Recombination radiation associated with A^+-centers in quantum dots in an external magnetic field

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The effect of an external magnetic field on the binding energy of a hole in an impurity complex $A^+ + e$ in a spherically symmetric quantum dot, as well as frequency dependence of the spectral intensity of recombination radiation of the quasi-zero-dimensional structure with impurity complexes $A^+ + e$ have been investigated. It is shown that in an external magnetic field there is a spatial anisotropy for the binding energy of A^+-state due to hybrid quantization in the quantum dot radial plane and dimensional quantization in the direction of an external magnetic field. It is shown that in an external magnetic field the spectral intensity curve of the recombination radiation shifts to the short-wavelength region of the spectrum and probability of the radiative transition of an electron to the level of A^+-center increases, which is caused by increase in the overlap integral of the envelope wave functions of a hole bound at the A^+-center and of an electron localized in the ground state of quantum dot.

1. Introduction

In recent years, interest in studying an external magnetic field's influence on the photoluminescence (PL) of structures with quantum wells (QWs) and quantum dots (QDs) has not been weakened [1–15]. This is due, first of all, to modification of the optical spectrum of nanostructures, impurity and exciton states, which leads to new interesting effects in the photoluminescence and optical absorption spectra under application of external magnetic field conditions. For example, circular polarization of the PL peak associated with A^+-centers was first measured in the case of the QWs of GaAs/ AlGaAs, and analysis of which has made it possible to determine the fine, spin and energy structure of the A^+-center [1]. In [5], PL spectra were studied in an external magnetic field of an ensemble of QD-InAs grown by method of the molecular-beam epitaxy on a (001) GaAs substrate disoriented in the (010)-direction. It was established in [5] that in an external magnetic field, the capture of the photo-borne carriers in an array of QDs, which have been formed as a result of coalescence, is suppressed, and as a result, an increase in the PL intensity has been observed. A magnetic field also exerts an influence on the kinetics of the QD-photoluminescence. Thus, in [3], an acceleration of the photoluminescence kinetics of the QD-InAs in the AlAs matrix in an external 5 T – magnetic field has been observed. The obtained results are explained in the framework of a model that takes into account the exchange and Zeeman splitting of the QD exciton levels in an external magnetic field [3].

The present work is devoted to the theoretical study of an external magnetic field's influence on the binding energy of a hole in an impurity complex $A^+ + e$ in a spherically symmetric QD, as well as on the frequency dependence of the recombination radiation spectral intensity (SIRI) of a quasi-zero-dimensional structure with impurity complexes $A^+ + e$.

2. Model

Let us consider the problem of the hole bound states in an impurity complex $A^+ + e$ of a semiconductor spherically symmetric QD in an external magnetic field. The potential of an infinitely deep spherically symmetric well has been used as a model of the QD confinement potential:

$$U(\rho) = \begin{cases} 0, & \text{if } \rho \leq R_0; \\ \infty, & \text{if } \rho \geq R_0, \end{cases} \tag{1}$$

where R_0 – the QD radius. Interaction of an electron in the ground state of QD with a hole localized at the A^0-center will be considered in the framework of the adiabatic approximation [4]. In this case, the electron potential, $V_{n,l,m}(r)$, acting on the hole, can be considered as averaged one over the electron motion [4]:

$$V_{n,l,m}(\vec{r}) = -\frac{e^2}{4\pi\varepsilon_0\varepsilon} \int\limits_0^{R_0} \frac{|\Psi_{n,l,m}(\vec{r}_e)|^2}{|\vec{r} - \vec{r}_e|} d\vec{r}_e, \tag{2}$$

Where e – the electron charge; ε – dielectric constant of the QD material; ε_0 – the electric constant; $\Psi_{n,l,m}(\vec{r})$ – the electron wave function in QD; $m = 0, \pm1, \pm2\ldots$ – the magnetic quantum number; $l = 0, 1, 2\ldots$ – the orbital quantum number.

In the first order of perturbation theory, for the ground state of an electron ($m = 0$, $l = 0$), potential (2) can be written in the next form:

$$V_{n,0,0}(\rho) = -\frac{e^2\beta_n}{4\pi\varepsilon_0\varepsilon R_0} + \frac{m_h^*}{2}\left(\omega_n^2 + \frac{\omega_B^2}{2}\right)\rho^2 + \frac{m_h^* z^2}{2}\omega_n^2, \tag{3}$$

where $\beta_n = \gamma_0 - \mathrm{Ci}(2\pi n) + \ln(2\pi n)$; $\hbar\omega_n = \left[\left(2\hbar^2\pi^2 n^2 e^2\right)/\left(3m_h^* R_0^3 4\pi\varepsilon_0\varepsilon\right)\right]^{1/2}$; ρ, φ, z – cylindrical coordinates; $\gamma_0 = 1.781$ – the Euler constant; $\mathrm{Ci}(x)$ – the integral cosine; n – the electron radial quantum number; m_h^* – the hole effective mass; $\omega_B = |e|B/m^*$ – the cyclotron frequency.

