NANOSYSTEMS: PHYSICS, CHEMISTRY, MATHEMATICS, 2017, 8 (2), P. 247–259

Kolmogorov equation for Bloch electrons and electrical resistivity models for nanowires

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PACS 02.30.Mv, 02.60.Nm, 05.20.Dd, 73.23.b **DOI 10.17586/2220-8054-2017-8-2-247-259**

The problem of a nanowires conductivity is studied from a kinetic point of view for quasiclassical Bloch electrons in an electric field. Few statements of problems with cylindrical symmetry for the integro-differential Kolmogorov equation are formulated: the dynamic Cauchy problem and two stationary boundary regime ones. The first is for an empty cylinder with scattering of the conduction electrons on walls, the second takes into account scattering on defects inside the wire. The integro-differential equations are transformed to integral ones and solved iteratively. There are two types of expansions with the leading terms in the right and left sides. The iteration series is constructed and its convergence studied.

Keywords: Kolmogorov kinetic equation, N-fold series, Bloch electrons, electrical resistivity, nanowires.

Received: 14 January 2017

Revised: 2 February 2017

1. Introduction

We consider a problem of a solid nanowires conductivity, which demonstrates an intriguing dependence on temperature and wire diameter that is fundamentally different from its bulk counterpart (e.g. for (Bi) see [1–3]). Conductivity is a non-equilibrium phenomenon, therefore we will use a kinetic description of charge carriers, following the idea of Bloch electrons as quasiclassical particles [4]. Having in mind the key element of the theory, a Bloch wave scattering problem [9], the distribution function (DF) f is interpreted as either a density number of electrons or probability density in phase space Γ with continuous wavenumbers (velocities) approximation.

There are few parameters, typical for electrons in solids, those are:

1. de Broglie wavelength $\lambda = h/p$;
2. v – typical velocity, e.g. one that enters Fermi–Dirac distribution function (FDF) as parameter;
3. τ_r – transport relaxation time;
4. mean free path: $l = v\tau_r$.

It is convenient to use dimensionless variables in the phase space Γ. In the next sections, we will imply that the position coordinates are measured in free path l units, while the velocity components are measured in units of v. One of aims of such description may be evaluation of correction to ballistic formula, or, more generally, to reproduce the Landauer regime [4].

The main *motivation* of this study is to link the conductivity parameters with temperature, that is achieved by a statement of problem formulation in such form, which includes the FDF distribution as an initial condition or a boundary one. A statement of the problem naturally implies that the conductor geometry is cylindrical for nanowires or other interesting cases, for example a point contact for tunnel microscopy.

For the distribution function, we take the integro-differential Kolmogorov equation [5] that was applied in [7] to the LIDAR problem, in [6] to the neutrons and to X-rays scattering [8]. Its form is presented in Sec. 2, where the collision terms are specified.

As the main mathematical tool of the basic equation solution, we derive an expansion in N-fold scattering series of the Bloch electron distribution function in a conducting domain. We also present a transition to integral equations and compact formulas for the distribution function, in particular, for the first and second iterations in this expansion. We consider a nanowire as a cylindrical waveguide via the choice of the domain geometry and reflective scattering by the walls.

In the Sec. 3, we describe the method of solution via the iterative scheme for the basic integro-differential equation, using the characteristic variables for the differential part, that allows us to transform the equation to an integral one. This problem admits studying transition regimes of switching and pulses of current.

In Sec. 4, we simplify the problem, by switching to the stationary case, which needs different characteristic variables to transform the differential part. We also formulate two problems, one for an empty cylinder with reflecting wall for the case in which the scattering inside the wire is much smaller than at boundary (Subsec. 4.2).

The second problem is closer to the normal temperature regime, when scatterers (e.g. phonons) fill the volume homogenously (Subsec. 4.3). To solve the problem, we expand the DF in series by number of collisions and derive the operator that links the neighboring terms (Subsec 4.5). In this problem, we choose the leading FDF term in the differential part as in [4] which results in the Fredholm equation for the next (first) term for the expansion.

Sec. 5 is devoted to the general iterative construction for the case of $\epsilon = 0$, specified to 1-fold and two-fold scattering solutions. Its explicit form allows one to evaluate the averaged values that give the formula for a current through a wire. The paper is concluded by a series convergence theorem at some conditions, that may permit one to estimate the number of terms and error of the corresponding calculation (Sec. 6).

2. Problem formulation

2.1. Kinetic equation and boundary conditions

Let us define the *collision integral* by the sum of losses by scattering with eventual account of absorption:

$$I_- = -f(\vec{r},\vec{v}) \int \sigma(\vec{r},\vec{v} \to \vec{v}')d\vec{v}' = -\sigma_t(\vec{r})f, \tag{1}$$

and the return term:

$$I_+ = \int \sigma(\vec{r},\vec{v}' \to \vec{v})f(\vec{r},\vec{v}')d\vec{v}', \tag{2}$$

so that *Kolmogorov kinetic equation* for Bloch electron in the phase space $\{\vec{r},\vec{v}\} \in \Gamma$ under action of electric field $\vec{E}$, directed along z, have the form:

$$\frac{\partial f}{\partial t} + \frac{e}{m}E\frac{\partial f}{\partial v_z} + \vec{v}\cdot\nabla f = -\sigma_t(z)f + I_+, \tag{3}$$

where $f(t,\vec{r},\vec{v})$ is the probability density function over phase space $\{\vec{r},\vec{v}\} \in \Gamma$, $\vec{v} = (v\sin\theta\cos\phi, v\sin\theta\sin\phi, v\cos\theta)$ and $\sigma_t(z)$, $\sigma(\vec{r},\vec{v}' \to \vec{v}) = \sigma(\gamma, z)$ are total and differential cross section densities per unit volume of a medium. In spherical coordinates of incident θ,ϕ and scattered θ',ϕ' particles the scattering angle is characterized as follows:

$$\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi - \phi'); \tag{4}$$

for elastic scattering ($v = v'$). We will discuss two problems: initial and boundary regime ones.

Initial problem. The function $f(t,\vec{r},\vec{v})$ is used as a distribution (generalized function) defined by action on the Schwartz space $\psi(\vec{r},v,\theta,\phi) \in \mathbf{S}$, via continuous linear functional $(f,\psi) \in \mathbb{R}$. The initial condition for (3) is also represented by a distribution. For example, for an initiation point with fixed velocity, we take:

$$f(0,\vec{r},v,\theta,\phi) = V\delta(\vec{r})\delta(\theta), \tag{5}$$

with a constant V as normalization factor. This means that we built a solution for the probability density as a weak limit (when $t \to 0$) to the δ-function at $t = 0$. The distribution $\delta(\theta)$ is chosen as:

$$(\delta(\theta), \psi(\vec{r},v,\theta,\phi)) = \int_0^{2\pi} \psi(\vec{r},v,0,\phi)d\phi, \tag{6}$$

$\psi \in \mathbf{S}$, and, in a conventional mode, $\delta(\vec{r}) = \delta(x)\delta(y)\delta(z)$.

