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Parameterization of an interaction operator of optical modes in a single-mode optical fiber

G. P. Miroshnichenko

ITMO University, St. Petersburg, 197101, Russia

gpmirosh@gmail.com

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The phenomenological parameters of the Hamiltonian for the photons produced in earlier studies [4] are associated with the parameters of the deformed optical fiber (OF). This Hamiltonian is necessary for the correct description of the propagation of photons through the quantum channel in a quantum communication protocols. Models of a compressing strain of the OF profile and a twisting deformation are considered. As a consequence, the phenomenological parameters of the Hamiltonian expressed in terms of such strains characteristics, as a relative compression of the profile, OF radius, the orientation angle of the deformed profile, rotation angle per unit length, elasto-optical tensor, and refraction coefficient.

Keywords: Hamiltonian of photons, deformed optical fiber, quantum channel, phenomenological parameters of the Hamiltonian.

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1. Introduction

For an electromagnetic wave, an optical fiber (OF) represents a non-uniform stochastic anisotropic environment. The principles of radiation quantization in the non-uniform anisotropic environment are stated, for example, in works [1–3]. The Hamiltonian for the photons in a single mode OF was obtained in [4], taking into account the small, isotropic, inhomogeneous, smoothly varying (along the OF) addition to the tensor of the dielectric permittivity. Photons are defined on the basis of the normal modes of an ideal single-mode OF. In [4], the problem of mode interactions in an OF was resolved using a properly chosen gauge. The transversality of the electric displacement vector is provided. The Hamiltonian conserves the number of photons. We used the approach of smooth perturbations (by analogy with the classical method of coupled waves). In this approach, researchers neglect the interaction of oppositely directed waves. The permittivity tensor was parameterized by three phenomenological parameters that determine the interaction of orthogonally polarized modes of the OF.

The classical theory of information transmission in an OF is well developed. Of particular interest is the single mode OF, in which two perpendicular polarized waves can propagate, phase front which is close to the plane (HE_{11} mode, [5, 6]). Currently, a single mode OF with low absorption (in windows of transparency) has been developed. They find broad application in optical communication systems over long distances with high speed information transfer [7]. The distortion features have been studied in detail for classical information in a single mode OF. In [8], the effect of optical activity in a twisted single mode OF was considered. The relationship of the single-mode OF curvature with the effect of the birefringence is discussed in [9]. The effect of tension on the occurrence of anisotropy in the one mode OF was studied in [10].

The new direction of modern informatics (the science of methods of communication, storage and processing of information) involve optical quantum information technologies. Quantum communication protocols – quantum cryptography, quantum teleportation, dense coding [11, 12] operate on the principles of quantum optics. The coding of information on a photon's polarization and degrees of freedom were used in [13]. A scheme using phase encoding was presented in [14], while in-phase light modulation was utilized in [15]. Additionally, a quantum key distribution protocol, using entangled photon polarization states, was proposed in [16]. In [17] the generation of entangled biphoton states with orbital-angular-momentum in triangular quadratic waveguide arrays with twisted geometry was considered.

In the following works, the phenomenological parameters of the photon Hamiltonian obtained in [4], are associated with parameters of the OF strain. The phenomenological parameters of the Hamiltonian were determined by the inverse permittivity tensor averaged over the volume of the OF segment. The dielectric permittivity of a deformed OF depends on the deformation parameters. Models for a compressing strain of the OF profile and a twisting deformation are considered. As a consequence, the phenomenological parameters of the Hamiltonian are expressed in terms of such characteristics, as a relative compression of the profile, OF radius, the orientation angle of the deformed profile, rotation angle per unit length, elasto-optical tensor, and refraction coefficient.