It can be shown that the wave function and energy spectrum corresponding to potential (3) have the next form:

$$\Psi_{n_1,m,n_2}(\rho,\varphi,z) = \frac{1}{a_1^2}\left(\frac{n_1!}{2^{n_2+1}\pi^{3/2}n_2!\,(n_1+|m|)!\,a_n}\right)^{1/2}\left(\frac{\rho^2}{2a_1^2}\right)^{|m|/2} \times$$
$$\exp\left[-\left(\frac{\rho^2}{4a_1^2} + \frac{z^2}{2a_n^2}\right)\right] H_{n_2}\left(\frac{z}{a_n}\right) L_{n_1}^{|m|}\left(\frac{\rho^2}{2a_1^2}\right)\exp(im\varphi), \quad (4)$$

where n_1, $n_2 = 0, 1, 2, \ldots$ – quantum numbers corresponding to Landau levels and to energy levels of an oscillating spherically symmetric well; $a_1^2 = a_n^2/\left(2\sqrt{1 + a_n^4/(4a_B^4)}\right)$; $a_n = \sqrt{\hbar/(m_h^*\omega_n)}$ – the characteristic oscillator length; $a_B = \sqrt{\hbar/(m^*\omega_B)}$ – the magnetic length; $H_n(x)$, $L_n^c(x)$ – the Hermite and Lagger polynomials, respectively.

$$E_{n_1,m,n_2}^{n,0,0} = -\frac{e^2}{4\pi\varepsilon_0\varepsilon R_0}\beta_n + \hbar\omega_n\left(n_2 + \frac{1}{2}\right) + \hbar\omega_n\left(2n_1 + |m| + 1\right)\sqrt{1 + \frac{\omega_B^2}{8\omega_n^2}} + \frac{\hbar\omega_B m}{2}. \tag{5}$$

The short-range impurity potential is described in terms of the zero-radius potential model:

$$V_\delta(\rho,\varphi,z;\,\rho_a,\varphi_a,z_a) = \gamma\frac{\delta(\rho - \rho_a)}{\rho}\delta(\varphi - \varphi_a)\,\delta(z - z_a)\left[1 + (\rho - \rho_a)\frac{\partial}{\partial\rho} + (z - z_a)\frac{\partial}{\partial z}\right], \tag{6}$$

where $\gamma = 2\pi\hbar^2/(\alpha m_h^*)$ – the zero radius potential power; α is determined by the bound state energy E_i of the same A^+-center in a bulk semiconductor; ρ_a, z_a – coordinates of the A^+-center in QD.

The one-hole Green function $G(\rho,\varphi,z,\rho_a,\varphi_a,z_a,E_{\lambda h})$ to the Schrödinger equation, corresponding to the source at the point $\vec{r}_1 = (\rho_1,\varphi_1,z_1)$ and to the energy $E_{\lambda h}$, can be written as

$$G(\rho,\varphi,z,\rho_a,\varphi_a,z_a,E_n) = -\sum_{n_1,m,n_2}\frac{\Psi_{n_1,m,n_2}^{(n)*}(\rho_1,\varphi_1,z_1)\,\Psi_{n_1,m,n_2}^{(n)}(\rho,\varphi,z)}{|E_{\lambda h}| + E_{n_1,m,n_2}^{n,0,0}}, \tag{7}$$

where $E_{\lambda h}$ – the hole binding energy, measured from the bottom of the electron adiabatic potential.

Using the expressions for the single-particle wave functions (4) and for the energy spectrum (5), for the Green function $G(\rho,\varphi,z,\rho_a,\varphi_a,z_a,E_n)$ in units of the effective Bohr energy $E_h = \hbar^2/\left(2m_h^* a_h^2\right)$ and the effective Bohr

radius of the hole $a_h = 4\pi\varepsilon_0\varepsilon\hbar^2/\left(m_h^* |e|^2\right)$, we obtain

$$
G\left(\rho, \varphi, z, \rho_a, \varphi_a, z_a, E_{\lambda h}\right) = \frac{-\beta_h}{2\pi^{3/2}a_n^3 E_h} \exp\left[-\left(\frac{\rho^2 + \rho_a^2}{4a_1^2} + \frac{z^2 + z_a^2}{2a_n^2}\right)\right] \times
$$

$$
\int\limits_0^{+\infty} dt \, \exp\left[-\left(\eta_{\lambda h}^2 \beta_h - \beta_0 + w + 1\right)t\right] \sum_{n_2=0}^{\infty} \left(\frac{e^{-t}}{2}\right)^{n_2} \frac{H_{n_2}\left(\frac{z_a}{a_n}\right) H_{n_2}\left(\frac{z}{a_n}\right)}{n_2!} \times
$$

$$
\sum_{m=-\infty}^{+\infty} \exp\left(-w\,|m|\,t\right) \left(\frac{\rho^2 \rho_a^2}{2a_1^2}\right)^{|m|} \exp\left(im\left((\varphi - \varphi_a) - \beta_h\left(a^*\right)^{-2}t\right)\right) \times
$$

$$
\sum_{n_1=0}^{+\infty} \frac{n_1!}{(n_1 + |m|)!} L_{n_1}^{|m|}\left(\frac{\rho^2}{2a_1^2}\right) L_{n_1}^{|m|}\left(\frac{\rho_a^2}{2a_1^2}\right) \exp\left[-2n_1 wt\right], \quad (8)
$$

here $\eta_{\lambda h}^2 = |E_{\lambda h}|/E_h$; $\beta_0 = \beta_n^s e^2/\left(4\pi\varepsilon_0\varepsilon R_0^* a_h E_h\right)$; $R_0^* = R_0/a_h$; $\beta_h = E_h/\hbar\omega_n$; $a^* = a_B/a_h$; $w = \sqrt{1 + \beta_h^2\left(a^*\right)^{-4}/2}$.