Boundary problem may be used when a conductor is in electric contact with a metal, that is characterized by some given FDF:

$$f(t,x,y,0,\vec{v}) = f_F(x,y,\vec{v}). \tag{7}$$

2.2. Distribution averaging. Electric current as number of particles rate

This is derived in direct applications as an integral by space variables which enter the solution as parameters, used as a receiver (anode) geometry description. Thus, we are concerned with:

$$J(\Delta,t) = \int_{\theta_0}^{\theta_1} (f(t,\vec{r},v,\theta,\phi),\psi)d\theta. \tag{8}$$

The action of the distribution f on $\mathbf{S}$ in the case of segmented continuous functions implies the integration with respect to ϕ, x, y, z. The applications relate to observations (measurements) as the result of averaging procedure, defined by (8). The expression (8) defines number of particles within a finite domain (Δ) of a measurement apparatus and having velocity direction between θ_0 and θ_1 restricted by aperture related to the apparatus window direction.

Our particular aim is the evaluation of number of particles per unit time which enter the round area of radius ρ_0 laying in the plane $z = z_c$ (receiver) with center in $x = y = 0$ and having velocity vectors inclined to z-axis within the angle interval $\theta \in [0, \theta_0]$. Here, an aperture angle θ_0, restricts possible velocities of particles directions. In the sample case we take here, the receiver domain Δ has cylindrical symmetry and for the initial direction along z, the function ψ does not depend on θ, ϕ, so it is defined as zero outside the receiver, and $\psi(x, y, z) = 1$ for internal points of the domain $x^2 + y^2 \leq \rho_0^2$, $z_0 \leq z \leq z_0 + \Delta t|\cos\theta|$ and zero outside, being z_0 the coordinate of anode interface, Δt – reaction time; for instant reaction:

$$I(t) = e \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_0^{\rho_0} \int_0^{\theta_0} \int_0^{2\pi} v_z(f(t, \rho, z, \theta, \phi), \psi(x, y, z)) \sin\theta d\phi d\theta d\rho. \tag{9}$$

Similar geometry was used in [8] for application to the problem of X-ray scattering.

3. Method of solution. N-fold scattering expansion

3.1. Cylindrical coordinates

A solution method depends on the conductor geometry and problem symmetry, that in our case of nanowires we have chosen as cylindrical one. Let the scattering cross section depend only on ρ and γ. For such a problem, we take (3) in cylindric coordinates neglecting $\dfrac{\partial f}{\partial \phi}$ term, having:

$$Lf = \frac{\partial f}{\partial t} + \epsilon \frac{\partial f}{\partial v_z} + v_z \frac{\partial f}{\partial z} + v_\rho \frac{\partial f}{\partial \rho} = -\sigma_t(\rho) f + \int_0^{\pi} \sigma(\gamma, \rho) f d\gamma, \tag{10}$$

with $\epsilon = \dfrac{e}{m} E$, $cos\gamma = (\vec{v}, \vec{v}')/v^2$.

For the first (Cauchy) problem, we take an initial condition, if the external field E is switched on at $t = 0$:

$$f(0, \rho, v_z, v_\rho) = f_F(\vec{v})\theta(\rho_0 - \rho), \tag{11}$$

where f_F is the FDF, θ – step (Heaviside) distribution.

A solution is searched as a N-fold scattering expansion:

$$f = f_0 + f_1 + f_2 + ... \tag{12}$$

We choose for the leading term f_0 the equation:

$$Lf_0 = \frac{\partial f_0}{\partial t} + \epsilon \frac{\partial f_0}{\partial v_z} + v_z \frac{\partial f_0}{\partial z} + v_\rho \frac{\partial f_0}{\partial \rho} = -\sigma_t(\rho) f_0, \tag{13}$$

which accounts only for losses, and initial condition:

$$f_0(0, \vec{\rho}, v_z, v_\rho) = f_F(\vec{v})\theta(\rho_0 - \rho). \tag{14}$$

The general approach for solution to the kinetic equation uses the transition to *characteristic variables*. Let us change variables in (13), putting $t' = t$, $v'_\rho = v_\rho$, and

$$v'_z = v_z + \epsilon(z - t) - v_z^2/2, \tag{15a}$$

$$\rho' = \rho - v_\rho t, \tag{15b}$$

$$z' = -\epsilon z + v_z^2/2. \tag{15c}$$

Equation (13) is transformed as:

$$\frac{\partial f_0}{\partial t'} = -\sigma_t(\rho' + v'_\rho t') f_0, \tag{16}$$

which is directly integrated including arbitrary *functional parameter G*:

$$f_0 = G(\vec{r}', \vec{v}') \exp\left[-\int_0^{t'} \sigma_t(\rho' + v'_\rho \tau) d\tau \right]. \tag{17}$$

It is useful to introduce a function Q via:

$$Q(t, \rho, v_\rho) = \exp\left[-\int_0^{t} \sigma_t(\rho + v_\rho \tau) d\tau \right]. \tag{18}$$

Going back to the original variables results in:

$$f_0 = G\left(-\epsilon z + v_z^2/2, \rho - v_\rho t, v_z + \epsilon(z-t) - v_z^2/2, v_\rho\right) \exp\left[-\int_0^t \sigma_t(\rho - v_\rho(t-\tau))d\tau\right]. \quad (19)$$

At $t = 0$,

$$f_0 = G\left(-\epsilon z + v_z^2/2, \rho, v_z + \epsilon z - v_z^2/2, v_\rho\right). \quad (20)$$

The function G is found from initial conditions (14):

$$G\left(-\epsilon z + v_z^2/2,\ \rho,\ v_z + \epsilon z - v_z^2/2,\ v_\rho\right) = f_F(\vec{v})\theta\left(\rho_0 - \rho\right). \quad (21)$$

3.2. Iterations construction

For $n \geq 0$, the expansion is defined by:

$$\frac{df_{n+1}}{dt'} = -\sigma_t f_{n+1} + \int_0^\pi \sigma(\gamma, \rho) f_n d\gamma, \quad (22)$$

