NANOSYSTEMS:
PHYSICS, CHEMISTRY, MATHEMATICS

Miroshnichenko G.P., et al. *Nanosystems: Phys. Chem. Math.*, 2022, **13** (6), 615–620.
http://nanojournal.ifmo.ru
DOI 10.17586/2220-8054-2022-13-6-615-620

Original article

Investigation of the method of current thermal modulation of the wavelength VCSEL

George P. Miroshnichenko[a], Alina N. Arzhanenkova[b], Michail Yu. Plotnikov[c]

ITMO University, St. Petersburg, Russia

[a]gpmirosh@gmail.com, [b]11arzh11@gmail.com, [c]plotnikov-michael@yandex.ru

Corresponding author: G. P. Miroshnichenko, gpmirosh@gmail.com

ABSTRACT **Subject of investigation.** In this paper, we analyzed in detail the thermal mode of the operation of a vertical-cavity surface-emitting laser (VCSEL) used in the experiment. Method. As a part of the work, we carried out a theoretical study of the recurrence relation describing the change in the VCSEL wavelength under the action of specially selected modulation current pulses. The high speed of the device working is determined by the optical demodulation scheme, which is based on using a phase-modulated carrier (the demodulation method used is arctangent demodulation: Phase Generated Carrier (PGC-ATAN)). **Main results.** Formulas were obtained that determine the frequency, phase, and modulation depth which leads to calculation of the principle of change in the modulated VCSEL wavelength at sampling points. Comparison with experimental data showed that the obtained formulas allow one to choose the optimal thermal mode of VCSEL operation and reliably calculate the characteristics of the modulation process in terms of the carrier phase. **Practical significance.** The obtained formulas make it possible to calculate the exact characteristics of the modulation process, and the precisely calculated phase of the modulation source. As a result, one can compensate it more effectively when demodulating phase of the interferometer.

KEYWORDS VCSEL, modulation, interferometric measurements, PGC- demodulation, PGC-ATAN.

FOR CITATION Miroshnichenko G.P., Arzhanenkova A.N., Plotnikov M.Yu. Investigation of the method of current thermal modulation of the wavelength VCSEL. *Nanosystems: Phys. Chem. Math.*, 2022, **13** (6), 615–620.

1. Introduction

Surface-emitting lasers with a vertical cavity (VCSEL) at a wavelength of 1550 nm is used not only in the telecommunications field, but also in constructing various fiber-optic sensor systems, in particular, based on Bragg gratings. Important problems in the development of such systems are as follows: the choice of the laser source of optical radiation, analysis of the noise of the measuring system, and the choice of the modulation method for the optical carrier. Reliable operation of fiber-optic interferometric measuring systems requires a small-sized source of optical radiation with the possibility of direct frequency modulation and the formation of optical pulses without external modulators. The aim of this work is to explore the thermal characteristics of VCSEL and analytically derive the parameters of the radiation source modulation principle. To select optimal method for modulating frequency of the radiation source, it is necessary to research principles of VCSEL operation when modulating current pulses of various shapes, with different durations and periods, and the possibility of controlling central wavelength. Source wavelength modulation is used to measure the phase produced in fiber optic interferometers. The construction of interferometers requires various metamaterials used in the parts of the interferometer. Particularly, one can mention mirrors in fiber-optic Fabry-Perot interferometer with certain metamaterials, the composition and properties of which can be found in article [1].

2. Review of previous studies

Surface-emitting lasers with vertical cavity form a separate type of semiconductor laser sources. The idea of creating such devices was proposed by the Japanese researcher Kenichi Iga in 1977 [2]. Due to a number of advantages, VCSEL finds application in modern radio-photonic devices designed for radio-electronic and combined systems of telecommunication and radar purposes, as well as in the development of various fiber-optic sensor systems. Modern VCSELs can operate in the wavelength range from 410 nm to 1000 nm. One observes the great interest to the sources in the telecom range – 850, 1300, 1550 nm. With rapid development of wireless communication networks, there is also an increasing need for simple, power-efficient and cost-effective transmission and distribution of radio frequency signals over optical fibers [2]. A theoretical analysis of VCSEL laser generation process at 1550 nm was carried out in [2–10]. For this purpose, the non-linear Statz-DeMars equations have been studied. These equations describe the dynamics and interaction of the carrier density and generation photon density under the action of the pump current for a set of VCSEL parameters at a wavelength of 1550 nm at a fixed temperature. Here parameters and combinations of parameters are defined that can be directly related to the experiment. Steady-state solutions and threshold values for the carrier density are found.