Summation over a quantum number n_2 can be performed using the Mehler formula:

$$
\sum_{n_2=0}^{\infty} \left(\frac{e^{-t}}{2}\right)^{n_2} \frac{H_{n_2}\left(\frac{z_a}{a_n}\right) H_{n_2}\left(\frac{z}{a_n}\right)}{n_2!} = \frac{1}{\sqrt{1 - e^{-2t}}} \exp\left\{\frac{2z_a z\, e^{-t} - \left(z_a^2 + z^2\right)e^{-2t}}{a_n^2\left(1 - e^{-2t}\right)}\right\}. \quad (9)
$$

Using the Hille–Hardy formula for the bilinear generating function of the Laguerre polynomials, it is possible to sum the series over the quantum number n_1:

$$
\sum_{n_1=0}^{\infty} \frac{n_1!}{(n_1 + |m|)!} L_{n_1}^{|m|}\left(\frac{\rho_a^2}{2a_1^2}\right) L_{n_1}^{|m|}\left(\frac{\rho^2}{2a_1^2}\right) \exp\left(-2n_1 wt\right) =
$$

$$
\left(\frac{\rho_a \rho}{2a_1^2}\right)^{-|m|} \exp\left[|m|\,wt\right] \times \left(1 - \exp\left(-2wt\right)\right)^{-1} \exp\left(-\exp\left(-2wt\right)\frac{\left(\rho_a^2 + \rho^2\right)}{2a_n^2\left(1 - \exp\left(-2wt\right)\right)}\right) \times
$$

$$
I_{|m|}\left(\frac{\rho_a \rho \exp\left(-wt\right)}{2a_1^2\left(1 - \exp\left(-2wt\right)\right)}\right). \quad (10)
$$

Summation over the magnetic quantum number m gives:

$$
\sum_{m=-\infty}^{+\infty} \exp\left[\left(i(\varphi - \varphi_a) - \beta_h\left(a^*\right)^{-2}t\right)m\right] I_{|m|}\left(\frac{\rho_a \rho \exp\left(-wt\right)}{2a_1^2\left(1 - \exp\left(-2wt\right)\right)}\right) =
$$

$$
\exp\left[\left(\exp\left[i(\varphi - \varphi_a) - \beta_h\left(a^*\right)^{-2}t\right] + \exp\left[-i(\varphi - \varphi_a) + \beta_h\left(a^*\right)^{-2}t\right]\right) \times \frac{\rho_a \rho \exp\left(-wt\right)}{2a_1^2\left(1 - \exp\left(-2wt\right)\right)}\right]. \quad (11)
$$

Taking into account (9) – (11), after separation of the diverging part, we obtain:

$$
G\left(\rho, \varphi, z, \rho_a, \varphi_a, z_a; E_{\lambda h}\right) =
$$

$$-\frac{1}{(2\pi)^{3/2}\sqrt{\beta_h}E_h a_h^3}\left\{\exp\left[-\frac{\left(\rho_a^2+\rho^2\right)w+z_a^2+z^2}{4\beta_h a_h^2}\right]\times\right.$$

$$\int\limits_0^{+\infty}dt\,\exp\left[-\left(\beta_h\eta_{\lambda h}^2-\beta_0+w+1/2\right)t\right]\times$$

$$\left[w\left(1-e^{-2t}\right)^{-1/2}\left(1-\exp\left[-2wt\right]\right)\times\exp\left\{\frac{2z_a z\,e^{-t}-\left(z_a^2+z^2\right)e^{-2t}}{2\beta_h a_h^2\left(1-e^{-2t}\right)}\right\}\times\right.$$

$$\exp\left[-\exp\left(-2wt\right)\frac{\left(\rho_a^2+\rho^2\right)w}{2\beta_h a_h^2\left(1-\exp\left[-2wt\right]\right)}\right]\times$$

$$\exp\left[\frac{1}{2}\left(\exp\left[i\left(\varphi-\varphi_a\right)-\beta_h\left(a^*\right)^{-2}t\right]+\exp\left[-i\left(\varphi-\varphi_a\right)+\beta_h\left(a^*\right)^{-2}t\right]\right)\times\frac{w\rho_a\rho\,\exp\left(-wt\right)}{\beta_h a_h^2\left(1-\exp\left(-2wt\right)\right)}\right]-$$

$$t^{-3/2}\exp\left[-\frac{\left(\rho-\rho_a\right)^2w+\left(z-z_a\right)^2}{4\beta_h a_h^2 t}\right]\right]+2\sqrt{\pi\beta_h}a_h\frac{\exp\left[-\sqrt{\frac{\left(2\beta_h\eta_{\lambda h}^2-2\beta_0+2w+1\right)\left(\left(\rho-\rho_a\right)^2w+\left(z-z_a\right)^2\right)}{2\beta_h a_h^2}}\right]}{\sqrt{\left(\rho-\rho_a\right)^2w+\left(z-z_a\right)^2}}\right\}. \quad (12)$$

The bound state energy of a hole in the total field (including the zero radius well located at a point $\vec{R}_a=\left(\vec{\rho}_a,\,z_a\right)$) is the pole of the Green's function, i.e. the equation solution:

$$\alpha=\frac{2\pi\hbar^2}{m^*}\left(\mathbf{T}G\right)\left(\rho_a,\varphi_a,z_a,\rho_a,\varphi_a,z_a;E_{\lambda h}\right), \quad (13)$$

where

$$\left(\mathbf{T}G\right)\left(\rho_a,\varphi_a,z_a,\rho_a,\varphi_a,z_a;E_{\lambda h}\right)=\lim_{\substack{\rho\to\rho_a\\\varphi\to\varphi_a\\z\to z_a}}\left[1+\left(\rho-\rho_a\right)\partial/\partial\rho+\left(z-z_a\right)\partial/\partial z\right]\times G\left(\rho,\varphi,z,\rho_a,\varphi_a,z_a;E_{\lambda h}\right).$$