2. The Hamiltonian of photons in a single mode OF

The Hamiltonians of photons in a single mode OF, expressed in terms of the operators $\widehat{b}^{\dagger}_{\beta\mu}$, $\widehat{b}_{\beta\mu}$, according to [4], has the form:

$$\widehat{H} = \sum_{\beta} \omega\left(\beta\right) \left(\sum_{\mu=H,V} \left(\widehat{b}^{\dagger}_{\beta\mu} \widehat{b}_{\beta\mu} + \frac{1}{2} \right) + \frac{1}{2} \sum_{\mu,\mu'=H,V} \widehat{b}^{\dagger}_{\beta\mu} \cdot \Delta\Xi^{(\beta)}_{\mu,\mu'} \cdot \widehat{b}_{\beta\mu'} \right). \tag{1}$$

The formula is written in the approximation of a smooth dependence of the tensor $\varepsilon\left(\mathbf{r}\right) = \varepsilon_0\left(\mathbf{r}\right) + \Delta\varepsilon\left(\mathbf{r}\right)$ on the longitudinal coordinate z. In this approximation, the Hamiltonian $\widehat{H}$ becomes single-mode, modes with different wave vectors do not interact. The matrix $\Delta\Xi^{(\beta)}$ has the form:

$$\Delta\Xi^{(\beta)}_{\mu,\mu'} = -\int dz \iint dxdy \left(\Delta\varepsilon\left(\mathbf{r}\right) \boldsymbol{\alpha}_{\beta,\mu}\left(x,y\right), \boldsymbol{\alpha}^{*}_{\beta,\mu'}\left(x,y\right) \right), \tag{2}$$

where $\Delta\varepsilon\left(\mathbf{r}\right)$ – a random correction to the dielectric permittivity $\varepsilon_0\left(\mathbf{r}\right)$ of the ideal OF. The emission quantized on the basis of modes for an ideal OF satisfy the following equation, boundary conditions and transversality conditions:

$$\begin{cases} \omega\left(\beta\right)^2 \varepsilon_0\left(\mathbf{r}\right) \boldsymbol{A}_{\beta,\mu}\left(\mathbf{r}\right) - c^2 \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{A}_{\beta,\mu}\left(\mathbf{r}\right) = 0, \\ \boldsymbol{\nabla}\cdot \left(\varepsilon_0\left(\mathbf{r}\right) \boldsymbol{A}_{\beta,\mu}\left(\mathbf{r}\right) \right) = 0, \end{cases} \tag{3}$$

where $\omega\left(\beta\right)$ – eigenvalues, β – longitudinal wave vector, $\boldsymbol{\nabla}\cdot$ – the divergence operation, $\boldsymbol{\nabla}\times$ – the rotor operation, $\boldsymbol{A}_{\beta,\mu}\left(\mathbf{r}\right) = \boldsymbol{\alpha}_{\beta,\mu}\left(x,y\right) \cdot \exp\left(-i\beta z\right)$ – basis of spatial modes, $\mu = H, V$ – polarization index. The property of the basis orthogonality is as follows:

$$\int \varepsilon_0\left(\mathbf{r}\right) \left(\boldsymbol{A}_{\beta,\mu}\left(\boldsymbol{r}\right), \boldsymbol{A}^{*}_{\beta,\mu'}\left(\boldsymbol{r}\right) \right) dV = \delta_{\mu,\mu'}.$$

We use a phenomenological description and parameterized of Hermitian matrix $\Delta\Xi^{(\beta)}$ (2):

$$\Delta\Xi^{(\beta)} = \begin{pmatrix} \varepsilon & \Gamma \cdot \exp(-i\delta) \\ \Gamma \cdot \exp(i\delta) & -\varepsilon \end{pmatrix}, \tag{4}$$

by the real parameters ε, Γ, δ. The anisotropy and gyrotropy of the OF which arises from the random correction $\Delta\varepsilon(\mathbf{r})$ are thought to be small. That is, the parameters ε, Γ satisfy the following relation:

$$\Omega = \sqrt{\varepsilon^2 + \Gamma^2} \ll 1. \tag{5}$$

In a typical OF, the parameter $\Omega \approx 10^{-5} \div 10^{-7}$. Parameter $\Gamma \sin\delta$ is typically referred to as the component optical activity vector (along the OF axis) [18]. If the axis of the laboratory coordinate system does not coincide with the axes of the OF, the photons in the laboratory frame of reference, determined by the operators $\widehat{b}^{\dagger}_{\beta\mu}$, $\widehat{b}_{\beta\mu}$, $\mu = H, V$ interact.