The frequency response of a laser transmitter when modulating a small signal is determined under the usual assumption of harmonic current modulation superimposed on a DC offset above the threshold value. The transfer function is found in frequency domain in the low-signal and the strong-signal approximations for modulation frequencies in the high-frequency GHz range. The FM response in the domain of high modulation frequencies from 10 MHz to 5.2 GHz is due to the carrier effect.

The theory of VCSEL modulation characteristics in the low frequency domain (up to several megahertz) is more complex. VCSEL allows one to tune the wavelength using the laser pump current. Up to a few megahertz, wavelength tuning is mostly a thermal effect: the current causes the VCSEL chip to heat up, and, as a result, the increasing of the refractive index of the material leads to the increase of the laser wavelength. To construct a theory for the dynamic processes under these conditions, it is necessary to improve the system of Statz-DeMars rate equations by adding the heat conduction equation with heat sources. The temperature model of VCSEL was described in works [2–7]. This model takes into account the effect of temperature increase in active domain depending on the injection current and the ambient temperature, and takes into account the temperature dependence of the gain, the pump current, and the barrier transparency. On the other hand, the frequency modulation at low modulation frequency exists due to the temperature modulation effect, which is successfully described by the dynamic thermal equation.

Studies of the thermal characteristics of VCSEL, as well as the principles of its operation, were done in works [2–4]. The coherent properties of the VCSEL radiation were also studied here. The experimental method for obtaining the coherence function of a laser diode with a vertical cavity is also used to obtain the dependences of the change in the central radiation wavelength and the width of the spectral components with direct current modulation of the laser at various frequencies and with various inverse duty cycles.

The method of homodyne demodulation based on the calculation of the values of arctangent function (PGC-ATAN) is one of the methods for processing the interference signals of fiber optic systems, where signal that enters the demodulation circuit has the following form:

$$\mathrm{I}_{in} = A + B \cdot \cos\left[C \cdot \cos\left(2 \cdot \pi \cdot \nu_0 \cdot t + \varphi_{gen}\right) + \varphi(t)\right]. \tag{1}$$

Here A and B are constants that are proportional to the power of the optical radiation, C is the depth of phase modulation, v_0 is the frequency of the signal from the reference oscillator, $\varphi(t)$ is the measured phase signal, φ_{gen} is the phase shift of the reference oscillator signal, which plays the role of the VCSEL modulation parameter. We need to determine φ_{gen} analytically, since inaccurate phase determination leads to unwanted demodulation errors, such as, for example, incorrect determination of the amplitude of the measured phase signal [11]. Articles [11–13] describe ways of finding the exact value φ_{gen}. It is proposed to look for the time delay between the signal from the reference oscillator and the carrier component of the interference signal after the bandpass filter [12]. In [13], it is suggested to replace the main carrier and the second harmonic with the meander of the carrier and the second harmonic, respectively.