Substituting (12) into (13), we obtain the dispersion equation for a hole, localized at the QD A^+-center in a magnetic field:

$$\sqrt{\eta_{\lambda h}^2-\beta_0\beta_h^{-1}+\left(2\beta_h\right)^{-1}+w\beta_h^{-1}}=\eta_i-\sqrt{\frac{2}{\pi\beta_h}}\int\limits_0^{+\infty}dt\,\exp\left[-\left(\beta_h\eta_{\lambda h}^2-\beta_0+w+1/2\right)t\right]\times$$

$$\left[\frac{1}{2t\sqrt{2t}}-w\left(1-e^{-2t}\right)^{-1/2}\left(1-\exp\left(-2wt\right)\right)^{-1}\exp\left[-\frac{\left(z_a^*\right)^2}{2\beta_h}\cot\left(\frac{t}{2}\right)\right]\times\right.$$

$$\left.\exp\left[-\frac{w\left(\rho_a^*\right)^2}{2\beta_h\left(1-\exp\left(-2wt\right)\right)}\left(1+\exp\left(-2wt\right)-2\exp\left(-wt\right)\cosh\left(\beta_h\left(a^*\right)^{-2}t\right)\right)\right]\right], \quad (14)$$

where $\eta_i^2=\left|E_i\right|/E_h$; E_i – the bound state energy of a hole localized at the same A^+-center in a bulk semiconductor; $z_a^*=z_a/a_h$; $\rho_a^*=\rho_a/a_h$.

Let us consider the process of the radiative transition of an excited electron to the A^+-center level. To calculate the frequency dependence of the recombination radiation spectral intensity (SIRR), it is necessary to obtain an expression for the wave function of an electron localized in the ground state of a spherically symmetric QD in a magnetic field. In the second order of perturbation theory, the energy spectrum of an electron in an external magnetic field can be written as

$$E=E^{(0)}+V_{n,l,m;\,n,l,m}+\sum_{n'\,l'\,m'}\frac{\left(R_0^*\right)^2\left|V_{n,l,m;\,n',l',m'}\right|^2}{\pi^2-\tilde{X}_{n',l'}^2}, \quad (15)$$

here $\tilde{X}_{n',l'}$ – the root of the Bessel function of a half-integer order $l+1/2$, $E^{(0)}=\tilde{X}_{n,l}^2 E_h/\left(R_0^*\right)^2$ – zero approach to electron energy in the size-quantized band, $V_{n,l,m;\,n',l',m'}$ – matrix element of the perturbation operator, which in spherical coordinates has the form:

$$\hat{V}_{n,l,m;\,n',l',m'}=-\frac{i\hbar\omega_B}{2}\frac{\partial}{\partial\varphi}+\frac{m_h^*\omega_B^2}{2}r^2\sin^2\theta. \quad (16)$$

In the second order of perturbation theory, the electron wave function is given by an expression of the form:

$$\Psi_{n,l,m}\left(r,\theta,\varphi\right) = \Psi_{n,l,m}^{(0)}\left(r,\theta,\varphi\right) + \sum_{n'l'm'} \frac{\left(R_0^*\right)^2 V_{n,l,m;\,n',l',m'}}{\pi^2 - \tilde{X}_{n',l'}^2}\Psi_{n',l',m'}^{(0)}\left(r,\theta,\varphi\right),\tag{17}$$

where $\Psi_{n,l,m}^{(0)}\left(r,\theta,\varphi\right)$ – the zero approximation wave function:

$$\Psi_{n,l,m}^{(0)}\left(r,\theta,\varphi\right) = Y_{l,m}\left(\theta,\varphi\right)\frac{J_{l+3/2}\left(\frac{\tilde{X}_{n,l}}{R_0^*}r^*\right)}{a_h^{3/2}\sqrt{2\pi}R_0^*\sqrt{r^*}J_{l+3/2}\left(\tilde{X}_{n,l}\right)}.\tag{18}$$

The matrix element $V_{n,l,m;\,n',l',m'}$ of the perturbation operator, taking into account (18), can be written as:

$$V_{n,l,m;\,n',l',m'} = \frac{\int\limits_0^{R_0^*}\int\limits_0^{\pi}\int\limits_0^{2\pi} Y_{l,m}^*\left(\theta,\varphi\right)J_{l+3/2}\left(\frac{\tilde{X}_{n,l}}{R_0^*}r^*\right)\hat{V}_{n,l,m;\,n',l',m'}Y_{l',m'}\left(\theta,\varphi\right)J_{l'+3/2}\left(\frac{\tilde{X}_{n',l'}}{R_0^*}r^*\right)r^*dr^*d\theta d\varphi}{2\pi R_0^* J_{l+3/2}\left(\tilde{X}_{n,l}\right)J_{l'+3/2}\left(\tilde{X}_{n',l'}\right)}.\tag{19}$$

Using the recurrence relations between the spherical functions and the properties of their orthogonality, the integrals over the variables θ and φ can be written as

$$\int\limits_0^{\pi}\int\limits_0^{2\pi} Y_{l,m}^*\left(\theta,\varphi\right)Y_{l',m'}\left(\theta,\varphi\right)\sin\theta d\theta d\varphi = \delta_{ll'}\delta_{mm'},\tag{20}$$

$$\int\limits_0^{\pi}\int\limits_0^{2\pi} Y_{l,m}^*\left(\theta,\varphi\right)Y_{l',m'}\left(\theta,\varphi\right)\sin^3\theta d\theta d\varphi =$$

$$\sqrt{\frac{\left(l-m+4\right)\left(l-m+3\right)\left(l-m+2\right)\left(l-m+1\right)}{\left(2l+1\right)\left(2l-1\right)^2\left(2l-3\right)}}\delta_{l,l'+2}\delta_{m,m'-2} \quad -$$

$$\frac{2}{\left(2l+3\right)\left(2l-1\right)}\sqrt{\left(l-m\right)\left(l-m-1\right)\left(l+m+2\right)\left(l+m+1\right)}\delta_{ll'}\delta_{m,m'-2} \quad +$$

$$\sqrt{\frac{\left(l+m+4\right)\left(l+m+3\right)\left(l+m+2\right)\left(l+m+1\right)}{\left(2l+5\right)\left(2l+3\right)^2\left(2l+1\right)}}\delta_{l,l'-2}\delta_{m,m'-2}.\tag{21}$$