3. Guided modes of weakly guiding fiber

We consider the case of the so-called weakly guiding cylindrical OF. Fiber parameters satisfy the inequality:

$$\frac{n_c^2 - n_{cl}^2}{2n_c^2} \ll 1, \tag{6}$$

where n_c, n_{cl} – the refractive indices of the core and cladding of OF. We will choose the axis z of the laboratory coordinate system along the axis of the OF segment. In an ideal OF, the dielectric permittivity $\varepsilon_0(\mathbf{r})$ depends on the transverse coordinates – x, y. Analysis of the solutions for equation (3) shows [19] that there are OF waveguide (guided) modes with a discrete spectrum of frequencies and not guided modes having a continuous spectrum. Under the condition (6), often implemented in practice, the structure of the guided modes is simplified. Of special interest for this practice is the so-called single-mode OF, in which one doubly-degenerate mode HE_{11} can be propagated. The refraction coefficient of the OF has the form of a step depicted in Fig. 1. We obtain the solution (3) for the mode HE_{11}. The first vector $\boldsymbol{A}_H$ is horizontal $q = H$:

$$\begin{gathered} \boldsymbol{A}_H = \boldsymbol{\alpha}_H(x,y) \cdot \exp(-i\beta z), \\ \boldsymbol{\alpha}_H(x,y) = \boldsymbol{\alpha}_H(x,y)_y \mathbf{j} + \boldsymbol{\alpha}_H(x,y)_z \mathbf{k}. \end{gathered}$$

The solution in the two regions (region 1: $r \le a$, region 2: $r > a$) has the form:

$$\begin{aligned} &\begin{cases} \boldsymbol{\alpha}_H^{(1)}(r,\phi)_x = 0, \\ \boldsymbol{\alpha}_{\mathrm{H}}^{(1)}(r,\phi)_y = A \cdot \dfrac{J_0(ur/a)}{J_0(u)}, & r \le a \\ \boldsymbol{\alpha}_H^{(1)}(r,\phi)_z = A \cdot \dfrac{iu}{a\beta}\dfrac{J_1(ur/a)}{J_0(u)} \sin\phi, \end{cases} \\ &\begin{cases} \boldsymbol{\alpha}_H^{(2)}(r,\phi)_x = 0, \\ \boldsymbol{\alpha}_H^{(2)}(r,\phi)_y = A \cdot \dfrac{K_0(wr/a)}{K_0(w)}, & r > a \\ \boldsymbol{\alpha}_H^{(2)}(r,\phi)_z = A \cdot \dfrac{iw}{a\beta}\dfrac{K_1(wr/a)}{K_0(w)} \sin\phi. \end{cases} \end{aligned} \tag{7}$$

The second vector $\boldsymbol{A}_V$ is vertical $q = V$:

$$\boldsymbol{A}_V = \boldsymbol{\alpha}_V(x,y)\cdot\exp(-i\beta z),$$
$$\boldsymbol{\alpha}_V(x,y) = \boldsymbol{\alpha}_V(x,y)_x\,\mathbf{i} + \boldsymbol{\alpha}_V(x,y)_z\,\mathbf{k}.$$

