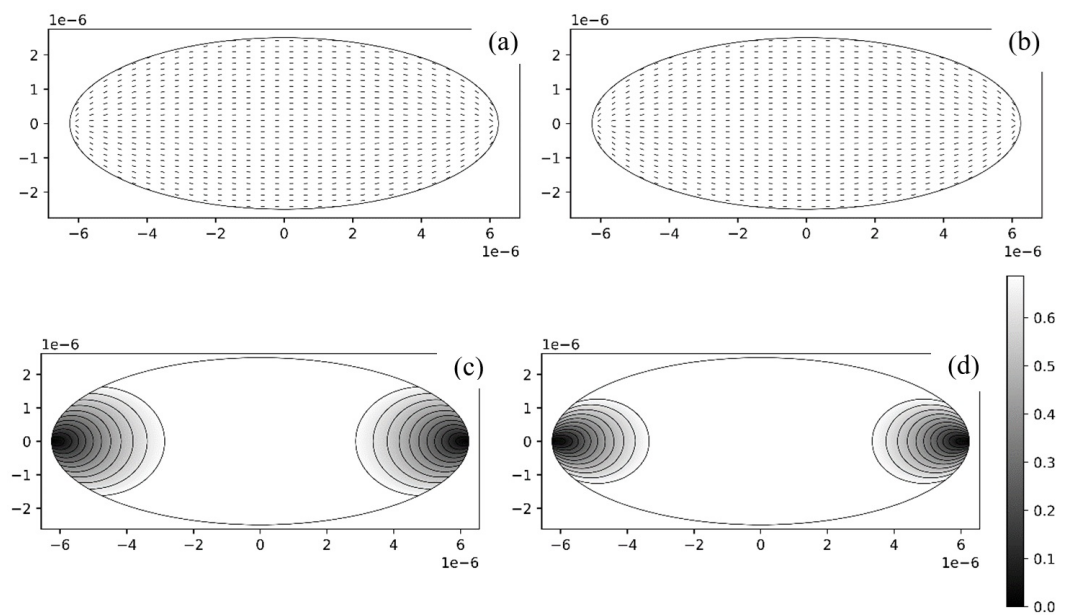


FIG. 2. Director field and order parameter for a strongly elongated droplet: FG model (a, c); DT model (b, d)

For $a/b = 2.5$, the DT model curve reaches a plateau $F \approx 1.7 \times 10^{-2}$ J. The FG model again yields a lower final value $F \approx 1.1 \times 10^{-2}$ J – a difference of about 35%.

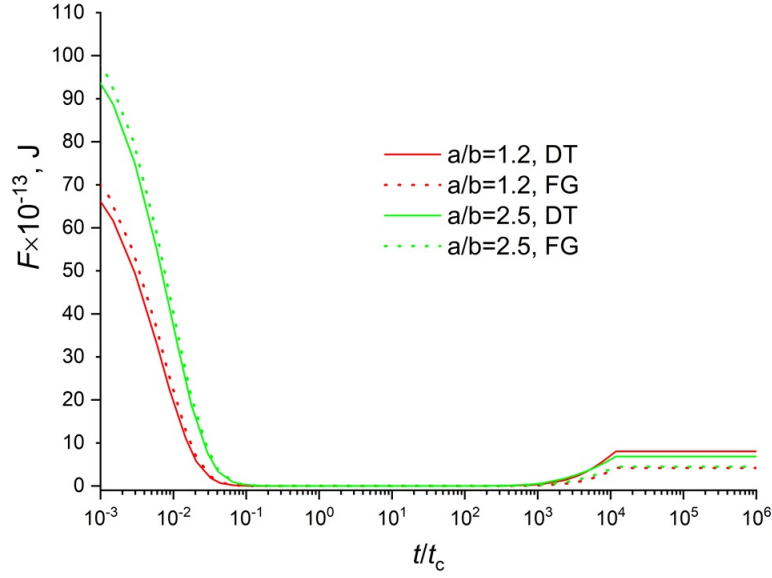


FIG. 3. Free energy relaxation plot for two droplet geometries and two anchoring models on a logarithmic time scale

Analytical estimates combined with numerical modeling provide a clear physical picture of how the form of surface anchoring affects director relaxation in an elliptical droplet. It was previously shown that in the FG model, the restoring torque at the ellipse poles for a vertical start is cubic with respect to the small deviation $\delta\psi$, whereas in the DT model it is linear. Numerical experiments indicate that this difference plays a dominant role in determining the mobility of surface defects and, consequently, the orientation of the resulting bipolar structure.