3. The principle of VCSEL wavelength modulation using a train of pump pulses

For the operation of the fiber-optic interferometric measuring systems that do not use an external modulator, it is necessary to know the principle of the change in the VCSEL wavelength with time due to the internal modulation. As has been noted above, such formula can be obtained by solving the coupled Statz-De Mars equations and the heat conduction equations [14–20]. These equations are parametrized by several source parameters, some of which are taken from the experimental data. Instead of this method, in the patent [21], a phenomenological approach is proposed. The authors used the experimentally obtained speed characteristics of the change in wavelength due to heating and cooling. The recurrent relations were derived that relate changes in the wavelength in the adjacent time intervals. In our work, these relations were solved explicitly. Let's divide the time axis into intervals $k = 0, 1, ..$, with the length of the time discretization $T_d = \nu^{-1} = 10^{-6}$ sec, where ν is the discretization frequency. In each interval k, one can observe two processes: 1 – VCSEL heating using a modulation current pulse with a duration T_k, 2 – VCSEL cooling after switching off the pulse for time T_d–T_k. The duration of the heating is adjustable and depends on the number of the interval according to the law of changing the duty cycle of the pulses:

$$T_k = T_{in} + T_a \cdot \sin\left(2 \cdot \pi \cdot \nu_0 \cdot T_d \cdot k\right) \tag{2}$$

Figure 1 is a plot of VCSEL wavelength during heating for various modulation currents [22–24].

These experimental curves make it possible to obtain the dynamic characteristics of thermal processes in the laser. Wavelength change by $\Delta\lambda$ nm in the heating process can be described by the formula:

$$\Delta\lambda = a_1 \cdot \left(1 - \exp\left(-b_1 \cdot t\right)\right) \tag{3}$$

Here the parameters obtained from the experiment are introduced: the heating rate b_1 sec^{-1} and the frequency shift due to heating a_1 nm. The wavelength shift $\Delta\lambda_k$ at the k-th interval (at the end of heating) is described by the following formula

$$\Delta\lambda_k = a_1 \cdot \left(1 - \exp\left(-b_1 \cdot T_k\right)\right) \tag{4}$$

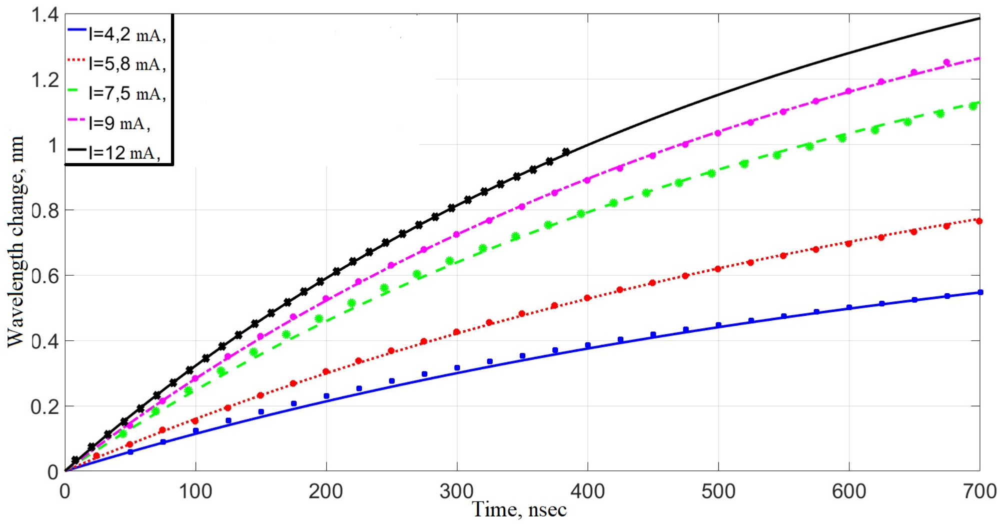

FIG. 1. VCSEL wavelength changes for different modulation amperages

The maximum wavelength shift $\Delta\lambda_k^{\max}$ at the k-th interval is as follows

$$\Delta\lambda_k^{\max} = \Delta\lambda_k^{\min} + \Delta\lambda_k \tag{5}$$

The VCSEL law of cooling at the k-th interval (after the heating stage) is determined by the speed parameter b_2 sec^{-1}.

$$\varepsilon = \exp\left(-b_2 \cdot t\right) \tag{6}$$

Parameter ε at the end of the k-th interval of the cooling stage is as follows:

$$\varepsilon_k = \exp\left(-b_2 \cdot (T_d - T_k)\right) \tag{7}$$

At the end of the k-th interval (at the beginning of the$(k+1)$-th interval) the minimal wavelength shift $\Delta\lambda_{k+1}^{\min}$ is as follows:

$$\Delta\lambda_{k+1}^{\min} = \Delta\lambda_k^{\max} \cdot \varepsilon_k \tag{8}$$

Figure 2 shows a plot with VCSEL cooling parameters [22–24].