Integration over a variable r^* gives

$$\int\limits_0^{R_0^*} J_{l+3/2}\left(\frac{\tilde{X}_{n,l}}{R_0^*}r^*\right)J_{l'+3/2}\left(\frac{\tilde{X}_{n',l'}}{R_0^*}r^*\right)r^*dr^* = \frac{R_0^*}{\left(\tilde{X}_{n,l}^2 - \tilde{X}_{n',l}^2\right)}\times$$

$$\left[R_0^*\tilde{X}_{n',l}J_{l+1/2}\left(R_0^*\tilde{X}_{n',l}\right)J_{l+3/2}\left(R_0^*\tilde{X}_{n,l}\right) - R_0^*\tilde{X}_{n,l}J_{l+1/2}\left(R_0^*\tilde{X}_{n,l}\right)J_{l+3/2}\left(R_0^*\tilde{X}_{n',l}\right)\right]\tag{22}$$

and

$$\int\limits_0^{R_0^*} J_{l+3/2}\left(\frac{\tilde{X}_{n,l}}{R_0^*}r^*\right)J_{l'+3/2}\left(\frac{\tilde{X}_{n',l'}}{R_0^*}r^*\right)r^{*3}dr^* =$$

$$\frac{\tilde{X}_{n,l}^{l+3/2}\tilde{X}_{n',l'}^{l'+3/2}}{2^{l+l'+3}\Gamma\left(l'+5/2\right)}\sum_{k=0}^{\infty}\frac{\left(-1\right)^k}{k!\Gamma\left(l+k+5/2\right)}\left(\frac{\tilde{X}_{n,l}^{l+3/2}}{2R_0^*}\right)^{2k}\frac{F\left(-k,-l-k-3/2,l+5/2,\frac{\tilde{X}_{n',l'}^2}{\tilde{X}_{n,l}^2}\right)}{l+l'+2k+4},\tag{23}$$

where $F\left(\alpha,\beta,x\right)$ – the degenerate hypergeometric function.

Taking into account (20) – (23), the matrix element of the considered optical transition can be represented as

$$
V_{n,l,m;\,n',l',m'} =
$$

$$
\sum_{n'=0}^{\infty} \Bigg\{ \frac{\hbar\omega_B m}{4\pi \left(\tilde{X}_{n,l}^2 - \tilde{X}_{n',l}^2 \right) J_{l+3/2}\left(\tilde{X}_{n,l} \right) J_{l+3/2}\left(\tilde{X}_{n',l} \right)} \left[R_0^* \tilde{X}_{n',l} J_{l+1/2}\left(R_0^* \tilde{X}_{n',l} \right) \, J_{l+3/2}\left(R_0^* \tilde{X}_{n,l} \right) - \right.
$$

$$
\left. R_0^* \tilde{X}_{n',l} J_{l+1/2}\left(R_0^* \tilde{X}_{n,l} \right) \, J_{l+3/2}\left(R_0^* \tilde{X}_{n',l} \right) \right] + \sum_{k=0}^{\infty} \frac{(-1)^k m_h^* \omega_B^2 R_0^* \left(\tilde{X}_{n,l}^{l+3/2} \right)^{2k+1}}{k! \Gamma\left(l + k + 5/2 \right) 2^{l+4+2k}} \times
$$

$$
\left[\sqrt{\frac{(l-m+4)(l-m+3)(l-m+2)(l-m+1)}{(2l+1)(2l-1)^2(2l-3)}} \frac{F\left(-k, -l-k-3/2, l+5/2, \frac{\tilde{X}_{n',l-2}^2}{\tilde{X}_{n,l}^2} \right)}{(2l+2k+2)\Gamma\left(l+1/2 \right)} - \right.
$$

$$
\frac{2F\left(-k, -l-k-3/2, l+5/2, \frac{\tilde{X}_{n',l}^2}{\tilde{X}_{n,l}^2} \right)}{(2l+3)(2l-1)(2l+2k+4)} \sqrt{(l-m)(l-m-1)(l+m+2)(l+m+1)} +
$$

$$
\left. \frac{F\left(-k, -l-k-3/2, l+5/2, \frac{\tilde{X}_{n',l+2}^2}{\tilde{X}_{n,l}^2} \right)}{2l+2k+6} \sqrt{\frac{(l+m+4)(l+m+3)(l+m+2)(l+m+1)}{(2l+5)(2l+3)^2(2l+1)}} \right] \Bigg\}. \quad (24)
$$

SIRR, taking into account the QD size dispersion, is determined by the expression of the next form:

$$
\Phi\left(\omega \right) =
$$

$$
\frac{4\omega^2 \sqrt{\varepsilon} e^2}{c^3 V} \left| \frac{\mathbf{P}_{eh} \mathbf{e}_0}{m_0} \right| \left| \int \sum_n \left| \int \Psi_{n,l,m}^* \left(\rho, \varphi, z \right) \Psi_\lambda \left(\rho, \varphi, z \right) d\rho d\varphi dz \right|^2 \times \; P\left(u \right) \delta\left(E_i - E_f - \hbar\omega \right) \, du, \quad (25)
$$

where m_0 – the free electron mass; $\mathbf{P}_{eh}$ – matrix element of the momentum operator on the band carriers Bloch amplitudes; ω – frequency of radiated electromagnetic wave with polarization $\mathbf{e}_0$; V – the QD volume; $P\left(u \right)$ – the Lifshitz–Slezov function.