Regardless of the chosen anchoring model and droplet elongation, the early stage of relaxation is universal. As the boundary conditions are smoothly applied (ramping), the equatorial torque, which is linear in both models, keeps the director near the vertical at the equator, and a vertical bipolar configuration with two boojums at the poles forms in the system. This stage concludes around $t/t_c \sim 300$.

The subsequent system behavior is governed by the migration of boojums along the boundary from the poles to the equator, driven by geometric anisotropy. The long sides of the ellipse create a larger boundary area for which tangential orientation is energetically favorable, and in the absence of a restraining force, the boojums shift, transforming the vertical bipolar configuration into a horizontal one.

In the FG model, the weak cubic polar torque provides substantially less resistance to boojum migration than the linear torque of the DT model. At both aspect ratios, boojums move along the boundary and reach the equator by the time anchoring is fully applied, forming a horizontal bipolar structure.

In the DT model, the linear polar torque actively resists boojum migration. At moderate elongation ($a/b = 1.2$), this resistance completely blocks the movement: the boojums remain at the poles throughout the entire simulation time, and the vertical bipolar structure is preserved as a stable state. At strong elongation ($a/b = 2.5$), the geometric anisotropy is large enough to overcome the linear torque, and the boojums migrate to the equator, forming a horizontal bipolar structure.

The significant difference in equilibrium energies indicates that the considered models lead to different stationary states. Since the respective configurations differ in boojum placement and bipolar axis orientation, these results point to a substantial modification of the system's energy landscape when transitioning from the DT formulation to the 2D reduction of the Fournier-Galatola model. A more detailed analysis of the structure of the respective minima requires a separate investigation.

The obtained results are consistent with previous studies showing that increasing aspect ratio promotes defect displacement toward the ends of the major axis [11]. However, unlike previously published works, the present study demonstrates that the choice of a specific degenerate planar anchoring model can alter not only the quantitative relaxation characteristics but also the orientation of the final bipolar structure.

Numerical calculations indicate that the direct use of the 3D Fournier-Galatola projector in 2D models leads to a qualitative difference in relaxation dynamics compared to the DT model: at moderate elongation, a horizontal bipolar structure forms instead of a vertical one, and for any geometry, the final energy is systematically underestimated. The DT formulation preserves a linear restoring torque along the entire boundary and may therefore provide a useful alternative framework for quasi-2D nematic modeling.

6. Conclusion

This work analyzes the validity of directly transferring the 3D formulation of Fournier-Galatola degenerate planar anchoring to 2D models of nematic liquid crystals. It is analytically shown that reducing the 3D projector to the 2D case yields a cubic dependence of the restoring torque on the director deviation at the poles of an elliptical droplet when starting from a vertical position. In the direct target tensor model, the torque remains linear at all boundary points. For characteristic deviations, the cubic torque in the FG model is roughly two orders of magnitude weaker than the linear torque of the DT model.

Numerical modeling of director relaxation in elliptical 5CB droplets with aspect ratios $a/b = 1.2$ and 2.5 revealed that this difference directly dictates the mobility of surface boojums and, consequently, the orientation of the final bipolar structure. In all cases, a vertical bipolar configuration forms during the early stage. In the FG model, the cubic polar torque does not provide substantial resistance to boojum migration toward the ends of the major axis, and the system transitions to a horizontal bipolar structure at any elongation. In the DT model, the linear polar torque blocks migration at $a/b = 1.2$, maintaining the vertical bipolar configuration, whereas at $a/b = 2.5$ the geometric anisotropy is strong enough to induce migration in this case as well.

The equilibrium energy in the FG model is systematically lower than in the DT model (by 35–48%), indicating a structural difference in the achieved minima. The findings demonstrate that directly utilizing the 3D projector in a 2D framework yields results that qualitatively differ from those of the direct target tensor model. The DT formulation, wherein the restoring torque is linear along the entire boundary, provides a more reliable basis for modeling quasi-2D nematic systems. A quantitative comparison of the predicted director dynamics with the experimental data reported in Ref. [4], including relaxation times and defect evolution, constitutes an important direction for future research.

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