$n = 0, 1, ...,$ with zero initial conditions for $n > 0$:

$$f_n\big|_{t=0} = 0. \quad (23)$$

Transforming (22) by (15) and plugging its inverse:

$$v_z = v_z' + z' + \epsilon t', \quad (24a)$$

$$\rho = \rho' + v_\rho' t', \quad (24b)$$

$$z = \epsilon^{-1}\left(-z' + (v_z' + z' + \epsilon t')^2/2\right), \quad (24c)$$

$$t = t', \quad (24d)$$

into the r.h.s. of (22) yields:

$$\frac{\partial f_{n+1}}{\partial t'} = -\sigma_t\left(\rho' + v_\rho' t'\right) f_{n+1} + \int_0^\pi \sigma\left(\gamma, \rho' + v_\rho' t'\right) f_n\Big(t',\ \epsilon^{-1}\left(-z' + (v_z' + z' + \epsilon t')^2/2\right),\ \rho' + v_\rho t',\ v_z' + z' + \epsilon t',\ v_\rho'\Big) d\gamma. \quad (25)$$

To have a more compact form, we define $f_{n+1} = Q\hat{f}_{n+1}$, which gives:

$$\frac{d\hat{f}_{n+1}}{dt'} = Q^{-1}\int_0^\pi \sigma\left(\gamma, \rho' + v_\rho' t'\right) f_n\Big(t',\ \epsilon^{-1}\left(-z' + (v_z' + z' + \epsilon t')^2/2\right),\ \rho' + v_\rho t',\ v_z' + z' + \epsilon t',\ v_\rho'\Big) d\gamma, \quad (26)$$

where the definition of $Q(t', \rho', v_{\rho'})$ by (18) and the relation $\dfrac{\partial Q}{\partial t'} = -\sigma_t Q$ are used. After integration, one has:

$$\hat{f}_{n+1} = Q^{-1} f_{n+1} = \int_0^{t'} \exp\left[\int_0^\tau \sigma_t(\rho' + v_\rho' \tau')d\tau'\right] \int_0^\pi \sigma\left(\gamma,\ \rho' + v_\rho' \tau\right) f_n\Big(\tau,\ \epsilon^{-1}\left(-z' + (v_z' + z' + \epsilon\tau)^2/2\right),\ \rho' + v_\rho \tau,\ v_z' + z' + \epsilon\tau,\ v_\rho'\Big) d\gamma d\tau. \quad (27)$$

In the original variables, it reads as:

$$f_{n+1} = Q\int_0^t \exp\left[\int_0^\tau \sigma_t\left(\rho - v_\rho(t-\tau')\right)d\tau'\right] \int_0^\pi \sigma\left(\gamma, \rho - v_\rho(t-\tau)\right) f_n\Big(\tau,\ \epsilon^{-1}\left(\epsilon z - v_z^2/2 + (v_z - \epsilon(t-\tau))^2/2\right),\ \rho - v_\rho(t-\tau),\ v_z - \epsilon(t-\tau),\ v_\rho\Big) d\gamma d\tau. \quad (28)$$

4. Stationary case

4.1. Boundary regime problem

For the next topic, *boundary regime problem*, we study the *stationary kinetic equation* in cylindrical coordinates:

$$v_z \frac{\partial f}{\partial z} + v_\rho \frac{\partial f}{\partial \rho} + \epsilon \frac{\partial f}{\partial v_z} = -\sigma_t(\vec{\rho}) f + \int \sigma(\vec{\rho}, \vec{v}', \vec{v}) f(\vec{\rho}, \vec{v}') d\vec{v}', \tag{29}$$

where $\frac{e}{m}E = \epsilon$, $f(z, \rho, \vec{v})$ is a DF over phase space $\{\vec{\rho}, \vec{v}\} \in \Gamma$, $\vec{v} = (v_z, v_\rho)$, $\vec{\rho} = (z, \rho)$ and $\sigma_t(z), \sigma(\vec{\rho}, \vec{v}', \vec{v})$ are total and differential cross section densities per unit volume. The function $f(\vec{\rho}, \vec{v})$ is used as a distribution defined by action on the Schwartz space $\psi(\vec{\rho}, \vec{v}) \in \mathbf{S}$, via continuous linear functional $(f, \psi) \in \mathbb{R}$. The boundary conditions for (29) is represented by a distribution, this paper choice is restricted by:

$$f(0, \rho, v_z, v_\rho) = f_F(\vec{v})\theta(\rho_0 - \rho), \tag{30}$$

where $f_F = \left(\exp\left[\frac{H - H_F}{k_B T}\right] + 1\right)^{-1}$ is the Fermi–Dirac electron DF. The energy $H = mv^2/2$ for quasi free Bloch electron, H_F – Fermi energy, T – temperature. This means that we have constructed a solution for the probability density with the boundary conditions as a weak limit (when $z \to 0$).

4.2. Stationary problem solution for empty cylinder with reflecting wall

Let us change the variables in (29) as:

$$z' = z - \epsilon\rho + v_\rho v_z, \tag{31a}$$

$$\rho' = \epsilon\rho - v_\rho v_z, \tag{31b}$$

$$v_z' = v_z^2 - 2\epsilon z. \tag{31c}$$

The inverse transformation reads as:

$$z = z' + \rho', \tag{32a}$$

$$\rho = \epsilon^{-1}\left(\rho' + v_\rho\sqrt{v_z' + 2\epsilon(z' + \rho')}\right), \tag{32b}$$

$$v_z = \sqrt{v_z' + 2\epsilon(z' + \rho')}. \tag{32c}$$

Hence, for a homogeneous along z elastic scattering at the cylinder wall $\rho = \rho_0$, with the differential cross-section:

$$\sigma(\rho, v_z, v_\rho, ...) = s\delta(\rho - \rho_0)\delta(v_z - v_z')\delta(v_\rho + v_\rho'), \tag{33}$$

as a function of the parameter v_ρ, simplifies the collision integral, giving the equation:

$$\begin{aligned} Lf(z, \rho_0, v_z, v_\rho) = \epsilon\frac{\partial f}{\partial v_z} + v_z\frac{\partial f}{\partial z} + v_\rho\frac{\partial f}{\partial \rho} = \\ - s\delta(\rho - \rho_0)f + \int s\delta(\rho - \rho_0)\delta(v_z - v_z')\delta(v_\rho + v_\rho')f(\vec{\rho}, \vec{v}')d\vec{v}' = \\ s\delta(\rho - \rho_0)\left[f(z, \rho_0, v_z, -v_\rho) - f(z, \rho_0, v_z, v_\rho)\right]. \end{aligned} \tag{34}$$

Taking the equation at the opposite v_ρ point and combining the results, yields:

$$\begin{aligned} L\big[f(z, \rho_0, v_z, v_\rho) + f(z, \rho_0, v_z, -v_\rho)\big] = Lf^+ = 0, \\ L\big[f(z, \rho_0, v_z, v_\rho) - f(z, \rho_0, v_z, v_\rho)\big] = Lf^- = \\ 2s\delta(\rho - \rho_0)\big[f(z, \rho_0, v_z, -v_\rho) - f(z, \rho_0, v_z, v_\rho)\big] = 2s\delta(\rho - \rho_0)f^-. \end{aligned} \tag{35}$$