The solution in the two regions (region 1: $r \le a$, region 2: $r > a$) has the form:

$$\begin{cases} \boldsymbol{\alpha}_V{}^{(1)}(r,\phi)_x = A\cdot\dfrac{J_0(ur/a)}{J_0(u)}, & \\ \boldsymbol{\alpha}_V^{(1)}(r\phi)_y = 0, & r \le a \\ \boldsymbol{\alpha}_V^{(1)}(r\phi)_z = A\cdot\dfrac{iu}{a\beta}\dfrac{J_1(ur/a)}{J_0(u)}\cos\phi, & \end{cases}$$
$$\begin{cases} \boldsymbol{\alpha}_V^{(2)}(r,\phi)_x = A\cdot\dfrac{K_0(wr/a)}{K_0(w)}, & \\ \boldsymbol{\alpha}_V^{(2)}(r,\phi)_y = 0, & r > a \\ \boldsymbol{\alpha}_V^{(2)}(r,\phi)_z = A\cdot\dfrac{iw}{a\beta}\dfrac{K_1(wr/a)}{K_0(w)}\cos\phi. & \end{cases} \tag{8}$$

The dispersion relation $\omega = \omega(\beta)$ for the mode HE_{11}, from the transversality conditions (3) has the form:

$$u\frac{J_1(u)}{J_0(u)} = w\frac{K_1(w)}{K_0(w)},$$

$$u = a\cdot\chi, \qquad w = a\cdot\gamma,$$
$$\chi^2 = \left(\frac{\omega}{c}n_c\right)^2 - \beta^2,$$
$$\gamma^2 = \beta^2 - \left(\frac{\omega}{c}n_{cl}\right)^2.$$

Here A is the the normalization factor. The normalization A has the form:

$$A = \frac{w^2}{\sqrt{\pi L}a\left(n_c w^2 + n_{cl}u^2\right)}\left(\frac{J_0(u)}{J_1(u)}\right)^2.$$

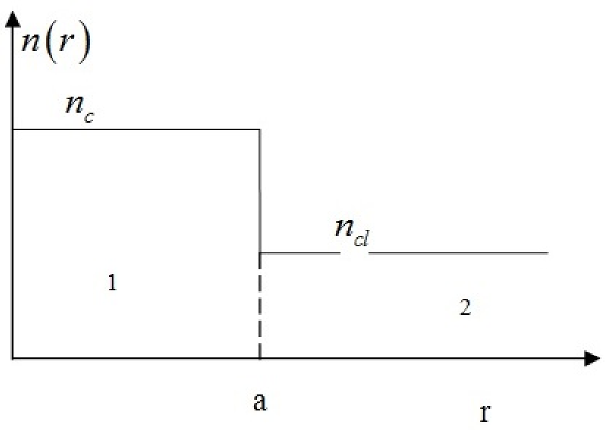

FIG. 1. The dependence of the refractive index $n(r)$ on the radial coordinate r in the cylindrical OF. n_c, n_{cl} – the refractive indices of the core and cladding of OF. 1, 2 – the core and cladding regions. a – the core radius

The dependence of frequency (for HE_{11} mode) on a dimensionless longitudinal wave vector $\beta \cdot a$ is shown in Fig. 2.