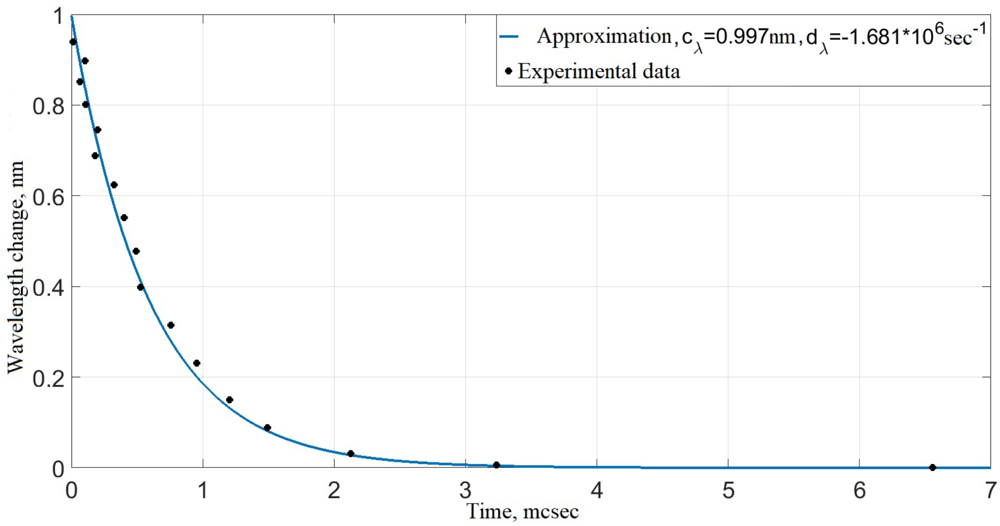

FIG. 2. Graph of VCSEL wavelength change during the cooling process

Figure 3 shows the plot of VCSEL wavelength change, calculated by recurrence relations with parameters selected from the experiment to study the process of the thermal modulation:

$a_1 = 1.641$ nm, $b_1 = 5.434{\cdot}10^5$ sec^{-1}, $b_2 = 1.177{\cdot}10^6$ sec^{-1}, $T_{in} = 10^{-8}$ sec, $T_a = 5{\cdot}10^{-9}$ sec, $\nu_0 = 5{\cdot}10^4$ sec^{-1}