Transition to the variables (31) first gives:

$$\frac{\partial f^+}{\partial z'} = 0, \tag{36}$$

and, secondly:

$$\frac{\partial f^-}{\partial z'} = 2s\frac{\delta\left(\epsilon^{-1}(\rho' + v_\rho\sqrt{v_z' + 2\epsilon(z' + \rho')}) - \rho_0\right)}{\sqrt{v_z' + 2\epsilon(z' + \rho')}} f^-\left(z' + \rho',\ \rho_0,\ \sqrt{v_z' + 2\epsilon(z' + \rho')},\ v_\rho\right). \tag{37}$$

The direct integration results in the first case as:

$$f^+ = \Phi^+(\rho', v_z', v_\rho), \tag{38}$$

while the second one yields in:

$$f^- = \int\limits_0^{z'} \frac{s\delta\left(\epsilon^{-1}(\rho' + v_\rho\sqrt{v_z' + 2\epsilon(\tau+\rho')}) - \rho_0\right)}{\sqrt{v_z' + 2\epsilon(\tau+\rho')}} f^-\left(z'+\rho',\ \rho_0,\ \sqrt{v_z' + 2\epsilon(\tau+\rho')},\ v_\rho\right) d\tau + \Phi^-(\rho', v_z', v_\rho'). \tag{39}$$

Solving the equation:

$$\rho' + v_\rho\sqrt{v_z' + 2\epsilon(\tau+\rho')} - \epsilon\rho_0 = 0$$

with respect to τ, we obtain:

$$\tau_0 = z - \epsilon\rho + v_\rho v_z + \frac{\epsilon}{2}\left\{\frac{(\rho_0-\rho)}{v_\rho}\right\}^2 - v_z\frac{(\rho_0-\rho)}{v_\rho},$$

which defines the only zero value of the δ-function argument. We plug it into (40), using $\delta(f(\tau)) = \dfrac{\delta(\tau-\tau_0)}{|f'(\tau_0)|}$, while:

$$\frac{d}{d\tau}\left[\epsilon^{-1}\left(\rho' + v_\rho\sqrt{v_z' + 2\epsilon(\tau+\rho')}\right) - \rho_0\right] = \frac{v_\rho}{\sqrt{v_z' + 2\epsilon(\tau+\rho')}},$$

and return to original variables by (32):

$$f^- = \int\limits_0^{z'} \frac{s\delta\left(\epsilon^{-1}\left(\rho' + v_\rho\sqrt{v_z' + 2\epsilon(\tau+\rho')}\right) - \rho_0\right)}{\sqrt{v_z' + 2\epsilon(\tau+\rho')}} f^-\left(z'+\rho',\ \rho_0,\ \sqrt{v_z' + 2\epsilon(\tau+\rho')},\ v_\rho\right) d\tau$$

$$= \begin{cases} \dfrac{s}{v_\rho} f^-\left(z,\ \rho_0,\ \dfrac{\epsilon\rho_0 - \epsilon\rho + v_\rho v_z}{v_\rho},\ v_\rho\right), & \text{if } \tau_0 \in [0,\ z - \epsilon\rho + v_\rho v_z] \\ 0, & \text{otherwise.} \end{cases} \tag{40}$$

We introduce $\hat{t} = \dfrac{\rho_0 - \rho}{v_\rho}$, then, inside the interval $\tau_0 \in [0,\ z - \epsilon\rho + v_\rho v_z]$, we write:

$$f^-(z, \rho, v_z, v_\rho) = \frac{s}{v_\rho} f^-\left(z, \rho_0, v_z + \epsilon\hat{t}, v_\rho\right), \tag{41}$$

and

$$\tau_0 = z - \epsilon\rho + v_\rho v_z + \frac{\epsilon}{2}\hat{t}^2 - v_z\hat{t}.$$

Its border values are:

$$\tau_0 = z - \epsilon\rho + v_\rho v_z + \frac{\epsilon}{2}\hat{t}^2 - v_z\hat{t} = z - \epsilon\rho + v_\rho v_z,$$

and

$$\tau_0 = z - \epsilon\rho + v_\rho v_z + \frac{\epsilon}{2}\hat{t}^2 - v_z\hat{t} = 0.$$

The first one gives the condition:

$$\frac{\epsilon}{2}\hat{t}^2 - v_z\hat{t} = 0,$$

which is equivalent to either relation: $\hat{t} = 0$, that fix the point

$$\rho = \rho_0,$$

or defines a hyperbolic curve in velocity space for each ρ

$$\frac{\epsilon}{2}\frac{\rho_0 - \rho}{v_\rho} - v_z = 0.$$

The second one also defines the curve:

$$v_z = \frac{z - \epsilon\rho + \frac{\epsilon}{2}\hat{t}^2}{\hat{t} - v_\rho}. \tag{42}$$

Both curves determine the integration domain for a mean value of DF in velocity subspace evaluation as in Sec. 2.3.

The symmetry of the boundary regime with respect to reflection $v_\rho \to -v_\rho$ yields:

$$f^-(0, \rho, v_z, v_\rho) = 0, \tag{43}$$

while (30) gives:

$$f^{+}(z,\rho,v_z,v_\rho) = \Phi^{+}\left(\epsilon\rho - v_\rho v_z, v_z^2 - 2\epsilon z, v_\rho\right) = 2f_F(\vec{v})\theta(\rho_0-\rho).$$

Taking the functional equation (40) at the point $z=0$, at $\tau_0 \in [0,\ z-\epsilon\rho+v_\rho v_z]$, the condition (45) reads:

$$f^{-}\left(0,\rho,v_z,v_\rho\right) = \frac{s}{v_\rho} f^{-}\left(0,\rho_0,v_z+\epsilon\hat{t},v_\rho\right) = 0. \tag{44}$$

The final step gives the DF:

$$f = (f^{-}+f^{+})/2. \tag{45}$$

4.3. Stationary problem for cylinder filled with scatterers

In this section, we suppose that the scattering inside the wire admits homogeneous distribution along z ($l \ll \lambda_B$), looking for the further simplification. Let $f(\rho,v_z,v_\rho)$ be a distribution function for Bloch electrons, that solves the equation (cf. (29)):

$$\epsilon\frac{\partial f}{\partial v_z} + v_\rho\frac{\partial f}{\partial \rho} = -\sigma_t(\vec{\rho})f + \int \sigma(\vec{\rho},\vec{v}',\vec{v})f(\vec{\rho},\vec{v}')d\vec{v}'. \tag{46}$$

Adding to the expression (33) the term that model elastic scattering with a given dependence on the only scattering angle inside the cylinder, we have:

$$\sigma(\rho,v_z,v_\rho,...) = s\delta(\rho-\rho_0)\delta(v_z-v_z')\delta(v_\rho+v_\rho') + \sigma_0(\gamma)\delta(v-v')\theta(\rho-\rho_0). \tag{47}$$