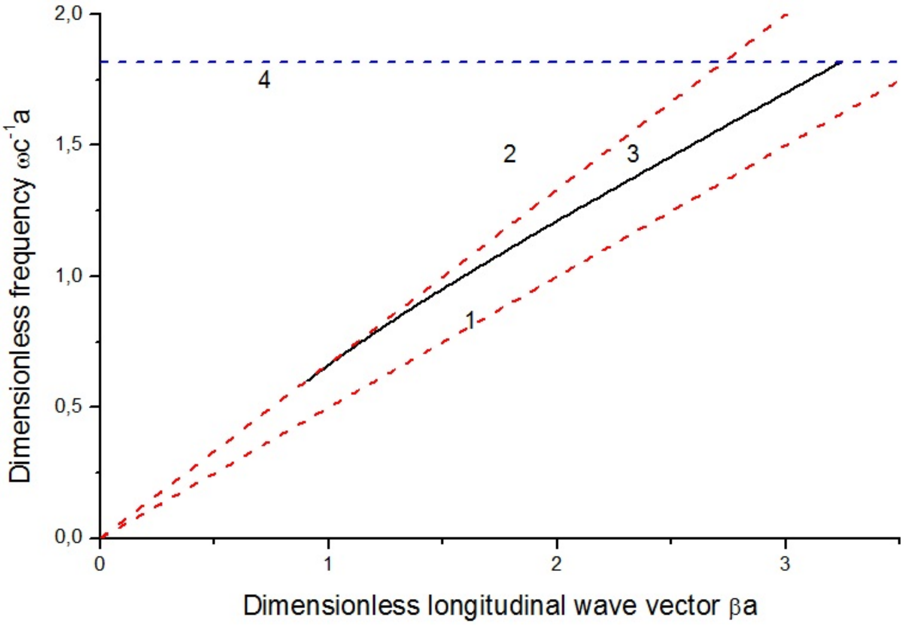

FIG. 2. The dispersion relation $\omega = \omega(\beta)$ for the mode HE_{11}, Plot 3. Plot 1 – the dispersion relation of the core $\omega = \beta a/n_c$. Plot 2 – the dispersion relation of the cladding $\omega = \beta a/n_{cl}$. Plot 4 – limit value of frequency $\omega a/c = 2.405/\sqrt{n_c^2 - n_{cl}^2}$. The wave vector multiplied by the radius of the core – βa – delayed along the horizontal axis. Value $\omega a/c$ is plotted along the vertical axis

4. Nonideal optical fiber

In the previous section, we obtained the basis of modes and dispersion relation for an ideal OF. In order to completely define a Hamiltonian for photons (1), it is necessary to connect phenomenological parameters ε, Γ, δ with the parameters of the OF. For this purpose we consider two models describing the dependence of the dielectric permittivity of the OF on a profile distortion and twisting of the OF [8–10]. The perturbation of the dielectric permittivity due to the deformation form of the OF is a scalar, and according to [8–10], has the form:

$$\Delta\varepsilon(r,\varphi) = -\eta \cdot r \cdot \varepsilon_0(r) \cos 2(\varphi - \varphi_B). \tag{9}$$

This perturbation connects z components (longitudinal components) of the two wave modes A_H and A_V. The graph of the OF cross section distorted by compression is submitted in Fig. 3. The oval cross section has the main axes turned in relation to laboratory system of coordinates on a angle φ_B. Calculation of a matrix $\Delta\Xi^{(\beta)}$ with use formulas (2), (7), (8), (9) yields the following result:

$$\Delta\Xi^{(\beta)} = \frac{\Delta(\beta)\eta a}{2}\begin{pmatrix} -\cos(2\varphi_B) & \sin(2\varphi_B) \\ \sin(2\varphi_B) & \cos(2\varphi_B) \end{pmatrix}.$$

Let's compare this matrix to matrix (4). We get a connection the phenomenological parameters ε, Γ, δ with the deformation parameters of the OF:

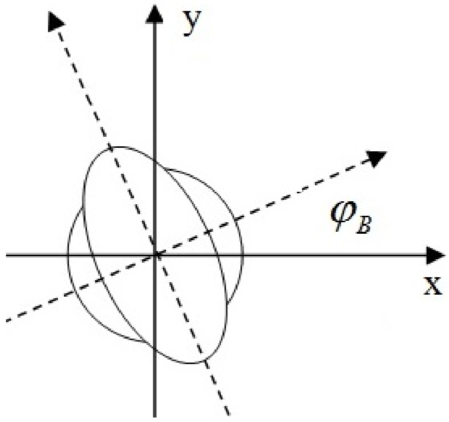

FIG. 3. Distortion of a OF core profile. η – relative compression of a profile $\varepsilon_0(r)$ at an angle φ_B

$$\begin{aligned}
&\delta = 0,\\
&\varepsilon = -\frac{\Delta(\beta)\,\eta a}{2}\cos(2\varphi_B),\\
&\Gamma = \frac{\Delta(\beta)\,\eta a}{2}\sin(2\varphi_B).
\end{aligned}$$

