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Original article

Three-dimensional extremely short optical pulses in a phonic crystal with a superlattice

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ABSTRACT Based on Maxwell's equations in Coulomb calibration, which describe the dynamics of extremely short optical pulses in a photonic crystal with a three-dimensional superlattice connected by strong tunneling along one axis, a phenomenological equation was obtained in the form of the classical $2+1$-dimensional sine-Gordon equation with variable coefficients for the case of cylindrical symmetry. The steady propagation of such pulses is established, as well as the dependence of the evolution of these pulses on the parameters of the photonic crystal.

KEYWORDS superlattice, extremely short optical pulses, photonic crystal.

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1. Introduction

One of the key tasks of nonlinear optics, nanophotonics and nanoelectronics and, as a consequence, based on their latest advances in science and technology, is to reduce the size of structural elements of devices, as well as to create new materials with which it will be possible to control the parameters of optical signals, as well as create their basis is the system of transmission, processing and storage of data. One of the suitable materials are the so-called superlattices. These are structures in which, in addition to the lattice, there is an artificially created potential (with a period significantly exceeding the lattice period) acting on electrons. Thus, it is possible to control the zone spectrum. One of the simplest ways to create a superlattice is the formation of a periodic system of quantum wells in a solid, which are bound by the tunnel effect, and as a result, collectivized states of electrons with a particular law of dispersion are formed. In this case, it is possible to control the size of the mini-zone, due to changes in the distance between quantum wells [1, 2].

As a medium for the formation of a superlattice, a photonic crystal with a spatially variable refractive index can be used. The choice of a system of quantum wells with tunneling as a material for a photonic crystal is due to the non-parabolic law of electron dispersion, which in turn determines the non-linearity of the response to the influence of electromagnetic moderate strengths, starting from the values of 10^3–10^4 V/cm [3, 4]. It should be noted that the inhomogeneity of the photonic crystal provides an ideal medium for the propagation of extremely short optical pulses, light bullets, and other soliton – like states [5–7].

Let us pay attention to three-dimensional extremely short optical pulses, which are localized in space electric field pulses with a duration corresponding to several periods of field oscillation, and all the energy of which remains concentrated in a finite limited region of space [8, 9].

The study of the propagation of electromagnetic pulses in photonic crystals is of great theoretical and practical importance in modern nonlinear optics and nanophotonics, so the problem considered in this paper is very relevant [10].

2. Physical model and basic equations

The geometry of the problem (Fig. 1) assumes that the current, the applied electric field, and the electric field of the pulse are directed along the *OY* axis, and the pulse moves along the *OZ* axis. Quantum dots form a system of quantum wells placed in a photonic crystal. It is assumed that the tunneling between quantum wells along the *OX* and *OZ* axes is small and can be neglected. It should be noted that the paper considers pulses containing few electromagnetic field cycles (1–5), so the longitudinal size of the wave packet is only a small number of wavelengths. That is, the wave packet under consideration is a light pancake with transverse dimensions significantly exceeding the longitudinal ones.

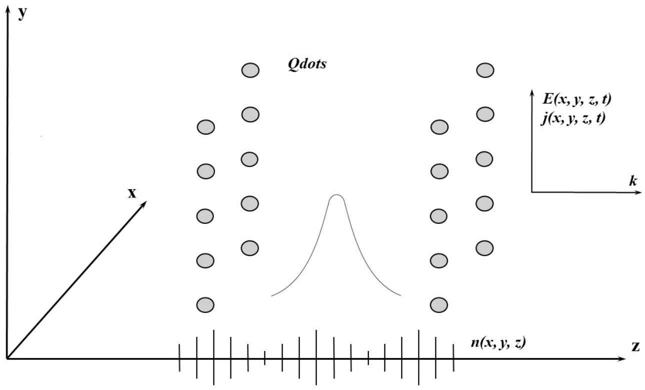


FIG. 1. Problem geometry

The Hamiltonian of a system of electrons, taking into account the geometry presented above can be represented in the form:

$$H = \sum_{jkl} t_0 a^{+}_{jkl\sigma} a_{jkl\sigma} + t\left(a^{+}_{jkl\sigma} a_{jkl\sigma+1} + a^{+}_{jkl\sigma+1} a_{jkl\sigma}\right) \tag{1}$$

where $a^{+}_{jkl\sigma}$ and $a_{jkl\sigma}$ – are the electron creation and annihilation operators in the quantum well with coordinates j, k, l along the axes *OX*, *OY*, and *OZ*, respectively, t_0 – is the electron energy of the quantum well, t – is the tunneling integral determined by overlapping of the electron wave functions in adjacent wells.