Simplifying the model by $\sigma_0(\gamma)=\sigma_0$ and plugging (47) in (29) gives:

$$\begin{aligned}\epsilon\frac{\partial f}{\partial v_z} + v_\rho\frac{\partial f}{\partial \rho} = &-\left[s\delta(\rho-\rho_0)+\sigma_0\theta(\rho-\rho_0)\right]f(\rho,v_z,v_\rho)- \\ &s\delta(\rho-\rho_0)\int \delta(v_z-v_z')\delta(v_\rho+v_{\rho'})f(\rho,v_z',v_\rho')dv_z'dv_\rho'+ \\ &\sigma_0\theta(\rho_0-\rho)\Big[f(\rho,v_z,v_\rho)+\int \delta(v^2-v'^2)f(\rho,v_z',v_\rho')dv_z'dv_\rho'\Big].\end{aligned} \tag{48}$$

After integration, in collision terms with $\delta(f(\tau)) = \sum_j \frac{\delta(\tau-\tau_j)}{|f'(\tau_j)|}$ account, we write:

$$\begin{aligned}\epsilon\frac{\partial f}{\partial v_z} + v_\rho\frac{\partial f}{\partial \rho} = &\ s\delta(\rho-\rho_0)\left[f(\rho,v_z,v_\rho)-f(\rho,v_z,-v_\rho)\right]+ \\ &\sigma_0\theta(\rho_0-\rho)\left[f(\rho,v_z,v_\rho)+\int \frac{f(\rho,v_{z+}',v_\rho')+f(\rho,v_{z-}',v_\rho')}{2\left|\sqrt{v_\rho^2+v_z^2-v_\rho'^2}\right|}dv_\rho'\right],\end{aligned} \tag{49}$$

where:

$$v_{z\pm}' = \pm\sqrt{v_\rho^2+v_z^2-v_\rho'^2}. \tag{50}$$

For this case, the *characteristic variables* are defined by the direct transform:

$$\rho' = \rho/v_\rho, \quad v_z' = v_z - \epsilon\rho/v_\rho, \tag{51}$$

and the inverse one as:

$$\rho = \rho' v_\rho, \quad v_z = v_z' + \epsilon\rho'. \tag{52}$$

This gives:

$$\begin{aligned}\frac{\partial f}{\partial \rho'} = &\ s\delta(\rho' v_\rho-\rho_0)\left[f(\rho' v_\rho,v_z'+\epsilon\rho',v_\rho)-f(\rho' v_\rho,v_z'+\epsilon\rho',-v_\rho)\right]+ \\ &\sigma_0\theta(\rho_0-\rho' v_\rho)\left[f(\rho' v_\rho,v_z'+\epsilon\rho',v_\rho)+\int \frac{f(\rho' v_\rho,v_{z+}',v_\rho')+f(\rho' v_\rho,v_{z-}',v_\rho')}{2\left|\sqrt{v_\rho^2+v_z^2-v_\rho'^2}\right|}dv_\rho'\right],\end{aligned} \tag{53}$$

where now:

$$v_{z\pm}' = \pm\sqrt{v_\rho^2+(v_z'+\epsilon\rho')^2-v_\rho'^2}. \tag{54}$$

Integrating, we arrive at the integral equation:

$$f(\rho', v_z', v_\rho) = s\int_0^{\rho'} \delta(rv_\rho - \rho_0)\Big[f(rv_\rho, v_z' + \epsilon r, v_\rho) - f(rv_\rho, v_z' + \epsilon r, -v_\rho)\Big]dr +$$

$$\sigma_0 \int_0^{\rho'} \theta(rv_\rho - \rho_0)\left[f(rv_\rho, v_z' + \epsilon r, v_\rho) + \int \frac{f(rv_\rho, v_{z+}', v_\rho') + f(rv_\rho, v_{z-}', v_\rho')}{2\left|\sqrt{v_\rho^2 + (v_z' + \epsilon r)^2 - v_\rho'^2}\right|} dv_\rho'\right] dr. \quad (55)$$

Now

$$v_{z\pm}' = \pm\sqrt{v_\rho^2 + (v_z - \epsilon\rho/v_\rho + \epsilon r)^2 - v_\rho'^2}. \quad (56)$$

In primary variables, it is:

$$f(\rho, v_z, v_\rho) = s\int_0^{\rho/v_\rho} \delta(rv_\rho - \rho_0)\Big[f(rv_\rho, v_z - \epsilon(\rho/v_\rho - r), v_\rho) - f(rv_\rho, v_z - \epsilon(\rho/v_\rho - r), -v_\rho)\Big]dr +$$

$$\sigma_0 \int_0^{\rho/v_\rho} \theta(\rho_0 - rv_\rho) \int \frac{f(rv_\rho, v_{z+}', v_\rho') + f(rv_\rho, v_{z-}', v_\rho')}{2\left|\sqrt{v_\rho^2 + (v_z - \epsilon\rho/v_\rho + \epsilon r)^2 - v_\rho'^2}\right|} dv_\rho' dr. \quad (57)$$

Changing the variable of integration as $rv_\rho = t$, $v_\rho > 0$, yields:

$$f(\rho, v_z, v_\rho) =$$

$$\frac{s}{v_\rho}\int_0^{\rho} \delta(t - \rho_0)\left[f\left(t, v_z - \frac{\epsilon}{v_\rho}(\rho - t), v_\rho\right) - f\left(t, v_z - \frac{\epsilon}{v_\rho}(\rho - t), -v_\rho\right)\right] dt +$$

$$\frac{\sigma_0}{v_\rho}\int_0^{\rho} \theta(\rho_0 - t)\int \frac{f(t, v_{z+}', v_\rho') + f(t, v_{z-}', v_\rho')}{2\left|\sqrt{v_\rho^2 + (v_z + \epsilon\frac{t-\rho}{v_\rho})^2 - v_\rho'^2}\right|} dv_\rho' dt =$$

$$\begin{cases} \dfrac{\sigma_0}{2v_\rho}\displaystyle\int_0^{\rho}\int \frac{f(t, v_{z+}', v_\rho') + f(t, v_{z-}', v_\rho')}{\left|\sqrt{v_\rho^2 + \left(v_z + \epsilon\frac{t-\rho}{v_\rho}\right)^2 - v_\rho'^2}\right|} dv_\rho' dt, & \text{if } \rho < \rho_0, \\ \dfrac{s}{2}\Big[f(\rho_0, v_z, v_\rho) - f(\rho_0, v_z, -v_\rho)\Big], & \text{if } \rho = \rho_0, \end{cases} \quad (58)$$

with v_{z+} under square root.