There, $\Delta(\beta)$ is the normalization function:

$$\Delta(\beta) = \frac{1}{4\pi a^2\beta^2}\frac{n_c u^2 J_0^{-2}(u)\int\limits_0^1 r^2 dr J_1^2(ur) + n_{cl} w^2 K_0^{-2}(w)\int\limits_1^\infty r^2 dr K_1^2(wr)}{n_c J_0^{-2}(u)\int\limits_0^1 r dr J_0^2(ur) + n_{cl} K_0^{-2}(w)\int\limits_1^\infty r dr K_0^2(wr)}. \tag{10}$$

The graph of normalized function (10) is shown in Fig. 4. Distortion of the OF's permeability tensor with twist deformation is described by formulas [8–10]:

$$\begin{aligned}
&\Delta\varepsilon(r,\varphi)_{yz} = \Delta\varepsilon(r,\varphi)_{zy} = -p_{44}n_0^4\tau\cdot x,\\
&\Delta\varepsilon(r,\varphi)_{xz} = \Delta\varepsilon(r,\varphi)_{zx} = p_{44}n_0^4\tau\cdot y.
\end{aligned} \tag{11}$$

where, x, y – transverse coordinates in the laboratory frame, τ – the angle of twist per unit length, n_0 – the average refractive index of the OF material, $p_{n,m}$ – elasto optical tensor. The scheme for twist deformation is shown in Fig. 5. Calculation of a matrix $\Delta\Xi^{(\beta)}$ with use formulas (2), (7), (8), (9) yields the following result:

$$\Delta\Xi^{(\beta)} = a\tau p_{44}n_0^4 S(\beta)\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Let's compare this matrix to matrix (4). We obtain a connection between the phenomenological parameters ε, Γ, δ with the deformation parameters of the OF:

$$\begin{aligned}
&\delta = \frac{\pi}{2},\\
&\varepsilon = 0,\\
&\Gamma = a\tau p_{44}n_0^4 S(\beta).
\end{aligned}$$

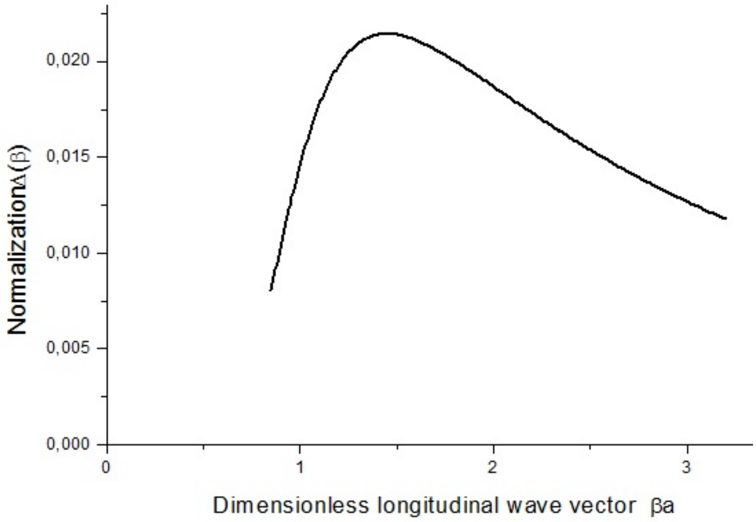

FIG. 4. The normalization function $\Delta(\beta)$, calculation by formula (10) for $n_c = 2$, $n_{cl} = 1.5$