The wave function of the A^+-state, as is known, differs only by a constant factor from the one-particle Green function:

$$
\Psi_{\lambda h}\left(\rho, \varphi, z \right) = C \exp\left(-\frac{w\rho^2 + z^2}{4\beta_h a_h^2} \right) \times
$$

$$
\int_0^{\infty} dt \exp\left[-\left(\beta_h \eta_{\lambda h}^2 - \beta_0 + w + \frac{1}{2} \right) t \right] \left(1 - e^{-2t} \right)^{-\frac{1}{2}} \left(1 - \exp\left[-2wt \right] \right)^{-1} \times
$$

$$
\exp\left\{ -\frac{z^2 \exp\left[-2t \right]}{2\beta_h a_h^2 \left(1 - \exp\left[-2t \right] \right)} \right\} \exp\left[-\exp\left[-2wt \right] \frac{w\rho^2}{4\beta_h a_h^2 \left(1 - \exp\left[-2wt \right] \right)} \right], \quad (26)
$$

where C – the normalization factor determined by an expression of the next form:

$$
C = \Bigg[-2^{-1/2} \pi^{-3/2} \beta_h^{3/2} a_h^3 w \Gamma\left(\frac{1}{2} - w \right) \frac{\Gamma\left(\frac{\beta_h \eta_{h\lambda}^2 + w}{2} + \frac{5}{4} \right)}{\left(\frac{\beta_h \eta_{h\lambda}^2 + w}{2} + \frac{1}{4} \right)^2 \Gamma\left(\frac{\beta_h \eta_{h\lambda}^2 - w}{2} + \frac{3}{4} \right)} \times
$$

$$
\left[\left(\frac{\beta_h \eta_{h\lambda}^2 + w}{2} + \frac{1}{4} \right) \left[\Psi\left(\frac{\beta_h \eta_{h\lambda}^2 + w}{2} + \frac{5}{4} \right) - \Psi\left(\frac{\beta_h \eta_{h\lambda}^2 - w}{2} + \frac{3}{4} \right) \right] - 1 \right] \Bigg]^{-1/2}. \quad (27)
$$

Taking into account (18), (24) and (26), the matrix element of the radiative recombination transition of an electron from the ground state of the size-quantized band to the level of the QD A^+-center in a magnetic field can be written

as:

$$
M_{1,\lambda} = \frac{C\,(2\beta_h)^{5/4}\,a_h^2}{2\pi R_0^{*2} J_{3/2}\left(\tilde{X}_{n,1}\right)} \int\limits_0^{+\infty}\int\limits_{-\infty}^{+\infty}\int\limits_0^{2\pi} \rho^* d\rho^* dz^* d\varphi \times
$$

$$
\int\limits_0^{+\infty} dt \exp\left[-\left(\beta_h \eta_{\lambda h}^2 - \beta_0 + w + \frac{1}{2}\right)t\right]\left(1 - e^{-2t}\right)^{-1/2}\left(1 - \exp\left[-2wt\right]\right)^{-1} \times
$$

$$
\exp\left[-\left(\frac{z^{*2}\left(1 + \exp\left[-2t\right]\right)}{2\beta_h\left(1 - \exp\left[-2t\right]\right)} + \frac{p^{*2}w\left(1 + \exp\left[-2wt\right]\right)}{2\beta_h\left(1 - \exp\left[-2wt\right]\right)}\right)\right] \times
$$

$$
\frac{\rho^*}{\left(\rho^{*2} + z^{*2}\right)^{3/4}}\left[\frac{J_{5/2}\left(\frac{\tilde{X}_{n,1}}{R_0^*}\sqrt{\rho^{*2} + z^{*2}}\right)}{J_{5/2}\left(\tilde{X}_{n,1}\right)} + \sum_{n'=1}^{\infty}\frac{R_0^{*2}V_{n,1,-1;n',1,-1}J_{5/2}\left(\frac{\tilde{X}_{n',1}}{R_0^*}\sqrt{\rho^{*2} + z^{*2}}\right)}{\left(\pi^2 - \tilde{X}_{n',1}^2\right)J_{5/2}\left(\tilde{X}_{n',1}\right)}\right], \quad (28)
$$

where $R_0^* = R_0/a_h^2$.

Performing integration in (28), we obtain

$$
M_{1,\lambda} = \frac{C\,(2\beta_h)^{5/4}\,a_h^2}{2\pi R_0^{*2} J_{3/2}\left(\tilde{X}_{n,1}\right)} \int\limits_0^{+\infty} dt \exp\left[-\left(\beta_h \eta_{\lambda h}^2 - \beta_0 + w + \frac{1}{2}\right)t\right]\left(1 - e^{-2t}\right)^{-1/2} \times
$$

$$
\left(1 - \exp\left[-2wt\right]\right)^{-1}\sum_{j=0}^{\infty}(-1)^j\left(\frac{1 - \exp\left[-2wt\right]}{1 + \exp\left[-2wt\right]}\right)^{2j+5/2} \times
$$

$$
\left[\frac{\sqrt{\pi}}{2}\Gamma\left(2j + \frac{5}{2}\right)\left(\frac{\tilde{X}_{n,1}\sqrt{2\beta_h}}{2R_0^*}\right)^{2j+3/2}\left(\frac{1 + \exp\left[-2t\right]}{1 - \exp\left[-2t\right]} - \frac{1 + \exp\left[-2wt\right]}{1 - \exp\left[-2wt\right]}\right)^{-1/2} - \right.
$$

$$
R_0^{*2}\sum_{n'=0}^{\infty}\frac{(-1)^{n'}(2j+3)!}{2n!\left(2j + \frac{5}{2} + n'\right)}\left(\frac{\tilde{X}_{n',1}\sqrt{2\beta_h}}{2R_0^*}\right)^{2j+3/2} \times
$$

$$
\left.\frac{V_{n,1,-1;n',1,-1}}{\pi^2 - \tilde{X}_{n',1}^2}\left(\frac{1 + \exp\left[-2wt\right]}{1 - \exp\left[-2wt\right]}\right)^{2j+5/2+n'}\left(\frac{1 + \exp\left[-2t\right]}{1 - \exp\left[-2t\right]} - \frac{1 + \exp\left[-2wt\right]}{1 - \exp\left[-2wt\right]}\right)^{-2j-3}\right]. \quad (29)
$$