Thanks to the Fourier transform for the operators $a^{+}_{jkl\sigma}$ and $a_{jkl\sigma}$, one can diagonalize the Hamiltonian H from (1) and obtain the dispersion relation for electrons in the superlattice, which is clearly shown, for example, in [3, 4, 15]:

$$\varepsilon(p) = t_0 + 2t \cdot \cos(ap), \tag{2}$$

where p – angular momentum, which is directed along the *OY* axis, a – the distance between adjacent quantum wells (along the *OY* axis).

It can be noted that a spectrum similar to (2) can be recorded under the conditions of electron-electron and electron-photon interactions, if t_0 and t are understood as the corresponding renormalized constants.

Using Maxwell's equations, using the Coulomb gauge ($E = -\partial$A/c∂t), the equation for the vector potential of the electromagnetic field of a three-dimensional extremely short optical pulse propagating inside a photonic crystal with a superlattice, in accordance with the geometry (Fig. 1), presented above will have the form [11]:

$$\frac{\partial^2 \mathbf{A}}{\partial x^2} + \frac{\partial^2 \mathbf{A}}{\partial y^2} + \frac{\partial^2 \mathbf{A}}{\partial z^2} - \frac{n^2(x,y,z)}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} + \frac{4\pi}{c}\mathbf{j} = 0, \tag{3}$$

where $A = (0, A(x,y,z,t), 0)$ – vector potential of the pulse electric field, $j = (0, j(x,y,z,t), 0)$ – electric current density, $n(x,y,z)$ – spatial change in refractive index, c – speed of light.

When constructing a model for the propagation of an extremely short pulse in a photonic crystal based on a superlattice, the following approximations were used: the electric field of the substrate is not taken into account; the continuum approximation is used; the length at which the refractive index of the photonic crystal changes is much larger than the spatial size of the pulse localization region; the Coulomb gauge is used to describe the magnetic and dielectric properties of the medium.

Since the typical relaxation time for electrons can be estimated as 10^{-12}–10^{-13} s [12]. The following approximation was used in equation (3): we neglected the term responsible for the inhomogeneity of the electron distribution function along the z axis. This is due to the general statement of the problem, in which the initial conditions along the z axis are chosen to be homogeneous, and the effects associated with the inhomogeneity of the electromagnetic field along this axis were not taken into account. In our opinion, this is justified by the introduced assumption about the homogeneity of the laser radiation wavefront.

The evolution of an ensemble of particles will be described by the classical kinetic Boltzmann equation in the relaxation time approximation:

$$\frac{\partial f}{\partial t} - \frac{q}{c}\frac{\partial A}{\partial t}\frac{\partial f}{\partial p} = \frac{F_0 - f}{\tau}, \tag{4}$$

where f – distribution function that has an implicit dependence on the coordinate; F_0 – Fermi distribution function, τ – relaxation time, q – charge, A – component of the vector potential of the electric field of the pulse.

For the current density, we use the standard expression [3]:

$$j = 2e\sum_{s=1}^{m}\int_{ZB} v_s(p)\cdot f(p,s)\,dp, \tag{5}$$

where the group velocity of electrons is introduced $v_s(p) = \dfrac{\partial\varepsilon_s(p)}{\partial p}$, $\varepsilon_s(p)$ – dispersion law describing the electronic properties of a system of quantum wells (equation (2)), e – electron charge. Integration is carried out over the first Brillouin zone.