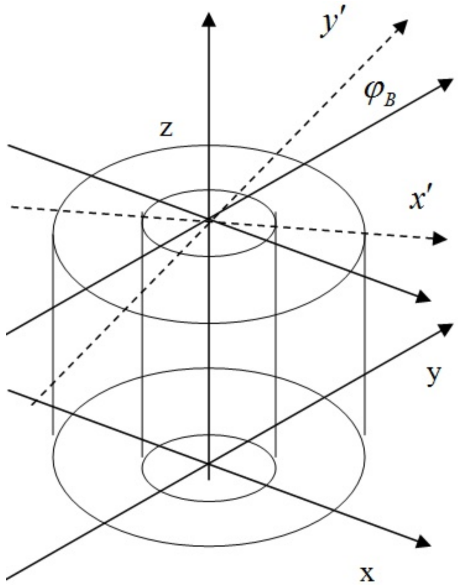

FIG. 5. The twist deformation of OF. There τ – angle of twist per unit length z. $n_0 = 1.46$ – the average refractive index of the OF material. $p_{44} = -0.075$ – elasto optical tensor, uniform along the fiber. The remaining coefficients of the permittivity tensor are zero. x, y – transverse coordinates of OF (laboratory frame), x', y' – transverse coordinates of OF (local frame), $\varphi_B = \tau z$

Here, the normalizing function $S(\beta)$ is defined in the following manner:

$$S(\beta) = \frac{1}{a\beta} \frac{n_c u J_0^{-2}(u) \int\limits_0^1 r^2 dr J_1(ur) J_0(ur) + n_{cl} w K_0^{-2}(w) \int\limits_1^\infty r^2 dr K_1(wr) K_0(wr)}{n_c J_0^{-2}(u) \int\limits_0^1 r dr J_0^2(ur) + n_{cl} K_0^{-2}(w) \int\limits_1^\infty r dr K_0^2(wr)}. \quad (12)$$

The graph of normalization function is shown in Fig. 6.

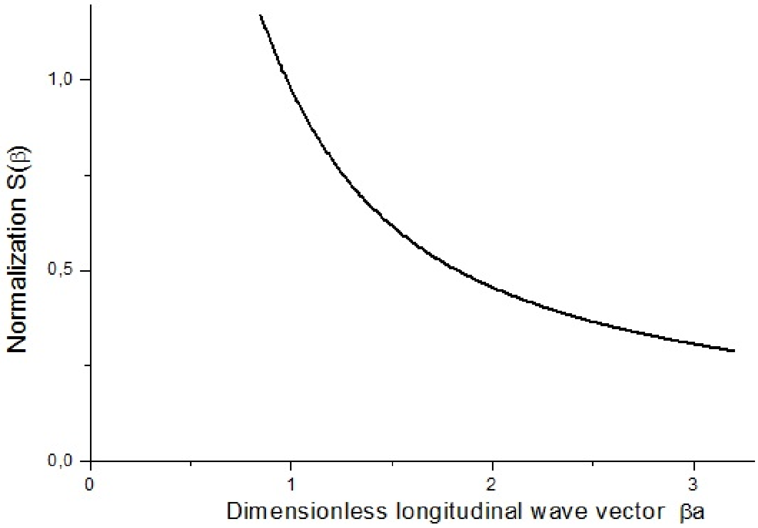

FIG. 6. The normalization function $S(\beta)$, calculation by formula (12) for $n_c = 2$, $n_{cl} = 1.5$

5. Conclusion

Quantum information, distributed by the OF, is encoded in the quantum states of photons. The transformation of information during the propagation of the photons through the OF is described by the Liouville equation for the density matrix of the photons. The Hamiltonian for the OF-based photons is required to write a quantum Liouville equation. The Hamiltonian of photons in the OF (1) contains phenomenological parameters ε, Γ, δ. These parameters depend on the longitudinal coordinate of the wave vector, β , and thus, determine the polarization mode for the dispersion of photons in the OF. To obtain the relationship between the phenomenological parameters ε, Γ, δ and β, we must use the distortion model for the optical and geometric characteristics of the OF against external influences. For this purpose, the model transverse compression and twist of the OF [8–10] were used in this work and the relationship between parameters ε, Γ, δ and β were determined in this work.

Acknowledgments

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