After integration in (29), we finally obtain:

$$
\Phi\left(X\right) = \Phi_0\frac{X^2\bar{\beta}_h^4 u_1\bar{w}}{\left(J_{3/2}\left(\tilde{X}_{n,0}\right)\right)^2\bar{R}_0^{*5}} \times \frac{\Gamma\left(\frac{1}{2} - \bar{w}\right)\Gamma\left(\Delta + 1\right)}{\Delta^2\Gamma\left(\Delta - \bar{w} + \frac{1}{2}\right)} \times
$$

$$
\left[\Delta\left[\Psi\left(\Delta + 1\right) - \Psi\left(\Delta - \bar{w} + \frac{1}{2}\right)\right] - 1\right]P\left(u_1\right) \times
$$

$$
\left|\int\limits_0^{\infty} dt \exp\left[-\left(\bar{\beta}_h\eta_{\lambda h}^2 u_1^{3/2} - \bar{\beta}_0 u_1^{-1} + \bar{w} + \frac{1}{2}\right)t\right]\left(1 - e^{-2t}\right)^{-1/2}\left(1 - \exp\left[-2\bar{w}t\right]\right)^{-1} \times\right.
$$

$$
\sum_{j=0}^{\infty}(-1)^j\left(\frac{1 - \exp\left[-2\bar{w}t\right]}{1 + \exp\left[-2\bar{w}t\right]}\right)^{2j+5/2} \times
$$

$$
\left[\frac{\sqrt{\pi}}{2}\Gamma\left(2j + \frac{5}{2}\right)\left(\frac{\tilde{X}_{n,1}\sqrt{2\beta_h}}{2\bar{R}_0^*}\right)^{2j+3/2}\left(\frac{1 + \exp\left[-2t\right]}{1 - \exp\left[-2t\right]} - \frac{1 + \exp\left[-2wt\right]}{1 - \exp\left[-2wt\right]}\right)^{-1/2} - \right.
$$

$$
\bar{R}_0^{*2}\sum_{n'=0}^{\infty}\frac{(-1)^{n'}(2j+3)!}{2n'!\left(2j + \frac{5}{2} + n'\right)}\left(\frac{\tilde{X}_{n',1}\sqrt{2\beta_h}}{2\bar{R}_0^*}\right)^{2j+3/2} \times
$$

$$
\left.\left.\frac{V_{n,1,-1;n',1,-1}}{\pi^2 - \tilde{X}_{n',1}^2}\left(\frac{1 + \exp\left[-2wt\right]}{1 - \exp\left[-2wt\right]}\right)^{2j+5/2+n'}\left(\frac{1 + \exp\left[-2t\right]}{1 - \exp\left[-2t\right]} - \frac{1 + \exp\left[-2wt\right]}{1 - \exp\left[-2wt\right]}\right)^{-2j-3}\right]\right|^2, \quad (30)
$$

where $\Phi_0 = 6\sqrt{\varepsilon}e^2 2^2 a_h^3 \pi^{-3/2} |\mathbf{P}_{eh}\mathbf{e}_0| / \pi^2 c^3 m_0$; $\Delta = \left(\bar{\beta}_h \eta_{h\lambda}^2 u^{3/2} + \bar{w}\right)/2 + 1/4$; $\bar{w} = \sqrt{1 + \bar{\beta}_h^2 u^3 a^{*-4}/2}$;

$\bar{\beta}_0 = \beta_n e^2 / 4\pi\varepsilon_0 \varepsilon a_h \bar{R}_0^*$; $\bar{\beta}_h = \left(3 E_h a_h \bar{R}_0^{*3} \pi \varepsilon_0 \varepsilon\right)^{1/2} / \left(\pi^2 e^2\right)^{1/2}$; $X = \hbar\omega/E_d$; u – is the root of a transcendental equation of the form:

$$\pi^2/R_0^{*2}u + \sum_{nn'=1}^{\infty} R_0^{*2} u \left(V_{1n'}(u)\right)^2 / \left(\pi^2 - X_{n',1}^2\right) = \eta_{\lambda h}^2 + X.$$

3. Dependence of the spectral intensity of recombination radiation on the energy of emitted photon and on the magnitude of an external magnetic field