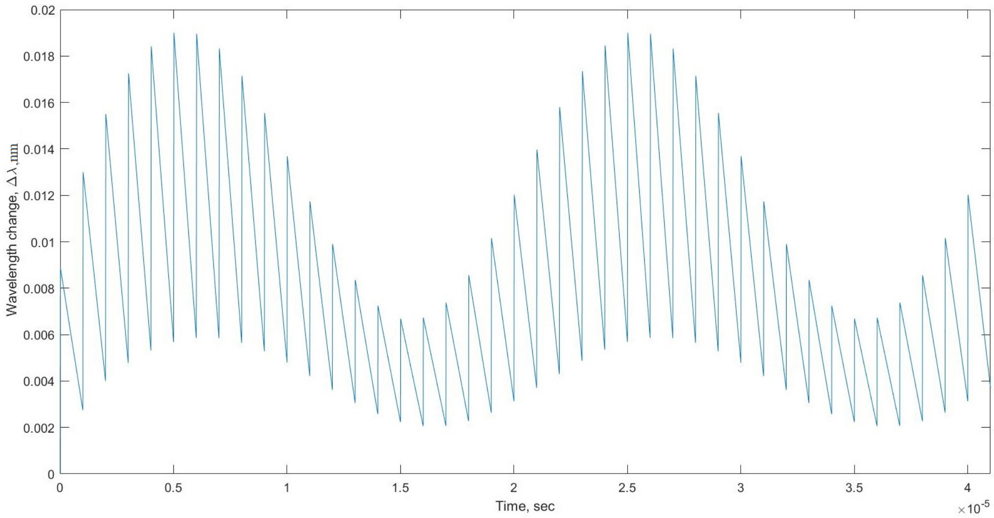

FIG. 3. VCSEL wavelength change with modulation amperage pulses

4. Analytical derivation of formulas for calculating the principle of change of VCSEL modulated wavelength

Let us denote by $\lambda_V = 1540$ nm the VCSEL generation wavelength in the absence of modulation pulses. Under the action of modulation pulses, the wavelength in each interval changes continuously, according to Fig. 3. The measuring system captures the VCSEL wavelength only at the sampling points $t_k = T_d \cdot k$, in which this length takes the values:

$$\lambda_V^{(k)} = \lambda_V + \Delta\lambda_k^{\min} \tag{9}$$

At the k-th wavelength sampling interval the wavelength takes the maximum value $\lambda_V + \Delta\lambda_k^{\max}$. Here $\Delta\lambda_k^{\min}$ is the wavelength λ_V correction at the k-th sampling point, $\Delta\lambda_k^{\max}$ is the maximum wavelength shift at the k-th sampling point. Under the action of modulation pulses, a periodic process of changing the wavelength is established. Using these parameters, we obtain a recursive relation describing the modulation process:

$$\begin{cases} \Delta\lambda_k^{\max} = \Delta\lambda_k^{\min} + \Delta\lambda_k \\ \Delta\lambda_{k+1}^{\min} = \Delta\lambda_k^{\max} \cdot \varepsilon_k \end{cases} \tag{10}$$

The recurrence relation for the desired correction $\Delta\lambda_k^{\min}$ has the following form:

$$\Delta\lambda_{k+1}^{\min} = \left(\Delta\lambda_k^{\min} + \Delta\lambda_k\right) \cdot \varepsilon_k \tag{11}$$

The initial condition is as follows: $\Delta\lambda_0^{\min} = 0$. The solution of the recurrence relation takes the form:

$$\begin{cases} \Delta\lambda_{k+1}^{\min} = \sum_{q=0}^{k} \left(\Delta\lambda_q \cdot \prod_{s=q}^{k} \varepsilon_s\right), \quad k = 0, 1, 2.... \\ \Delta\lambda_0^{\min} = 0 \end{cases} \tag{12}$$

In order to evaluate the product

$$\prod_{s=q}^{k} \varepsilon_s = \exp\left(-b_2\left(T_d - T_{in}\right)(k - q + 1)\right) \exp\left(b_2 T_a \sum_{s=q}^{k} \sin\left(2\pi\nu_0 T_d s\right)\right) \tag{13}$$

one can use the well-known formula:

$$\sum_{s=0}^{k} \sin(\varphi \cdot s) = \frac{\sin\left(\frac{k}{2} \cdot \varphi\right) \cdot \sin\left(\frac{k+1}{2} \cdot \varphi\right)}{\sin\left(\frac{\varphi}{2}\right)} \cdot, \quad \varphi = 2\pi\nu_0 T_d\ . \tag{14}$$

Keeping in mind that $|b_1 \cdot T_k| \ll 1$, $|b_2 \cdot T_a| \ll 1$ and assuming that $\Delta\lambda_q \approx a_1 \cdot b_1 \cdot T_q$, one simplifies the product as follows:

$$\prod_{s=q}^{k} \varepsilon_s \approx \exp\left(-b_2 \cdot (T_d - T_{in}) \cdot (k + 1 - q)\right) \tag{15}$$

Hence, relation (12) is simplified as follows:

$$\Delta\lambda_{k+1}^{\min} \approx a_1 b_1 e^{-b_2(T_d - T_{in})(k+1)} \sum_{q=0}^{k} \left(T_q e^{b_2(T_d - T_{in})q} \right) \tag{16}$$