4.4. Alternative expansion for stationary case

Neglecting the longitudinal inhomogeneity (the second term) in (34) let us study it as the basic equation. The structure of the integro-differential equation and physical sense of its terms in conditions of constant current suggest an alternative expansion with the leading term in the l.h.s. [4]. A solution is searched as an expansion (12), but we choose for the link between f_0 and f_1, the equation:

$$Lf_0 = \epsilon\frac{\partial f_0}{\partial v_z} + v_\rho\frac{\partial f_0}{\partial \rho} = -\sigma_t f_1 + \int \sigma(\vec{\rho}, \vec{v}', \vec{v}) f_1(\vec{\rho}, \vec{v}') d\vec{v}, \quad (59)$$

taking the FDF distribution for f_0 inside the cylinder of radius ρ_0:

$$f_0(\rho, v_z, v_\rho) = f_F(v_z, v_\rho)\theta(\rho_0 - \rho). \quad (60)$$

Differentiating in the l.h.s., and switching to angle variables in collision integral as in (13), one has:

$$\epsilon\frac{\partial f_F(v_z, v_\rho)}{\partial v_z}\theta(\rho_0 - \rho) + v_\rho f_F(v_z, v_\rho)\delta(\rho - \rho_0) = -\sigma_t f_1 + \int_0^{\pi} \sigma(\vec{\rho}, \gamma) f_1(\vec{\rho}, \vec{v}') d\gamma. \quad (61)$$

Equation (61) is the Fredholm II integral equation with continuous kernel for the function f_1. The theory of such equations guarantee the existence and methods of its solution outside the integral operator spectrum, which depends on details of the kernel $\sigma(\vec{\rho}, \gamma)$ behavior and needs special investigation after the kernel (cross-section) is specified for given material.

5. Approximate solutions

5.1. N-fold iteration ($\epsilon = 0$)

In this section, we consider the problem of zero field transport for conductors contact pulse current excitations as a Cauchy problem. Let us return to the spherical coordinates for the basic equation (3). Integration and transformation yields:

$$f_{n+1}(x, y, z, \theta, \phi) = \int_0^t Q(\tau, z, \theta) \int_0^{\pi} \int_0^{2\pi} \sigma(\cos\gamma, z - \tau\cos\theta)$$
$$f_n\Big(t - \tau,\ x - \tau\sin\theta\cos\phi,\ y - \tau\sin\theta\sin\phi,\ z - \tau\cos\theta,\ \theta',\ \phi'\Big) \sin\theta' d\theta' d\phi' d\tau, \quad (62)$$

where

$$Q(t, z, \theta) = \exp\left[-\int_0^t \sigma_t(z - \tau\cos\theta)\right] d\tau. \quad (63)$$

This expression defines the recurrence operator:

$$f_{n+1} = K_n f_n, \quad (64)$$

the form of which determines the properties of approximate solutions and convergence of the multiple scattering series (12). The basic equation for f_0 is integrated as:

$$f_0 = G(\vec{r}, \vec{v}) \exp\left[-\int_0^t \sigma_t(z') d\tau\right]. \quad (65)$$

The functional parameter G is found from initial conditions (see (5)), modeling a point pulse contact:

$$f_0(0, \vec{r}, \theta, \phi) = V\delta(x)\delta(y)\delta(z)\delta(\theta), \quad (66)$$

that results in:

$$f_0(t, \vec{r}, \theta, \phi) = V\delta(z - t)\delta(x)\delta(y)\delta(\theta)Q(t, z, \theta). \quad (67)$$

For the first iteration, f_1 one obtains from (62):

$$f_1 = V \int_0^t Q(\tau, z, \theta) E(t - \tau, z - \tau\cos\theta, 0)$$
$$\sigma(\cos\theta, z - \tau\cos\theta)\delta(x - \tau\cos\theta - (t - \tau))\delta(y - \tau\sin\theta\sin\phi)\delta(z - \tau\sin\theta\cos\phi) d\tau. \quad (68)$$

Similar expressions are obtained and interpreted in the case of N-fold scattering terms, the two-fold one is presented in the following sections.

5.2. One-fold scattering for a point receiver

As mentioned above, the integrand in (68) is considered as distributions on Schwartz space $\mathbf{S}$ of functions x, y, z which depend on ϕ, θ, τ as parameters. For example, f_1 acts on an element $\psi(\vec{r}) \in \mathbf{S}$ as:

$$\Big(f_1(t, \vec{r}, \theta, \phi),\ \psi\Big) = V \int_0^{2\pi} \int_0^t Q(\tau, \tau\cos\theta + t - \tau, \theta) E(t - \tau, t - \tau, 0)$$
$$\sigma(\cos\theta, \tau\cos\theta + t - \tau)\psi(\tau\sin\theta\cos\phi, \tau\sin\theta\sin\phi, \tau\cos\theta + t - \tau) d\tau d\phi, \quad (69)$$

where ψ describes a receiver. It is determined in Sec. (2.2) as $\psi(\vec{r}) = 1$ at:

$$x^2 + y^2 \le \rho_0^2, \quad z_0 \le z \le z_0 + \Delta t|\cos\theta|$$

and zero outside, being z_0 the boundary coordinate of the cylindrical contact.

From the definition (8) for the forward scattering and aperture angle θ_0 we have:

$$J_1(t) = \int_0^{\theta_0} (f_1, \psi) \sin\theta d\theta. \tag{70}$$

Therefore, we get for (68) when going to the intensity of a point receiver that we do as the limit:

$$I_{1p}(t,0,0,z_0) = -\lim_{\Delta t\to 0}\lim_{\rho_0\to 0} \int_{z_0}^{z_0+\cos\theta_0\Delta t}\int_0^{2\pi}\int_0^{\rho_0}\int_0^{\theta_0}\int_0^{t} Q(\tau,z_0,\theta)\int \sigma(\cos\theta\cos\theta', z-\tau\cos\theta)$$
$$f_0(t-\tau, x-\tau\sin\theta\cos\phi, y-\tau\sin\theta\sin\phi, z-\tau\cos\theta,\theta)\sin\theta' d\theta' d\tau d\theta d\rho d\phi dz, \tag{71}$$

or, plugging f_0, one has:

$$I_{1p}(t) = -\lim_{\Delta t\to 0}\lim_{\rho_0\to 0}\frac{1}{\Delta t}\int_0^{\rho_0}\int_0^{\theta_0}\int_0^{t}\int_{z_0}^{z_0+\cos\theta_0\Delta t} Q(\tau,z,\theta_0)Q(t-\tau, z-\tau\cos\theta_0,\theta_0)$$
$$\int \sigma(\cos\theta_0\cos\theta', z-\tau\cos\theta_0)\psi(\tau\sin\theta_0\cos\phi, \tau\sin\theta_0\sin\phi, \tau\cos\theta_0)\sin\theta' d\theta' d\tau d\phi d\rho dz. \tag{72}$$