Taking into account the formula for the current density (equation 5), we can obtain the generalized sine-Gordon equation [13], which describes the dynamics of an extremely short optical pulse in a photonic crystal with a superlattice:

$$\begin{aligned}
&\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} - \frac{n^2(x,y,z)}{c^2}\frac{\partial^2 A}{\partial t^2} + \frac{qb}{\pi h\tau}\sin\left(\frac{aq}{c}A\right) = 0,\\
&b = \int_{-q0}^{q0} dp\cdot\cos(ap)\,F_0(p),\\
&F_0(p) = \frac{1}{1+\exp\{\varepsilon(p)/k_bT\}},
\end{aligned} \tag{6}$$

where k_b – Boltzmann constant, T – temperature, q_0 – boundaries of the first Brillouin zone.

A detailed derivation of equation (6) is shown in earlier works of the authors [14, 15]. Since the effect of charge accumulation can be neglected [16], it can be assumed that the cylindrical symmetry in the field distribution is preserved. Taking into account the above, the final equation for the vector potential in a cylindrical coordinate system will take the form:

$$\begin{aligned}
&\frac{\partial^2 A}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right) - \frac{n^2(z,r)}{c^2}\frac{\partial^2 A}{\partial t^2} + \frac{qb}{\pi h\tau}\sin\left(\frac{aq}{c}A\right) = 0,\\
&r = \sqrt{x^2+y^2}.
\end{aligned} \tag{7}$$

The initial conditions for the vector potential correspond to the Gaussian profile of the pulse for one oscillation of the electric field and the refractive index of the medium of a photonic crystal made of CNTs are set as follows:

$$A_{t=0} = A_0\exp\left\{-\frac{r^2}{\gamma^2}\right\}\exp\left\{-(z-z_c)^2/\beta^2\right\},$$

$$\left.\frac{dA}{dt}\right|_{t=0} = \frac{2vz}{\gamma^2}A_0\exp\left\{-\frac{r^2}{\gamma^2}\right\}\exp\left\{-(z-z_c)^2/\beta^2\right\},$$

$$n(z,r) = 1 + \alpha\cos(2\pi z/\chi).$$

Here β, γ – parameters defining the pulse width of the axes z and r, respectively, A_0 – initial pulse amplitude, v – initial pulse velocity when entering the medium, z_c – start coordinate, α – refractive index modulation depth, χ – refractive index modulation period.

3. Numerical simulation results

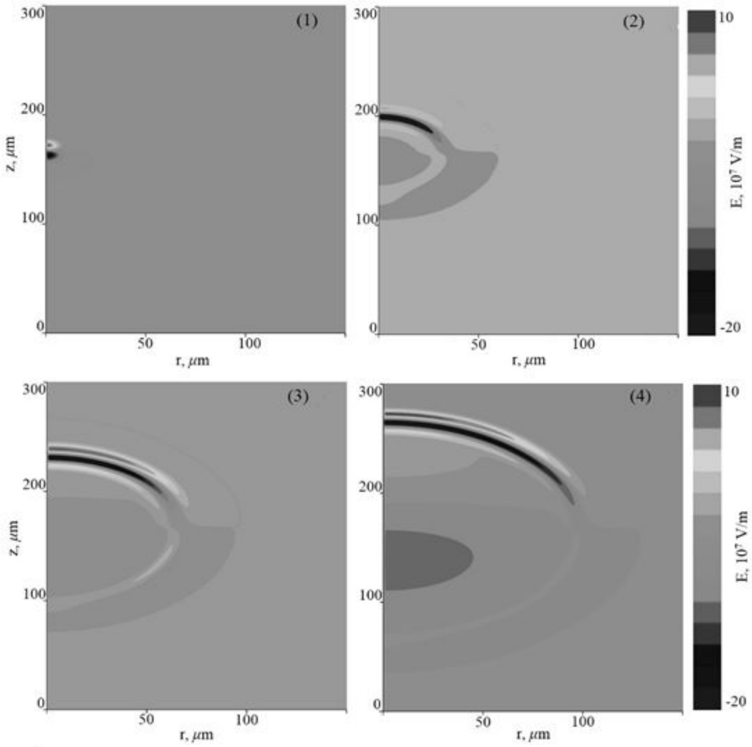
The investigated equation (7) was solved numerically using an explicit difference scheme of the "cross" type [17]. In numerical simulation of the system under study, its parameters were chosen as follows: $m = 13$, $T = 293$ K, relaxation time in the superlattice $\approx 10^{-11}$ s; pulse duration $\approx 10^{-14}$ s. The values of the parameters that determine the pulse width, as well as the initial pulse velocity upon entering the medium, were set as follows ($\beta = \gamma = \sqrt{1-v^2}$, $v = 0.95c$).