For large k ($k \to \infty$) one can deal with asymptotics (the asymptotic behavior is observed quite quickly, at about a hundredth step):

$$\begin{aligned} &\Delta\lambda_k^{\min} = \alpha - \beta\cos\left(2\pi\nu_0 T_d k + \Delta\right), \ \Delta\lambda_0^{\min} = 0, \ \alpha = \frac{a_1 b_1 T_{in}}{\exp\left(b_2\left(T_d - T_{in}\right)\right) - 1}, \ \beta = \frac{a_1 b_1 T_a}{Z}, \\ &Z = \sqrt{1 + \exp\left(2b_2\left(T_d - T_{in}\right)\right) - 2\cos\left(2\pi\nu_0 T_d\right)\exp\left(b_2\left(T_d - T_{in}\right)\right)}. \end{aligned} \tag{17}$$

Here the phase shift Δ is the solution of the following system of equations:

$$\begin{cases} \sin\left(\Delta\right) = \dfrac{1}{Z}\left(\cos\left(2\pi\nu_0 T_d\right)\exp\left(b_2\left(T_d - T_{in}\right)\right) - 1\right), \\ \cos\left(\Delta\right) = \dfrac{1}{Z}\sin\left(2\pi\nu_0 T_d\right)\exp\left(b_2\left(T_d - T_{in}\right)\right). \end{cases} \tag{18}$$

Finally, we calculate the time-dependent source wavelength modulated by the frequency ν_0:

$$\lambda\left(t\right) = \lambda_V + \alpha - \beta\cos\left(2\pi\nu_0 t + \Delta\right). \tag{19}$$

5. Results

In Fig. 4, we can see a comparison of the exact wavelength, measured at the sampling points $\lambda_V^{(k)} = \lambda_V + \Delta\lambda_k^{\min}$ with an asymptotic solution $\lambda\left(t\right)$. In general, despite a slight discrepancy at the beginning, the asymptotic solution is an agreement with the exact values.

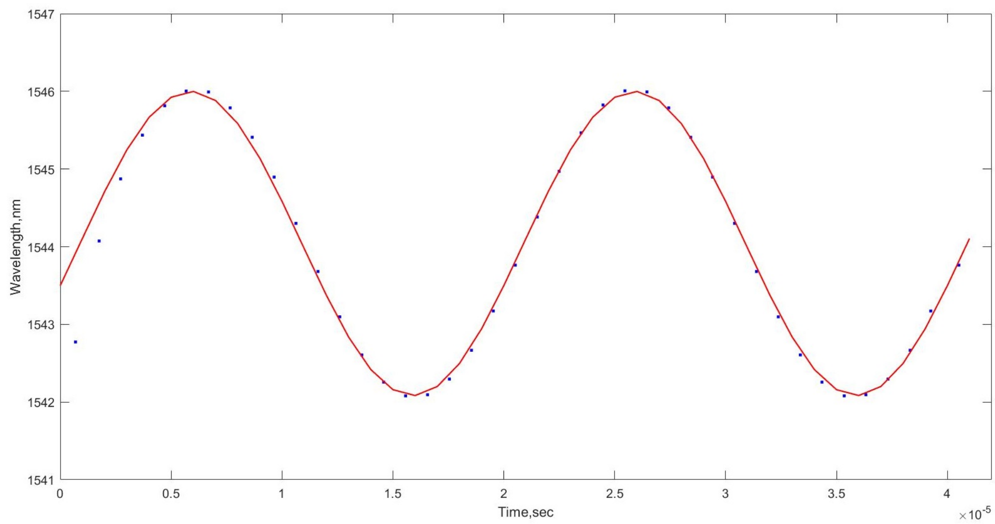

FIG. 4. Comparison of the exact wavelength (blue dots) with the asymptotic solution (red curve)

It follows from formulas (17) that the wavelength modulation law is determined by four parameters: the central wavelength $\lambda_V + \alpha$, where α is the central wavelength shift due to the heating procedure, β is the modulation depth, $\varphi_{gen} = \Delta - \pi$ is the modulation phase principle (desired phase shift of the reference oscillator signal), ν_0 is the modulation frequency. According to formulas (3), these parameters are calculated through the thermal parameters of VCSEL.