The area of integration lies between the horizontal lines $z = z_0$, $z_0 + \Delta z$ and inclined lines $z = \cos\theta_0 + a$ and $z = \cos\theta_0 + b$, where a, b are boundaries of domain, filled with scatterers under consideration: e.g. a "cloud" of defects. The vertical line marks a current pulse arrival time $z = t$. In the case of a fixed angle θ_0, $z' = \tau(\cos\theta_0 - 1) + t$, $\tau = (\cos\theta_0 - 1)^{-1}(z' - t)$, hence the argument of the δ-function is:

$$z_0 + t - \tau - z' = z_0 + t - (\cos\theta_0 - 1)^{-1}(z'-t) - z' = z_0 - bt + az' = a\left(z' - \frac{b}{a}\right)t + \frac{z_0}{a},$$

where

$$a = -1 - (\cos\theta_0 - 1)^{-1} = \frac{\cos\theta_0}{1-\cos\theta_0}, \quad b = 1 + (\cos\theta_0 - 1)^{-1}.$$

The second argument of the scattering amplitude σ is therefore:

$$z_0 - \tau\cos\theta_0 = z_0 - (\cos\theta_0 - 1)^{-1}(z'-t)\cos\theta_0.$$

The result of 1-fold scattering for zero angle for the point receiver is almost trivial from a geometrical point of view, the arriving pulse is infinitely short. The expression for intensity contains natural spherical divergence, exponential decay due to absorption and forward scattering in a level inside the layer:

$$J_1^p(t) = \lim_{\Delta_i\to 0}\frac{1}{\prod\Delta_i}\int_{\theta_0}^{\pi}\int_0^{2\pi}(f_1,\psi)\sin\theta d\phi d\theta, \quad i = 1,2,3, \tag{73}$$

where the function ψ have nonzero components if:

$$|x| \le \Delta x = \Delta_1, \quad |y| \le \Delta y = \Delta_2, \quad |z| \le \Delta t|\cos\theta| = \Delta_3,$$

and 0 outside the domain. Plugging f_1 from (68) yields:

$$J_1^p(t) = \lim_{\Delta_i\to 0}\frac{1}{\prod\Delta_i}\int_{\theta_0}^{\pi}\int_0^{2\pi}\int_0^{t} E(\tau, \tau\cos\theta + t - \tau, \theta)E(t-\tau, t-\tau, 0)$$
$$\sigma(\cos\theta, \tau\cos\theta + t - \tau)\psi(\tau\sin\theta\cos\phi, \tau\sin\theta\sin\phi, \tau\cos\theta + t - \tau)d\tau\sin\theta d\phi d\theta. \tag{74}$$

5.3. Alternative variables of integration. One- and two-fold solutions for a point receiver

Let us change the variables of integration, having in mind more convenient (compared to [7]) description of the integration domain and limiting procedure:

$$\begin{aligned} x &= \tau\sin\theta\cos\phi, \\ y &= \tau\sin\theta\sin\phi, \\ x &= \tau\cos\theta + t - \tau. \end{aligned} \tag{75}$$

The inverse ones are found as:

$$\tau = \frac{x^2+y^2+z^2-2zt+t^2}{2(t-z)},$$
$$\cos\phi = \frac{x}{\sqrt{x^2+y^2}}, \tag{76}$$
$$\cos\theta = \frac{x^2+y^2+z^2-2zt+t^2-2(t-z)^2}{x^2+y^2+z^2-2tz+t^2)}.$$

We calculate the Jacobian:

$$J\sin\theta = \frac{2}{x^2+y^2+z^2-2zt+t^2}. \tag{77}$$

Let us define the integration intervals by means of:

$$0 \le \tau \le t, \quad 0 \le \phi \le 2\pi, \quad \pi-\theta_0 \le \theta \le \pi,$$
$$0 \le \tau\cos\theta + t - \tau \le \Delta t, \tag{78}$$
$$|\tau\sin\theta\cos\phi| \le \Delta x, \quad |\tau\sin\theta\sin\phi| \le \Delta y, \quad t-\tau+\tau\cos\theta \ge 0,$$

then $|x| \le \Delta x$, $|y| \le \Delta y$, $0 \le z \le \Delta t$.

1-fold again. The explicit expression for the integral J_1, corresponding to (74), in new variables takes the form:

$$J_1 = \lim \frac{1}{2\Delta x\Delta y\Delta z}\int\limits_{-\Delta x}^{\Delta x}\int\limits_{-\Delta y}^{\Delta y}\int\limits_{-\Delta t}^{0}\frac{Q'Q''\sigma}{x^2+y^2+z^2-2zt+t^2}dxdydz, \tag{79}$$

where $Q' = Q(\tau, z, \theta)$, $Q'' = Q(t-\tau, z-\tau\cos\theta, 0)$, is defined by (63). In arguments of Q', Q'', σ the variables τ, θ expressed in new ones (76).

Performing the limiting transition $\Delta x \to 0$, $\Delta y \to 0$, $\Delta z \to 0$, $\Delta t \to 0$ for a point receiver, we obtain the simple formula for back scattering:

$$I_1^p(t) = \frac{2}{t^2}E'\left(\frac{t}{2}, 0, \pi\right)E''\left(\frac{t}{2}, \frac{t}{2}, 0\right)\sigma\left(-1, \frac{t}{2}\right). \tag{80}$$

This reproduces the one-fold LIDAR formula for a small receiver in convenient form.

The dependence of $I_1(t)$ is shown for the homogeneous distribution of scatterers at Fig. 1 and for a layer of scatterers and for a layer of scatterers at Fig. 2.

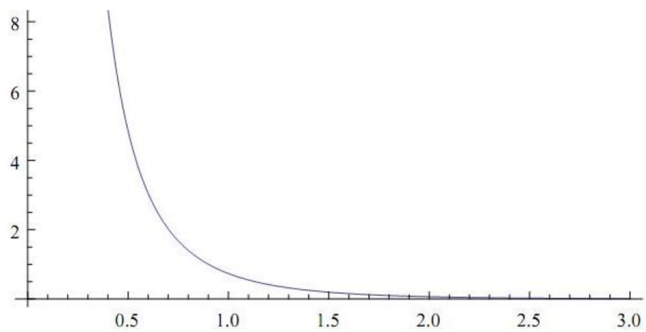

FIG. 1. Homogeneous scatterers distribution case $I_1(t)$

5.4. 2-fold case

From (62), similar to (68), when using the correspondingly modified transformations (75), (76), one arrives at the distribution term f_2:

$$f_2 = \int\limits_0^t Q(\tau_2, z, \theta)\int\limits_0^{2\pi}\int\limits_0^{\pi}\sigma(\cos\gamma, z-\tau_2\cos\theta)\int\limits_0^{t-\tau_2} Q1(\tau, z-\tau_2\cos\theta, \theta')Q(t-\tau_2-\tau, t-\tau-\tau_2, 0)\sigma(\cos\theta', t-\tau-\tau_2)$$
$$\delta(x-\tau_2\sin\theta\cos\phi-\tau\sin\theta'\cos\phi')\delta(y-\tau_2\sin\theta\sin\phi-\tau\sin\theta'\sin\phi')$$
$$\delta(z-\tau_2\cos\theta-\tau\cos\theta'-(t-\tau_2-\tau))d\tau\sin\theta' d\theta' d\phi' d\tau_2. \tag{81}$$