The evolution of an electromagnetic field pulse during its propagation in a medium with a spatially variable refractive index (photonic crystal) with a superlattice, in the case of one electric field oscillation, is shown in Fig. 2.

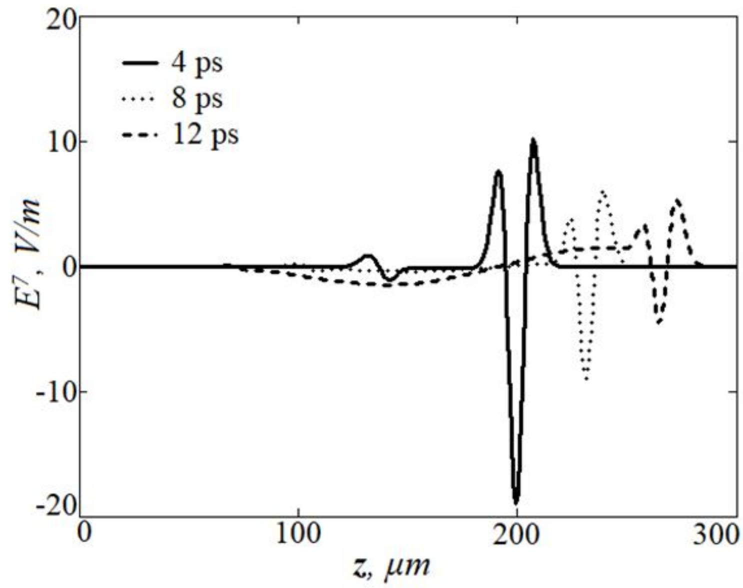
The figure presented earlier, we can conclude that the pulse energy remains localized in a limited spatial area, that the impulse spreads steadily. However, due to diffraction and dispersion effects, there is a damping effect, and a "tail" appears at the trailing edge.

The evolution of a three-dimensional extremely short optical pulse, at a time instant of 4 ps, in a photonic crystal with a superlattice, with various parameters of the photonic crystal, is shown in Fig. 3,4.

It can be seen from Fig. 3,4 that, by changing the modulation depth of the refractive index, the pulse shape changes, but its energy remains in a limited spatial region. In turn, by varying the modulation period of the refractive index, it is possible to control the change in the group velocity of the wave packet of an extremely short optical pulse. Moreover, the longer the period, the faster the impulse spreads. This is due to the fact that the processes of reflection and interference occur less frequently. Note that from a practical point of view, this result is important because it allows one to control the pulse rate by changing the parameters of the photonic crystal. At the same time, the propagation of extremely short optical pulses in a photonic crystal with a superlattice has a number of important differences from the case of a medium with a constant refractive index. Perhaps the most important difference is that pulses in such a medium have a more complex



(A)



(B)

FIG. 2. Problem geometry A) Dynamics of a three-dimensional extremely short optical pulse in a photonic crystal with a superlattice, at different times (refractive index modulation parameters: modulation depth $\alpha = 0.1$; modulation period $\chi = 2.5$ μm): 1) 0.2 ps, 2) 4 ps, 3) 8 ps, 4) 12 ps; B) Cuts of an extremely short optical pulse at different times (4 ps, 8 ps, 12 ps)