6. Conclusion

In this work, we explored the thermal regime of the VCSEL radiation source. With the help of the experimental data, four parameters were analytically obtained for the optimal calculation of the VCSEL modulation principle: the central wavelength, the modulation depth, the phase and the frequency of modulation. In Fig. 4, we can see that precisely calculated wavelength $\lambda(t)$ practically coincides with the asymptotic solution having a small discrepancy at the beginning only. In the previous researches, the thermal parameters of VCSEL were not calculated, although it was pointed out that unreliably calculated characteristics of signal from the reference generator can lead to errors in the interference signal, which is necessary for further processing. Attempts have been made to calculate the exact value of φ_{gen}, in order to improve the quality of the interference signal entering the demodulation circuit, but the analytical solution hasn't been considered. The obtained formulas for exact calculation of VCSEL source parameters will improve the quality of phase demodulation from the interferometer signal and will allow one to avoid errors from the source in determining the interference signal.

References

[1] Kivshar Yu.S. From metamaterials to metasurfaces and metadevices. *Nanosystems: physics, chemistry, mathematics*, 2015, **6**(3), P. 346–352.
[2] Iga K. Surface-Emitting Laser–Its Birth and Generation of New Optoelectronics Field. *IEEE J. Sel. Top. In Qu. El.*, 2000, **6**(6), P. 1201.
[3] Rashidul Hasan R., Basak R. Characteristics of a Designed 1550 nm AlGaInAs/InP MQW VCSEL. *Int. J. Multidis. Scien. Eng.*, 2013, **4**(1), P. 5–9.
[4] Jungo M., Monti di Sopra F., Erni D., Baechtold W. Scaling effects on vertical-cavity surface-emitting lasers static and dynamic behavior. *Journal of Applied Physics*, 2002, **91**(9), P. 5550–5557.
[5] Bjerkan L., Royset A., Hafskjaer L., and Myhre D. Measurement of Laser Parameters for Simulation of High-speed Fiberoptic Systlems. *J. Lightwave Technol.*, 1996, **15**, P. 839.
[6] Tucker R.S. High-speed Modulation of Semiconductor Lasers. *IEEE Trans. El. Dev.*, 1985, **ED-32**(1)2, P. 2572.
[7] Chen J., Hangauer A., Strzoda-Markus-Christian Amann R. Experimental characterization of the frequency modulation behavior of vertical cavity surface emitting lasers. *Appl. Phys. Lett.*, 2007, **91**, P. 141105.
[8] Dutta N.K., Tayahi M. and Choquette K.D. Transmission experiments using oxide confined vertical cavity surface emitting lasers. *Electronics Letters*, 1997, **33**(13), P. 1147–1148.
[9] Meng Xun, Guanzhong Pan, Zhuangzhuang Zhao, Yun Sun, Chengyue Yang, Qiang Kan, Jingtao Zhou, and Dexin Wu. Analysis of Thermal Properties of 940-nm Vertical Cavity Surface Emitting Laser Arrays. *IEEE Trans. El. Devices*, 2021, **68**(1), P. 158–163.
[10] Larsson A., Carlsson C., Gustavsson J., Haglund E., Modh P., Bengtsson J. Direct high-frequency modulation of VCSELs and applications in fibre optic RF and microwave links. *New J. Phys.*, 2004, **6**, P. 176.
[11] Nikitenko A., Plotnikov M., Volkov A. PGC-ATAN demodulation scheme with the Carrier Phase Delay Compensation for Fiber-Optic Interferometric sensors. *IEEE Sensors Journal*, 2018, **18**(5), P. 1985–1992.