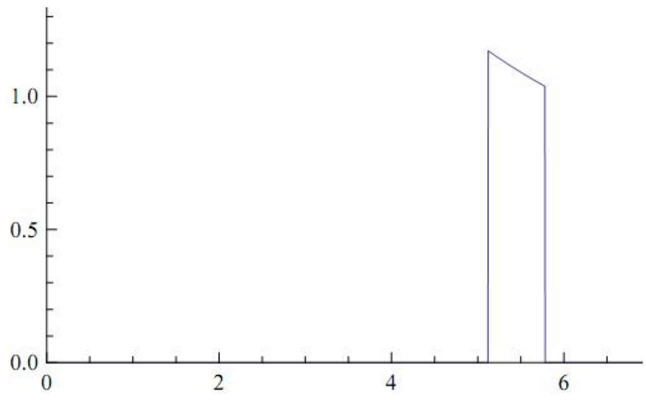

FIG. 2. $I_1(t)$, 1-fold scattering from a layer of scatterers

In new variables, (75) the integral J_2 takes the form:

$$J_2 = \lim_{\Delta V \to 0} \frac{1}{2\Delta x \Delta y \Delta z} \int\limits_{-\Delta x}^{\Delta x} \int\limits_{-\Delta y}^{\Delta y} \int\limits_{0}^{\Delta t} \int\limits_{0}^{t/2} \int\limits_{0}^{2\pi} \int\limits_{\pi-\theta_0}^{\pi} \frac{Q^* \sigma(\cos\gamma, \tau_1(\cos\theta_1 - 1) + t - \tau_2)\sigma(\cos\theta_1, t - \tau_1 - \tau_2)}{(\tau_2 \sin\theta_2 \cos\phi_2 - x)^2 + (\tau_2 \sin\theta_2 \sin\phi_2 - y)^2 + (t - (1 - \cos\theta_2)\tau_2 - z)^2} \sin\theta_2 d\theta_2 d\phi_2 d\tau_2 dz dx dy, \quad (82)$$

where:

$$Q^* = Q(\tau, z - \tau_2 \cos\theta, \theta')Q(t - \tau_2 - \tau, t - \tau - \tau_2, 0). \quad (83)$$

The corresponding intensity from 2-fold scattering for a layer of scatterers is shown at Fig. 3.

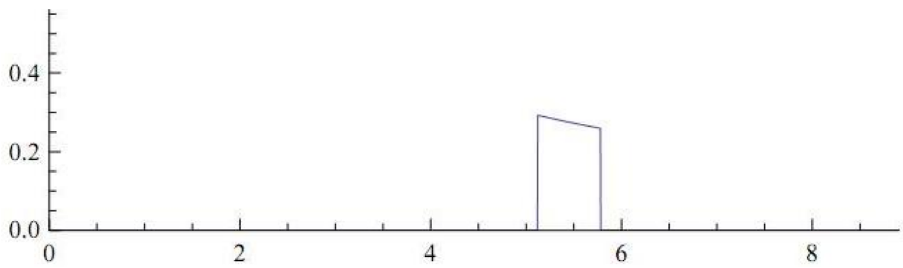

FIG. 3. $I_2(t)$, intensity: 2-fold scattering from a layered medium

6. Convergence theorem

6.1. Resulting estimation

Let the norm of σ be denoted as $\|\sigma\| = \max\limits_{\gamma,z} \sigma$:

$$J_N \leq 2^{3n-1}\|\sigma\|^n \exp[-\sigma_{\min} t]\pi^{n-1}t^{n-3}\frac{(n-1)^2}{((n-1)!)^{1/3}}\theta_0^{2/3}. \quad (84)$$

The series generated by r.h.s. of the inequality (84) converge because:

$$\lim_{n\to\infty} \sqrt[n]{2^{3n-1}\|\sigma\|^n \exp[-\sigma_{\min} t]\pi^{n-1}t^{n-3}\frac{(n-1)^2}{((n-1)!)^{1/3}}} \rightarrow 0. \quad (85)$$

(root test). Therefore, the radius of convergence is infinity.

On proof. Divide the integration domain D to subdomains D_1, D_2 so that for a positive ϵ_n, the following holds:

$$\left(t - \sum_2^n \tau_i(1-\cos\theta_i)\right)^2 + \left(\sum_2^n \tau_i \cos\theta_i \sin\phi_i\right)^2 + \left(\sum_2^n \tau_i \cos\theta_i \cos\phi_i\right)^2 \\ \leq \epsilon_n \sim D_1, \\ \geq \epsilon_n \sim D_2. \tag{86}$$

This determines the choice of ϵ_n.

6.2. Error estimation

If $8\pi\|\sigma\|t < 1$, the following estimate for error is the following:

$$\left| I(t) - \sum_1^n I_k(t) \right| \leq 4\|\sigma\| e^{-\sigma_{\min} t} \theta_0^{2/3} t^2 \sum_{k=n}^{\infty} (8\pi\|\sigma\|t)^{k-1} = 4t^2\theta_0^{2/3}\|\sigma\| \frac{e^{-t\sigma_{\min}}}{1-8\pi\|\sigma\|t}. \tag{87}$$

Having such an estimation, one can decide what number of N-fold contributions should be taken into account for a given error.

7. Conclusion

We do understand that the approximations for the differential cross-sections as (47) are too rough. We have chosen such models to move the theory in explicit form as far as possible. Modifications of the formulas should improve the description and may be compared with the presented results. The convergence theorem is certainly generalized for nonzero field.

In transport problems, the electron *Bloch wave* scatters either on phonons or on a solid crystal lattice inhomogeneities (point defects or dislocations). Such contribution may be incorporated into the suggested scheme. One of the simplest, to say a "textbook" version of the model, accomplished in a sense of possibility to plot the dependences of the conductivity on temperature and wire radius, see our contribution with Botman, see also [9].

Account of electron-electron scattering implies transition to Boltzmann equation. The quantum corrections also use its generalizations [10].

Acknowledgment

The kinetic method based on the distributions application was inspired by fruitful discussions with B. S. Pavlov and I. Y. Popov [11].

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