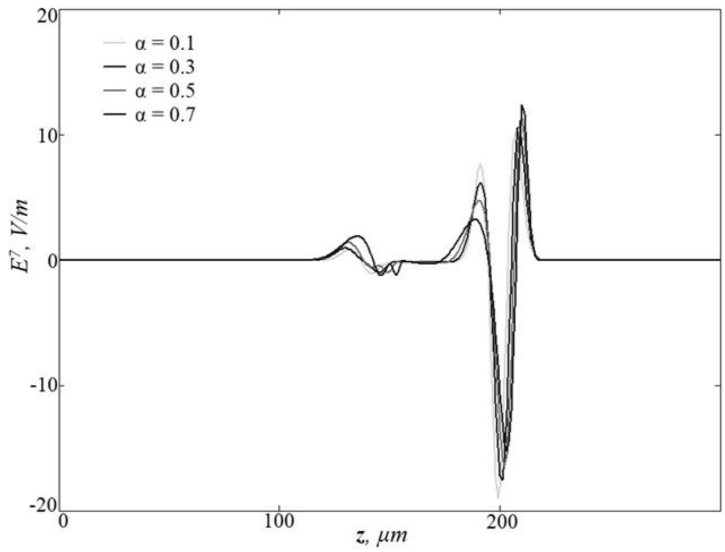


FIG. 3. Sections of a three-dimensional extremely short optical pulse in a photonic crystal made of CNTs with a superlattice, at a time instant of 4 ps, with different values of the modulation depth of the refractive index, at a fixed modulation period $\chi = 2.5$ μm: $\alpha = 0.1$, 0.3, 0.5, 0.7

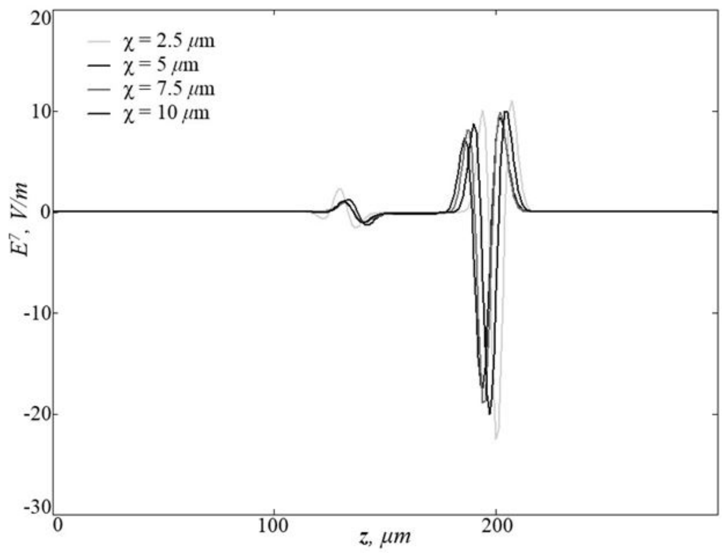


FIG. 4. The dynamics of propagation of a three-dimensional extremely short optical pulse in a photonic crystal of CNTs with a superlattice, at a time instant of 4 ps, with different values of the refractive index modulation period, at a fixed modulation depth $\alpha = 0.3$: $\chi = 2.5$ μm, 5 μm, 7.5 μm, 10 μm

transverse structure, which, in our opinion, is associated with the excitation of internal vibration modes of an extremely short pulse when interacting with an inhomogeneity of the refractive index of the medium.

The influence of the parameters of a photonic crystal on the shape and group velocity of the wave packet of an extremely short pulse has been repeatedly tested for extremely short pulses and light bullets of various initial shapes, including Gaussian, super-Gaussian, as well as on diffractionless pulses, Bessel and Airy cross sections, in the case of carbon nanotubes with a similar dispersion law for electrons [18–20].

4. Conclusions

1. A physical model was constructed that describes the dynamics of a three-dimensional extremely short optical pulse in a photonic crystal with a superlattice (a system of quantum wells that are coupled by tunneling).

2. It was found that pulses in such a medium propagate stably, keeping their energy in a limited spatial region. The shape of the pulse undergoes insignificant changes; after its passage, a "tail" is formed, due to the influence of dispersion and diffraction effects.

3. The period and depth of modulation of the refractive index affect the shape and group velocity of an extremely short optical pulse.

The results obtained in this work are important from a theoretical and practical point of view, since they can be applied to create, for example, all-optical delay lines, as well as form the basis of the elemental base of optoelectronic and nanoelectronic devices, storage devices, transmission and processing of data, which in its turn is relevant for infocommunication technologies.

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