[12] Volkov A.V., Oskolkova E.S., Plotnikov M.Yu., Mekhrengin M.V., Shuklin P.A. Phase shift influence research of the reference oscillator signal on the output signal in homodyne demodulation scheme. *Scientific and Technical Journal of Information Technologies, Mechanics and Optics*, 2015, **15**(4), P. 608–614.
[13] Huang S.-C., Lin H. Modified phase-generated carrier demodulation compensated for the propagation delay of the fiber. *Applied Optics*, 2007, **46**(31), P. 7594–7603.
[14] Carlsson C., Martinsson H., Schatz R., Halonen J., Larsson A. Analog Modulation Properties of Oxide Confined VCSELs at Microwave Frequencies. *J. Light. Tech.*, 2002, **20**(9), P. 1740–1749.
[15] Yang Yi, Ruan Yu, Li Zhengjia. Rate-equation-based Thermal VCSEL Model and its Equivalent Circuit Application in Transceiver Module Design. *J. Opt. Commun.*, 2002, **236**, P. 205–208.
[16] Mena P.V., Morikuni J.J., Kang S.-M., Harton A.V., and Wyatt K.W. A Simple Rate-Equation-Based Thermal VCSEL Model. *J. Ligt. Tech.*, 1999, **17**(5), P. 865–872.
[17] Garciaand V.G., Farzane M. Transient thermal imaging of a vertical cavity surface-emitting laser using thermoreflectance microscopy. *Journal of Applied Physics*, 2016, **119**, P. 045105.
[18] Chen G., Tien C.L., Wu X., Smith J.S. Thermal Diffusivity Measurement of GaAs/AlGaAs Thin-Film Structures. *Journal of Heat Transfer, Transactions ASME*, 1994, **116**(2), P. 325–331.
[19] Zhao Y.-G. and McInemey J.G. Transient Temperature Response of Vertical-Cavity Surface-Emitting Semiconductor Lasers. *IEEE J. Quant. El.*, 1995, **31**(9), P. 1668–1673.
[20] Kobayashi S., Yamamoto Y., Ito M., Kimura T. Direct Frequency Modulation in AlGaAs Semiconductor Lasers. *IEEE Transactions on Microwave Theory and Techniques*, 1982, **MTT-30**(4), P. 428.
[21] Kireenkov A.Yu., Aleinik A.S., Plotnikov M.Yu., Mekhrengin M.V. Patent No. 2646420. Method for frequency-pulse modulation of a semiconductor laser source of optical radiation for interrogating optical interferometric sensors. Russian Federation, IPC: H01S 5/042, G01B 9/02, G02F1/00. Priority from 11/23/2016.
[22] Belikin M.N., Kulikov A.V., Strigalev V.E., Aleinik A.S., and Kireenkov A.Yu. Study of a compact radiation source for fiber-optic interferometric phase sensors. *J. Opt. Technol.*, 2015, **82**(12), P. 805–809.
[23] Belikin M.N. Small-sized high-speed spectral response recorder for fiber-optical Bragg- gratings sensors. PhD dissertation in technical sciences, Saint Petersburg, 2016, P. 95–104.
[24] Kireenkov A.Yu. Fiber-optic interferometric methods for constructing measuring systems based on a surface-emitting laser. PhD dissertation in physical sciences, Saint Petersburg, 2017, P. 90–108.

Submitted 20 October 2022; revised 29 November 2022; accepted 30 November 2022

Information about the authors:

George P. Miroshnichenko – ITMO University, Kronverkskiy, 49, St. Petersburg, 197101, Russia; ORCID 0000-0002-4265-8818; gpmirosh@gmail.com

Alina N. Arzhanenkova – ITMO University, Kronverkskiy, 49, St. Petersburg, 197101, Russia; ORCID 0000-0003-4869-2838; 11arzh11@gmail.com

Michail Yu. Plotnikov – ITMO University, Kronverkskiy, 49, St. Petersburg, 197101, Russia; ORCID 0000-0003-2506-0379; plotnikov-michael@yandex.ru

Conflict of interest: the authors declare no conflict of interest.