Figure 1 shows the frequency dependence of the spectral intensity of recombination radiation, as well as its dependence on the magnitude of an external magnetic field. The spectral intensity of recombination radiation in an external magnetic field increases, which is associated with an increase in the overlap integral of the envelope wave functions of a hole bound at the A^+-center and of an electron localized in the ground state of quantum dot. Fig. 2(a,b) shows the coordinate dependence of the wave function modulus square, for the A^+-state and for the electronic wave function of the ground state, respectively, for different values of the magnitude of an external magnetic field "B". It can be seen that as the value of B increases, the degree of localization both as for the hole (see Fig. 2a) and as for the electron wave functions increases, and accordingly the overlap integral increases.
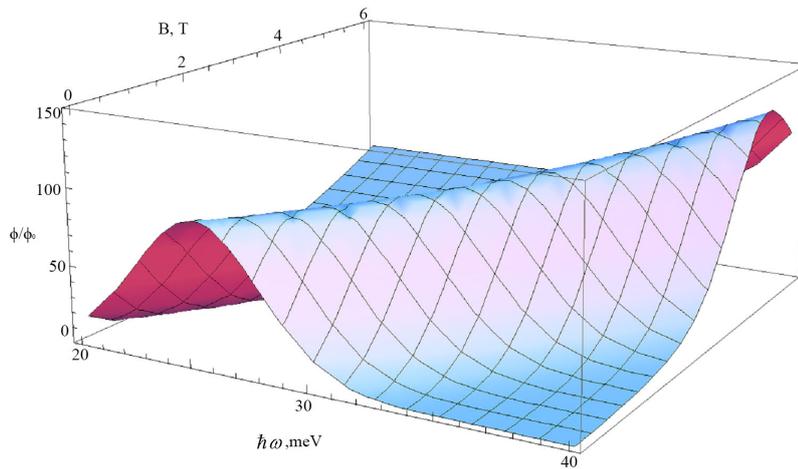

FIG. 1. Dependence of the spectral intensity of recombination radiation (in relative units) on the emitted photon energy and on the magnitude of an external magnetic field B, for the quasi-zero-dimensional structure of InSb–QD at $R_0 = 55$ nm

4. Conclusions

Dependence of the binding energy of a hole in the $A^+ + e$ complex on the magnitude of an external magnetic field has been investigated by the zero-range potential method in the adiabatic approximation. It is shown that in an external magnetic field there is a spatial anisotropy of the binding energy for A^+-state due to hybrid quantization in the QD radial plane and due to dimensional quantization in the direction of an external magnetic field. In the dipole approximation, the frequency dependence calculation of the spectral intensity of recombination radiation for a quasi-zero-dimensional structure in an external magnetic field has been performed, taking into account dispersion of the QDs radius. It is shown that in an external magnetic field the spectral intensity of recombination radiation curve shifts to the short-wave region of the spectrum and probability of the electron radiative transition to the A^+-center level increases, which is associated with an increase in the overlap integral of the envelope wave functions of a hole bound at the A^+-center and of an electron localized in the ground state of quantum dot. The obtained results can be used in the development of IR sources or terahertz radiation (depending on the QD radius), on the basis of quasi-zero-dimensional structures with impurity complexes, with parameters controlled in an external magnetic field.
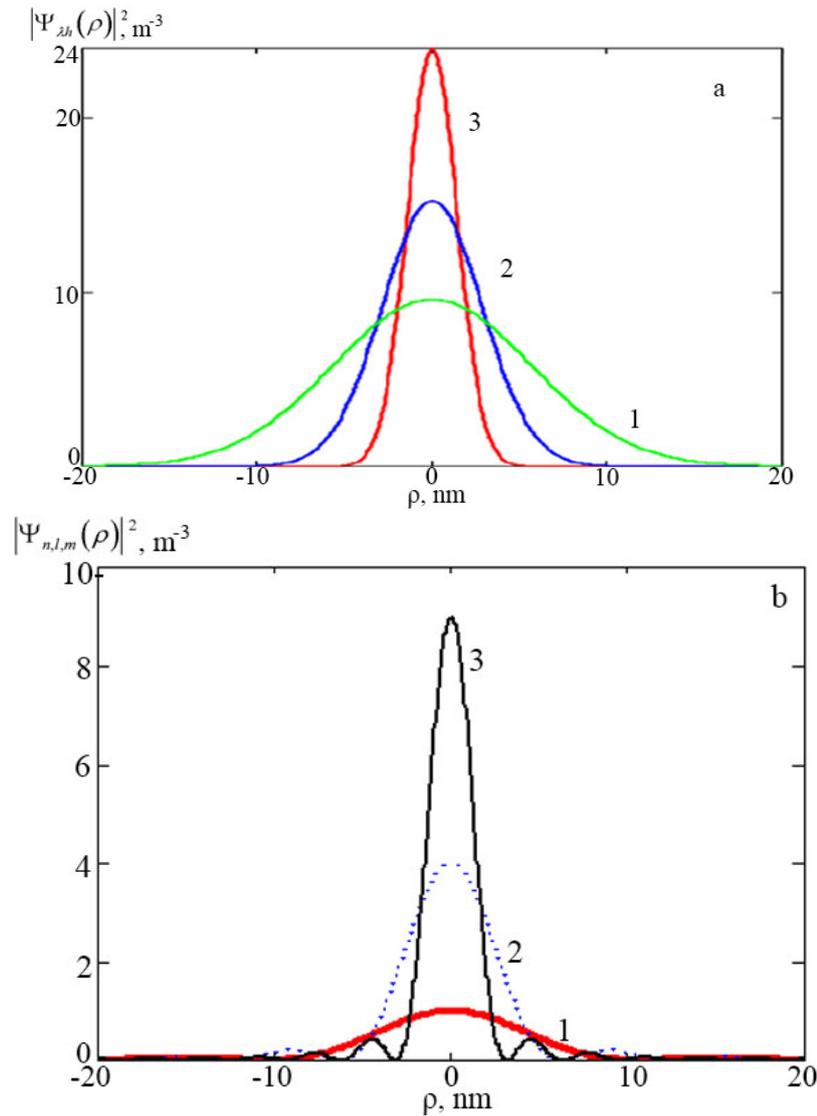
FIG. 2. The coordinate dependence of the wave function modulus square: (a) for A^+-state and (b) for the electronic wave function for various values of the magnetic field intensity B. 1: $B = 0$; 2: $B = 2T$; 3: $B = 5T$, at $R_0 = 20$ nm

Acknowledgements

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